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9/7/17

I ☆ SIMPLE STRESSES & STRAINS

Stress: when an external force acts on a body, the body tends to under go some deformation. Due to cohesion between the molecules, the body resists the deformation.

* This resistance by which material of the body opposes the deformation is known as "strength of material."

//. Within the certain limit (elastic limit) the resistance offered by the material is proportional to the deformation brought out on the material by the external force. Also within this limit resistance is equal to the external force but beyond the elastic stage, the resistance offered by the material is less than the applied load, in such cases the deformation continues, until the failure occur.

→ within elastic stage, the resisting force = external load

and this resisting force per unit area = stress
= intensity of stress.

$$\therefore \boxed{\text{stress} = \frac{\text{Applied load}}{\text{Area of c/s}}}$$

UNIT : N/m^2, N/mm^2,

$$\boxed{1\ N/m^2 = 1\ \text{pascal} = 10^{-6}\ N/mm^2}$$

STRAIN :→ The ratio of change of dimension of the body to the original dimension is known as strain.

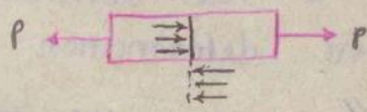
(1) Tensile strain:- $\dfrac{\text{increase in length}}{\text{original length}} = e$

(2) compressive strain:- $\dfrac{\text{Decrease in length}}{\text{original length}}$

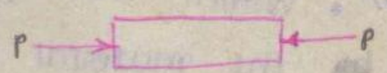
(3) volumetric strain :- $= \dfrac{\text{change in volume}}{\text{original volume}}$

(4) shear strain :- strain due to shear stress $= \dfrac{\text{Transverse displacement}}{\text{original disp.}}$

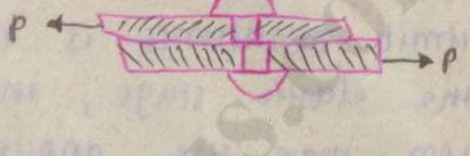
(1) Tensile stress :- $\dfrac{\text{Tensile load (P) (pull)}}{\text{Area of C/S}}$



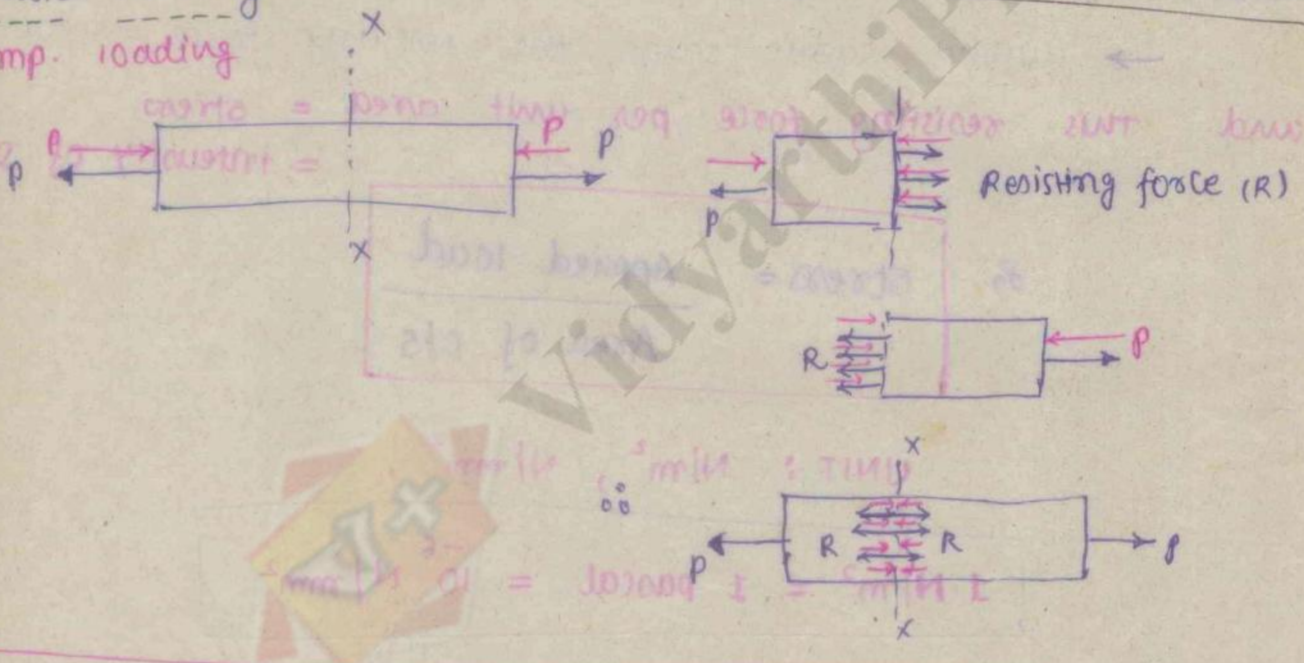
(2) Comp. stress :- $\dfrac{\text{comp load (P) (push)}}{\text{Area of C/S}}$



(3) shear stress :- $\dfrac{\text{shear load (P)}}{\text{Area of C/S.}}$



Tensile loading ----
comp. loading ------



factor of safety $= \dfrac{\text{Ultimate stress}}{\text{safe stress.}}$

Elastic limit

- The property by virtue of which certain materials return back to their original position (shape) after the removal of the force is called elasticity.
  - And this is only possible beyond (upto) elastic limit
  - if the external force exceeds the elastic limit the material loses its elasticity and permanent deformation takes place.

→ safe stress = stress within elastic limit

Hooke's law

"if material is loaded within elastic limit, the stress is proportional to the strain produced by the stress."

Modulus of elasticity (YOUNG'S MODULUS) (E)

$$\boxed{E = \frac{\text{stress}}{\text{strain}}} \quad N/m^2$$

← for tensile & comp. loading

Modulus of rigidity (SHEAR MODULUS)

$$\boxed{C \text{ (or } G \text{ or } N) = \frac{\text{shear stress}}{\text{shear strain}}}$$

* stress at elastic limit = $\dfrac{\text{load at elastic limit}}{\text{Area of c/s.}}$

* percentage elongation = $\dfrac{\text{Total increase in length}}{\text{original (gauge) length}} \times 100$

  = ___ %

* percentage decrease in Area = $\dfrac{\text{original Area} - \text{Area at failure}}{\text{original area}} \times 100$

Analysis of Bar of Varying sections

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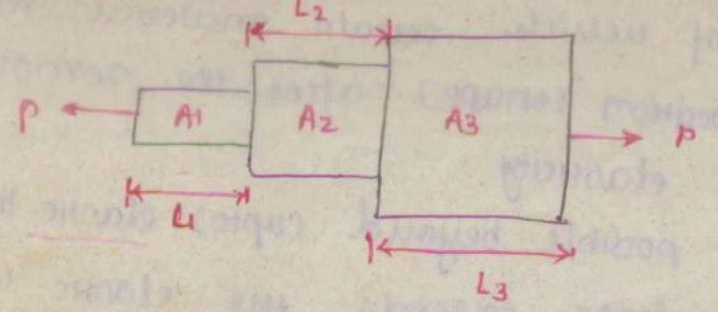


- $e_1 = \frac{dL_1}{L_1}$

$\frac{\text{stress}}{E} = \frac{dL_1}{L_1}$

$\frac{P}{A_1 \cdot E} = \frac{dL_1}{L_2}$

$\therefore \; dL_1 = \frac{PL_1}{A_1 \cdot E}$

& $dL_2 = \frac{PL_2}{A_2 E}$, $dL_3 = \frac{PL_3}{A_3 E}$

$\therefore$ Total change in length

$$\boxed{= \frac{P}{E}\left(\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right)}$$

when E is same (because of same material)

when E is different

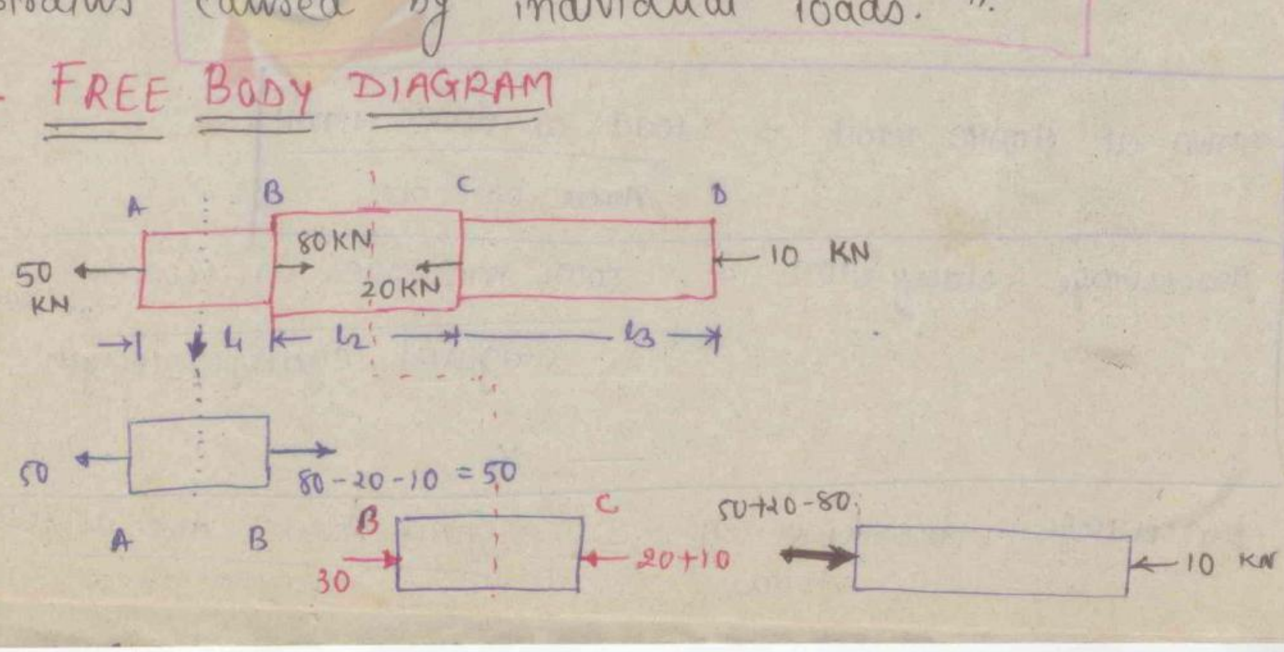
Total change in length

$$\boxed{= P\left(\frac{L_1}{A_1E_1} + \frac{L_2}{A_2E_2} + \frac{L_3}{A_3E_3}\right)}$$

PRINCIPLE OF SUPER POSITION

" When a number of loads are acting on a body the resulting strain will be the algebraic sum of strains caused by individual loads. "

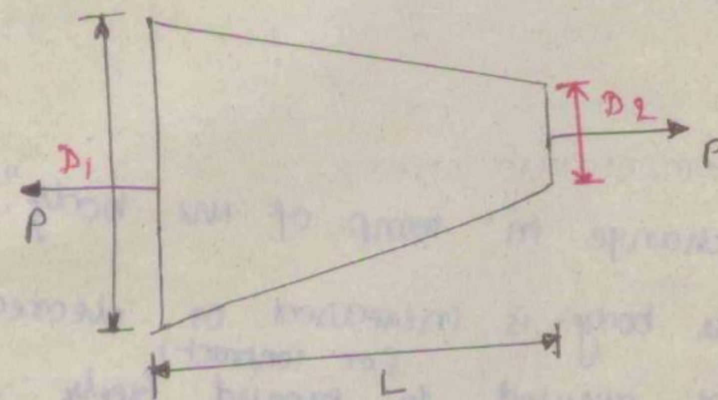
FREE BODY DIAGRAM



☆ Total change in length = Total Extension due to tensile load − Total decrease in length due to compressive load.

$$= \delta_{AB} - \delta_{BC} - \delta_{CD}$$

Analysis of Uniformly tapered circular rod



Total Extension of tapered rod

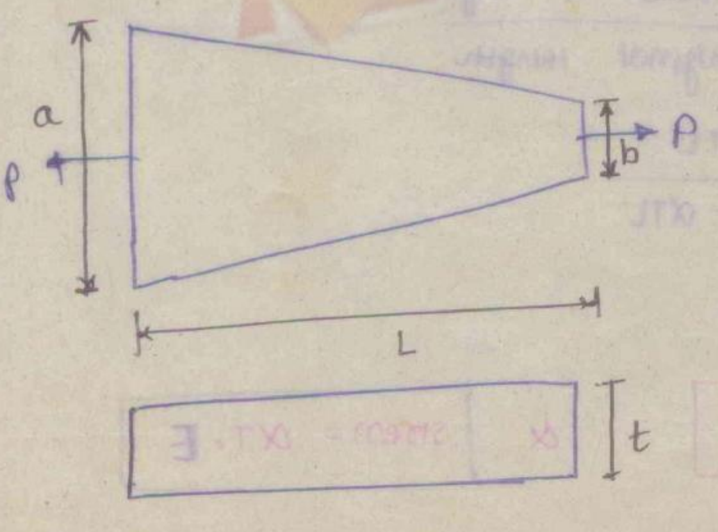
$$= \frac{4PL}{\pi E D^2}$$ if rod is uniform

$$\frac{PL}{AE}$$

$$A = \frac{\pi}{4} D^2$$

$$\boxed{\text{Total Extension} = \frac{4PL}{\pi E D_1 D_2}}$$

Analysis of uniformly tapered rectangular bar

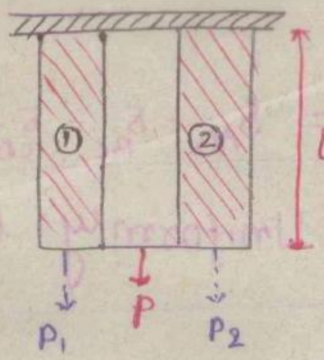


Total Extension

$$= \frac{PL}{E \cdot t\,(a-b)} \ln \frac{a}{b}$$

Composite Sections

- strain in each bar is equal
- $P = P_1 + P_2$ • length is equal



$\frac{E_1}{E_2}$ = modular ratio.

$\frac{P}{E}$ = strain = constant

THERMAL STRESSES (temperature stress)

stresses induced due to change in temp. of the body".

• when the temp. of the body is increased or decreased, and the body is not allowed to expand (or contract) freely, stress will be set up and if body is allowed to expand or contract freely, no stresses will be set up in the body.

→ if the rod is free to expand, then the extension of the rod

$$\boxed{dL = \alpha \cdot T \cdot L}$$

& comp. strain = $\frac{\text{Decrease in length}}{\text{original length}}$

$$= \frac{\alpha TL}{L - \alpha TL}$$

$$\simeq \frac{\alpha TL}{L}$$

$\boxed{\text{comp. strain} \simeq \alpha T}$ & $\boxed{\text{stress} = \alpha T \cdot E}$

∴ Thermal strain = αT ← rise in temp

Thermal stress = $\alpha T E$, α = coeff. of linear expansion.

→ **Stress and strain when the support yeild**

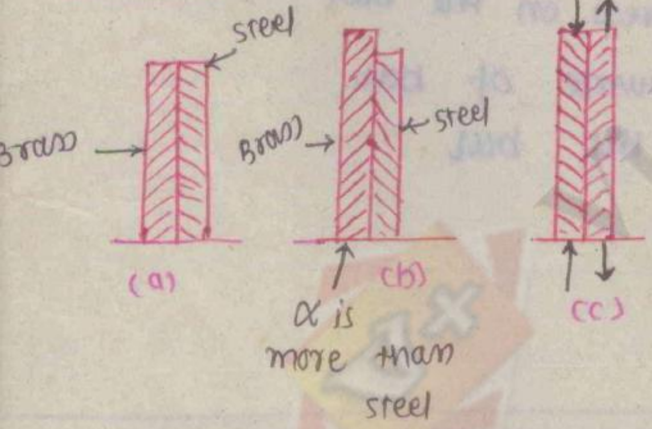
Actual expansion = Expansion due to rise in temp − amount of yeild (δ)

$$= \alpha T . L - \delta$$

$$\text{Actual } \boxed{\text{strain} = \frac{\alpha . T . L - \delta}{L}}$$

$$\& \ \boxed{\text{stress} = \text{strain} \times E}$$

Thermal Stresses in composite Bars.



As both the bars are not free to expand as shown in fig (c), steel bar & brass bars will be subjected to tensile & compressive load respectively

☆ for equilibrium, Tension in steel bar = compression in brass

∴ load on brass = load on steel

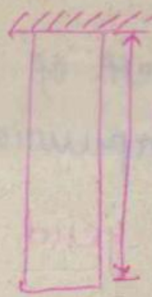
$$\therefore \boxed{p_b \times A_b = p_s \times A_s}$$

$$\bigstar \ \boxed{\alpha_s T + \frac{p_s}{E_s} = \alpha_b T - \frac{p_b}{E_b}}$$

more (lower α) expansion

less expansion (Higher α)

Elongation of bar due to self weight :-

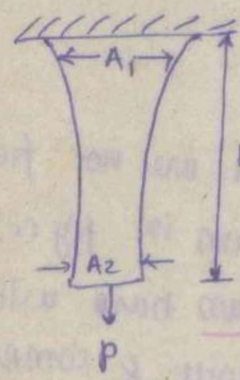


$$\boxed{\text{Total elongation} = \frac{W \cdot L}{2E}}$$

Analysis of bar of uniform strength

- The stress due to self weight of bar is not constant but the stress increases with the increase of distance from the lower end.

— if the self weight is neglected and a bar of uniform section is subjected to an axial load, then the stress in the bar would be uniform



A_1 = Area of upper end

A_2 = Area of lower end

p = uniform stress on the bar

w = weight / volume of bar.

L = length of the bar

$$\boxed{A_1 = A_2 \, e^{\frac{wL}{p}}}$$

- In case of nut and bolt used on the tube with washers, the tensile load on the bolt is equal to the compressive load on the tube.

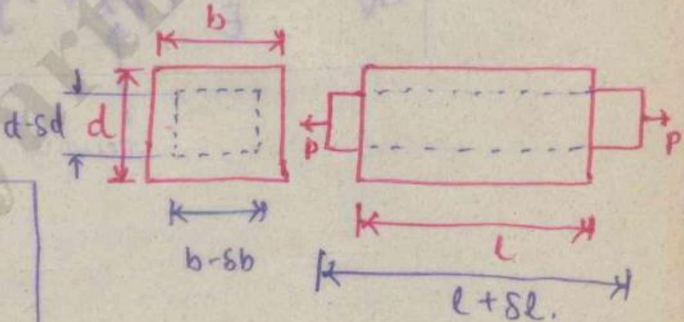
Material	coefficient of thermal expansion
Steel	$\longrightarrow 12 \times 10^{-6}$ °C
Copper	$\longrightarrow 17.5 \times 10^{-6}$ °C
Stainless steel	$\longrightarrow 18 \times 10^{-6}$ °C
Brass, bronze	$\longrightarrow 19 \times 10^{-6}$ °C
Aluminium	$\longrightarrow 23 \times 10^{-6}$ °C

2. ELASTIC CONSTANTS

When a body is subjected to an axial load, there is an increase in the length but at the same time, there is decrease in other dimensions of the body at right angles to the line of action of the applied load.

- longitudinal strain: $\frac{\delta L}{L}$

- lateral strain: $\frac{\delta b}{b}$ or $\frac{\delta d}{d}$



[*] if longitudinal strain is tensile, the lateral strain will be compressive & vice-versa.

POISSON'S RATIO

$$\boxed{\mu = \frac{1}{m} = \frac{\text{lateral strain}}{\text{longitudinal strain}}}$$

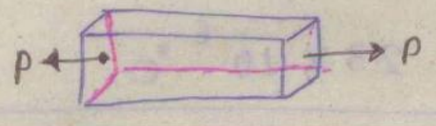
$\frac{1}{m}$ lies mostly bet^n $\frac{1}{3}$ to $\frac{1}{4}$ but for rubber

$\frac{1}{m} = \frac{1}{2.22}$ to $\frac{1}{2}$

☆ Volumetric strain

$$\boxed{e_v = \frac{\delta V}{V}}$$

☆ Volumetric strain of a rectangular bar which is subjected to an axial load p in the direction of its length.

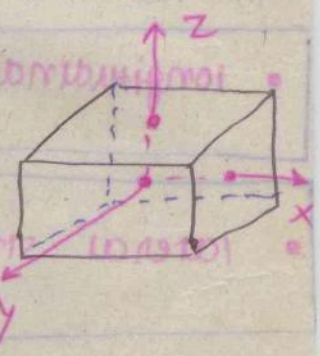


$$\boxed{e_v = \text{longitudinal strain} \left(1 - \frac{2}{m}\right)}$$

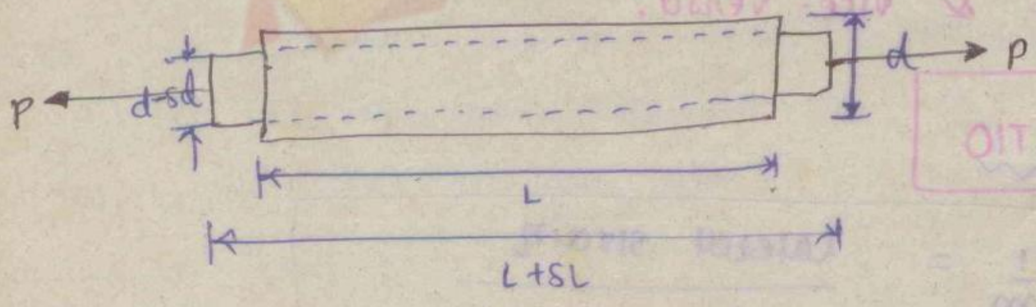
☆ Volumetric strain of a rectangular bar subjected to three forces which are mutually perpendicular.

$$\boxed{e_v = \frac{1}{E}\left(p_x + p_y + p_z\right)\left(1 - \frac{2}{m}\right)}$$

p_x — tensile stress in x direction.



☆ Volumetric strain of cylindrical rod



$$\boxed{e_v = \frac{\delta L}{L} - \frac{2\delta d}{d}}$$

e_v = strain in length − twice the strain in dia.

Bulk modulus :-

$$K = \frac{\text{Direct stress}}{\text{Volumetric strain}}$$

$$\boxed{K = \frac{p}{e_v}}$$

Relation bet^n E & K

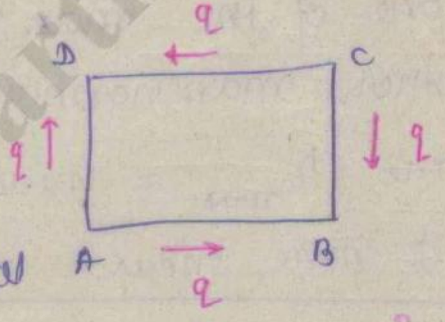
$$\boxed{E = 3K\left(1 - \frac{2}{m}\right)}$$

$$\boxed{E = 2C\left(1 + \frac{1}{m}\right)}$$ Relation bet^n C & E

where C = Modulus of Rigidity $= \frac{\text{shear stress}}{\text{shear strain}}$

PRINCIPLE OF COMPLEMENTARY SHEAR STRESSES

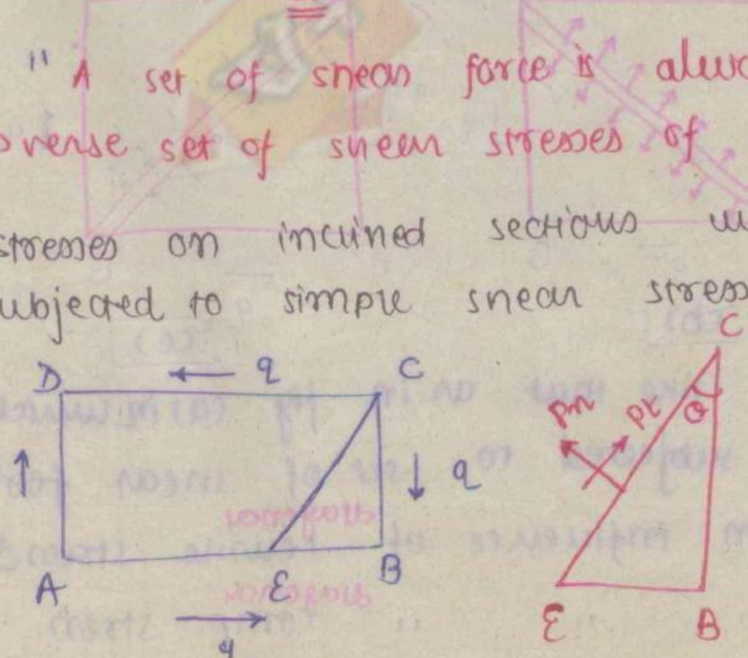
"A set of shear stress across a plane is always accomplished by a set of balancing shear ~~force~~ stresses across the plane & normal to it."



OR

"A set of shear force is always accomplished by a transverse set of shear stresses of the same intensity."

⟹ stresses on inclined sections when the element is subjected to simple shear stresses.



$$p_n = q\sin 2\theta$$

$$p_t = q\cos 2\theta$$

→ p_n is max when $\sin 2\theta = 1$

$\therefore 2\theta = \frac{\pi}{2}$ $\therefore$ $\boxed{\theta = \frac{\pi}{\pm 4}}$

& $\boxed{p_n = \pm q}$

& $\boxed{p_t = 0}$.

& when $\theta = 0, 180$, $p_t = max$. & $p_n = 0$

The planes having max. shear stress are having zero p_n

Hence. The planes which carry the max. normal stresses are having zero shear stresses.

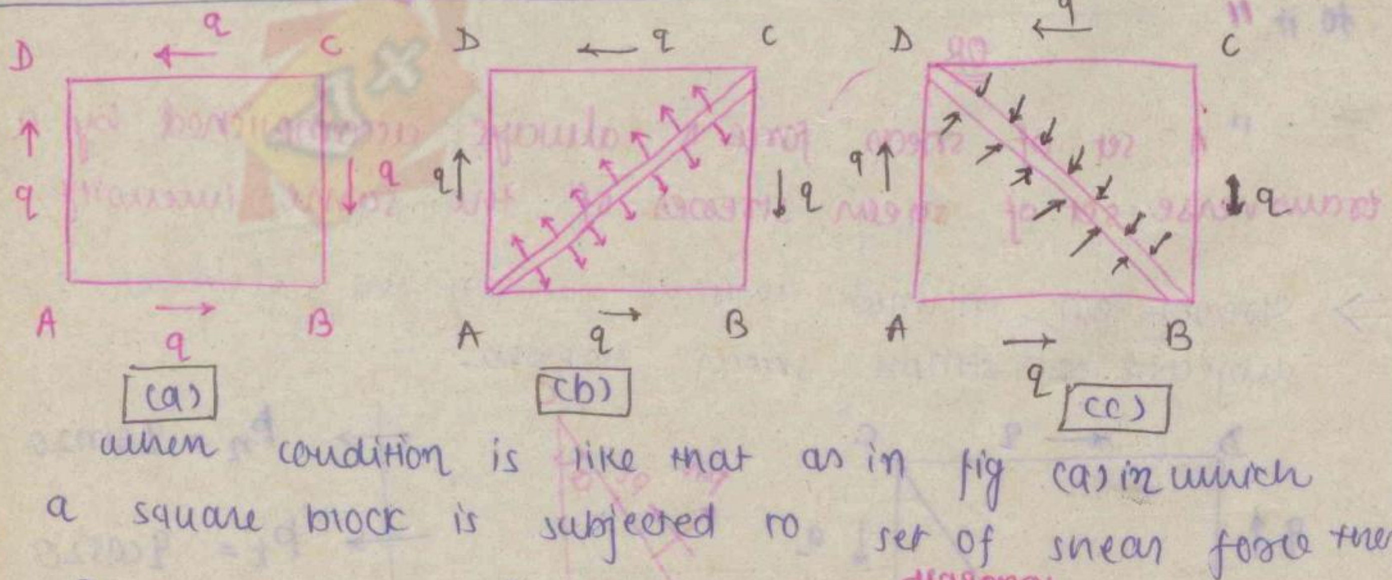
- When an element is subjected to a set of shear stress, then

(1) The planes of $p_{n_{max}}$ are perpendicular to each other.

(2) The planes of $p_{n_{max}}$ are at $45°$ to the plane of pure shear.

(3) One of the max. normal stress is tensile while the other max. normal stress is compressive.

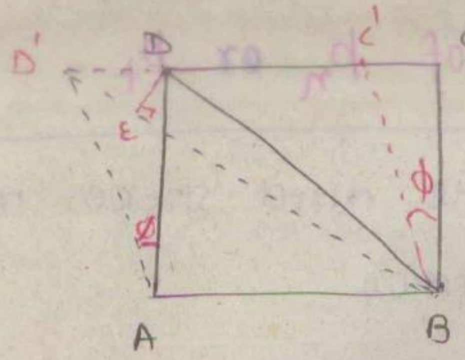
(4) The $p_{n_{max}}$ = Intensity of shear stress on the plane of pure shear.



(a) (b) (c)

When condition is like that as in fig (a) in which a square block is subjected to set of shear force then Diagonal AC is in influence of diagonal tensile stress & that of BD " " " " diagonal comp. stress.

Direct (Tensile & compressive) strains of the diagonals



Tensile strain in diagonal

$BD = \frac{q}{E}\left(1+\frac{1}{m}\right)$

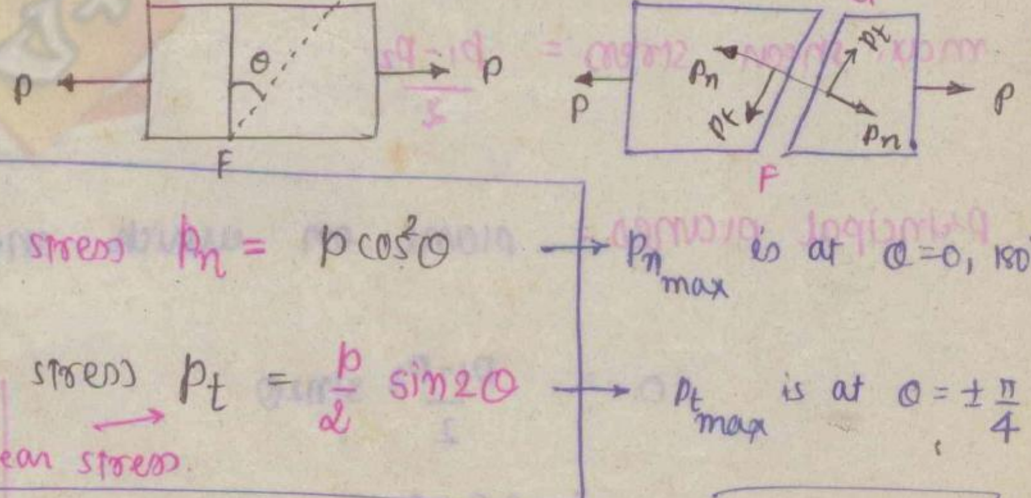
$= \frac{1}{2} \times$ shear strain

= comp strain in diagonal AC

3. PRINCIPAL STRESSES & STRAINS

- The planes which have no shear stress are known as principal stresses.
- The normal stresses acting on a principal plane are known as principal stresses.

3.1 Analytical method for determining the stresses on oblique section.



principal stress $p_n = p\cos^2\theta$ → $p_{n_{max}}$ is at $\theta = 0, 180^\circ$

principal stress $p_t = \frac{p}{2}\sin 2\theta$ → $p_{t_{max}}$ is at $\theta = \pm\frac{\pi}{4}$

shear stress

$p_{n(max)} = p$ (tensile or comp)

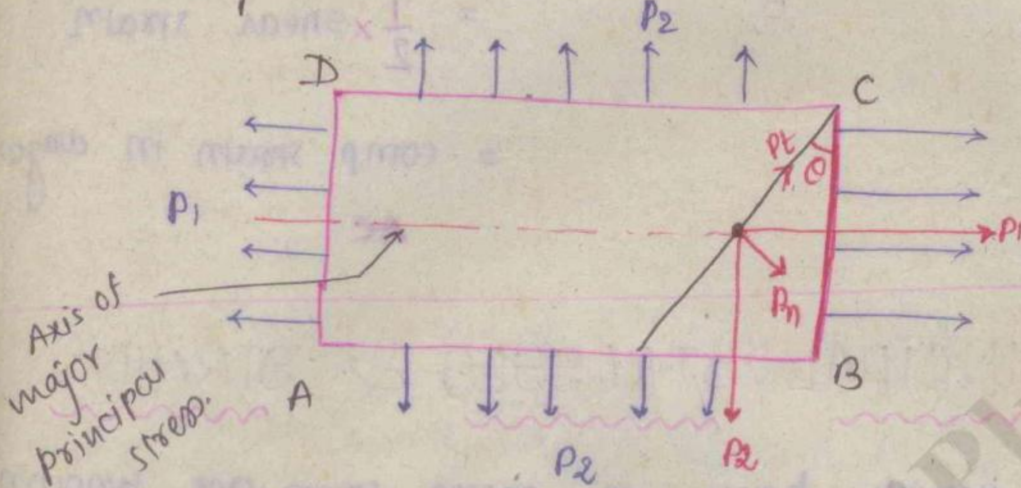
$p_{t(max)} = p/2$ (shear)

& $\boxed{p = \frac{P}{A}}$

Safe stress along inclined plane

= least of p_n or p_t

3.2 A member subjected to like direct stresses in two mutually perpendicular direction.



CB = ~~major~~ minor principal plane.

$$\boxed{p_n = \left(\frac{p_1+p_2}{2}\right) + \left(\frac{p_1-p_2}{2}\right)\cos 2\theta}$$

$$\boxed{p_t = \left(\frac{p_1-p_2}{2}\right)\sin 2\theta} \quad \& \quad \boxed{p_R = \sqrt{p_n^2 + p_t^2}}$$

- max. shear stress = $\frac{p_1 - p_2}{2}$

Principal planes = plane on which shear stress is zero

$$0 = \frac{p_1-p_2}{2}\sin 2\theta$$

$\therefore 2\theta = 0$

$\therefore \boxed{\theta = 0 \text{ or } 90^\circ}$

when $\theta = 0$

$p_n = p_1$

when $\theta = 90^\circ$

$p_n = p_2$

e.g ||| The principal stresses at a point in a bar are 200 N/mm² (tensile) and 100 N/mm² (compressive).

means

$p_1 = 200\ N/mm^2$ = major principal stress. (tensile)

$p_2 = -100\ N/mm^2$ = minor principal stress (comp.)

$$\boxed{\text{OBLIQUITY} = \tan\phi = \frac{p_t}{p_n}}$$

A member subjected to a simple shear stress.

3.3

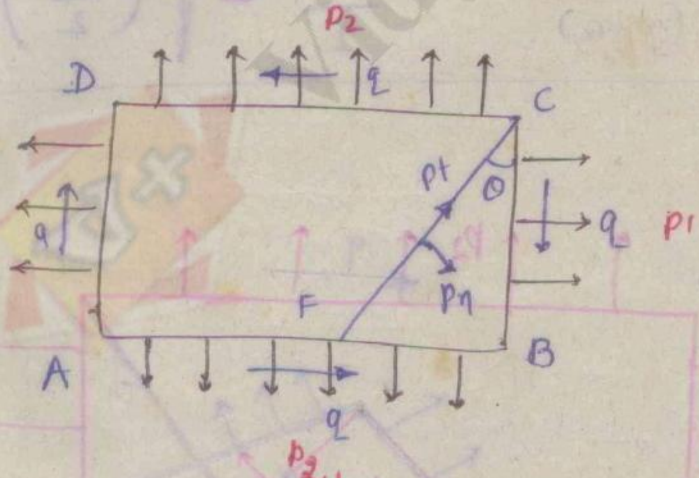
$$\boxed{\begin{aligned} p_n &= q\sin 2\theta \\ p_t &= -q\cos 2\theta \end{aligned}}$$

q = shear stress.

θ = Angle at which plane is considered.

A MEMBER SUBJECTED TO DIRECT STRESSES IN TWO MUTUALLY PERPENDICULAR DIRECTIONS ACCOMPLISHED BY A SIMPLE SHEAR STRESS.

3.4



$$\boxed{p_n = \left(\frac{p_1+p_2}{2}\right) + \left(\frac{p_1-p_2}{2}\right)\cos 2\theta + \left(q\sin 2\theta\right)} \longrightarrow (1)$$

$$\boxed{p_t = \left(\frac{p_1-p_2}{2}\right)\sin 2\theta - \left(q\cos 2\theta\right)} \longrightarrow (2)$$

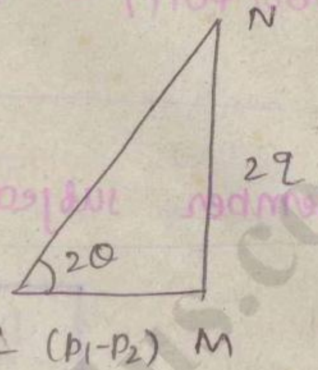
position of principal stress:-

when $p_t = 0$ means eq^n (2) = 0

$$\therefore \boxed{\tan 2\theta = \frac{2q}{p_1 - p_2}} \quad \text{---} \quad (3)$$

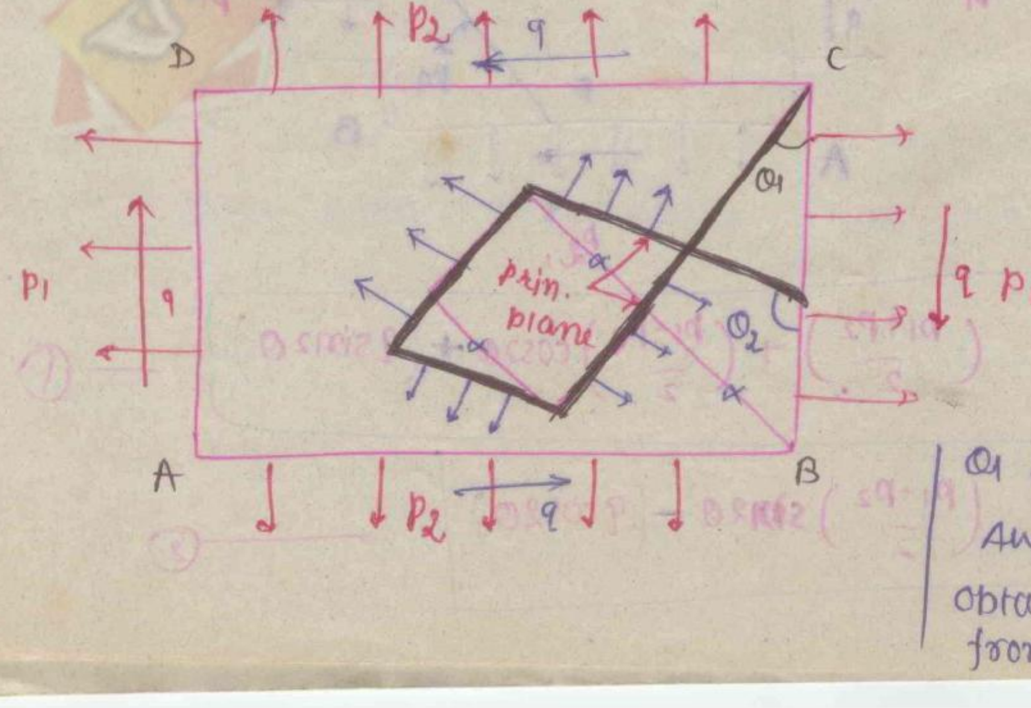
- $\sin 2\theta = \dfrac{2q}{\sqrt{(p_1-p_2)^2 + 4q^2}}$

- $\cos 2\theta = \dfrac{(p_1-p_2)}{\sqrt{(p_1-p_2)^2 + 4q^2}}$



Major principal stress (max.) $= \dfrac{p_1+p_2}{2} + \sqrt{\left(\dfrac{p_1-p_2}{2}\right)^2 + q^2}$

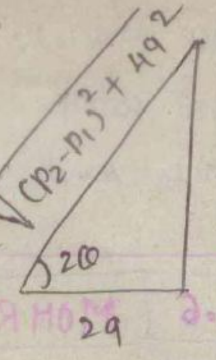
minor principal stress (min) $= \dfrac{p_1+p_2}{2} - \sqrt{\left(\dfrac{p_1-p_2}{2}\right)^2 + q^2}$



θ_1 & θ_2 Angle obtained from (3) eq^n

Max. shear stress :-

$$\boxed{\tan 2\theta = \frac{p_2 - p_1}{2q}}$$



$$\boxed{p_{t_{max}} = \frac{1}{2}\sqrt{(p_1-p_2)^2 + 4q^2}}$$

☆ if a hole is made at the intersection of both the diagonal i.e diagonal BD & AC then after the application of the forces,(stress) hole is elongated & becomes ellipse then - major axis & minor axis of ellipse can be found out by.

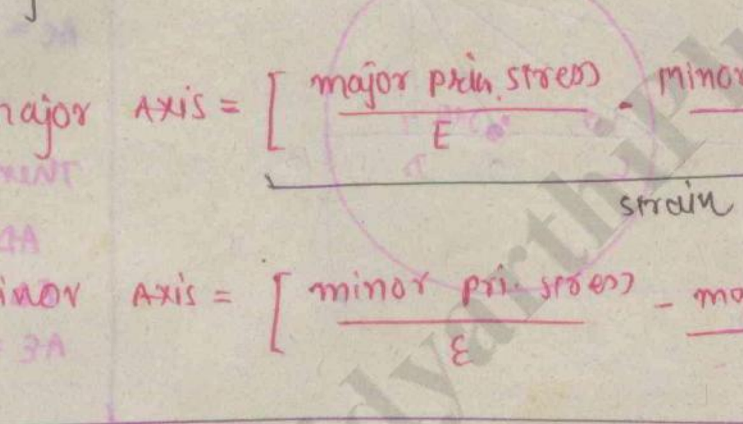
$$\text{major Axis} = \underbrace{\left[\frac{\text{major pri. stress}}{E} - \frac{\text{minor pri. stress}}{mE}\right]}_{\text{strain}} \times \text{Dia. of hole}$$

$$\text{Minor Axis} = \left[\frac{\text{minor pri. stress}}{E} - \frac{\text{major pri. stress}}{mE}\right] \times \text{Dia. of hole.}$$

3.5 Graphical Method.

(To the scale)

★ Graphical Method for 3.2



$OA = p_1$

$OB = p_2$

θ = Angle with stress p_2

Then

$OG = p_n$

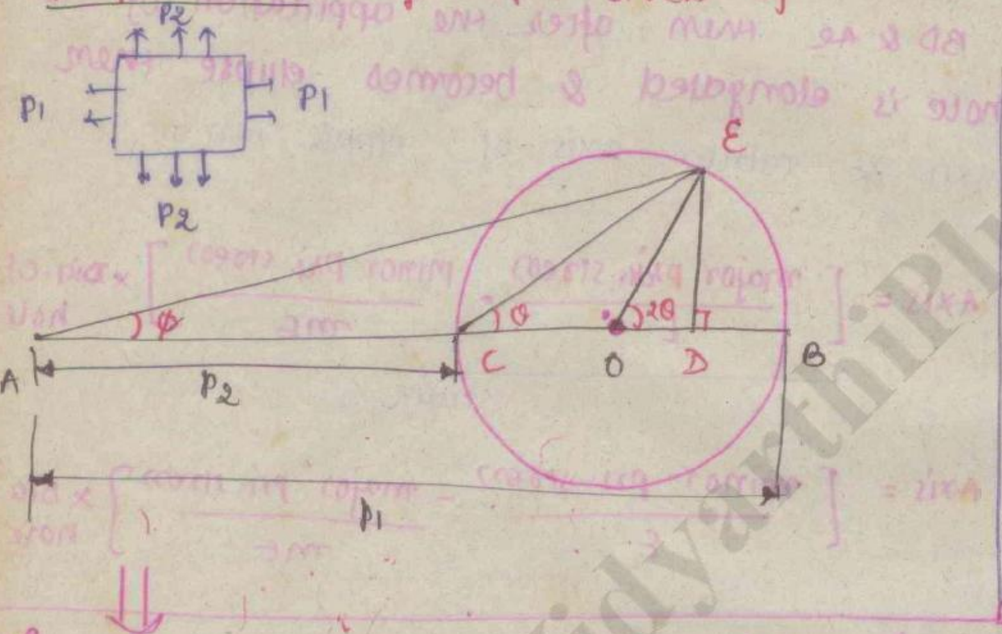
$GF = p_t$

OF = Resultant stress

* if p_1 & p_2 are tensile stress then pt. F will be in 1st quadrant
* if p_1 is comp. and p_2 is tensile then F will be in 2nd quadrant
* if p_1 & p_2 are comp. then F will be in 3rd quadrant.
* if p_1 is tensile and p_2 is comp. then F will be in 4th quadrant

3.6 MOHR'S CIRCLE

* Mohr's circle when body is subjected to two mutually perpendicular tensile principal stress of unequal intensities.



AB = p_1 = major tensile stress

AC = p_2 = minor tensile stress

Then

AD = p_n

ED = p_t

AE = $R = \sqrt{p_n^2 + p_t^2}$

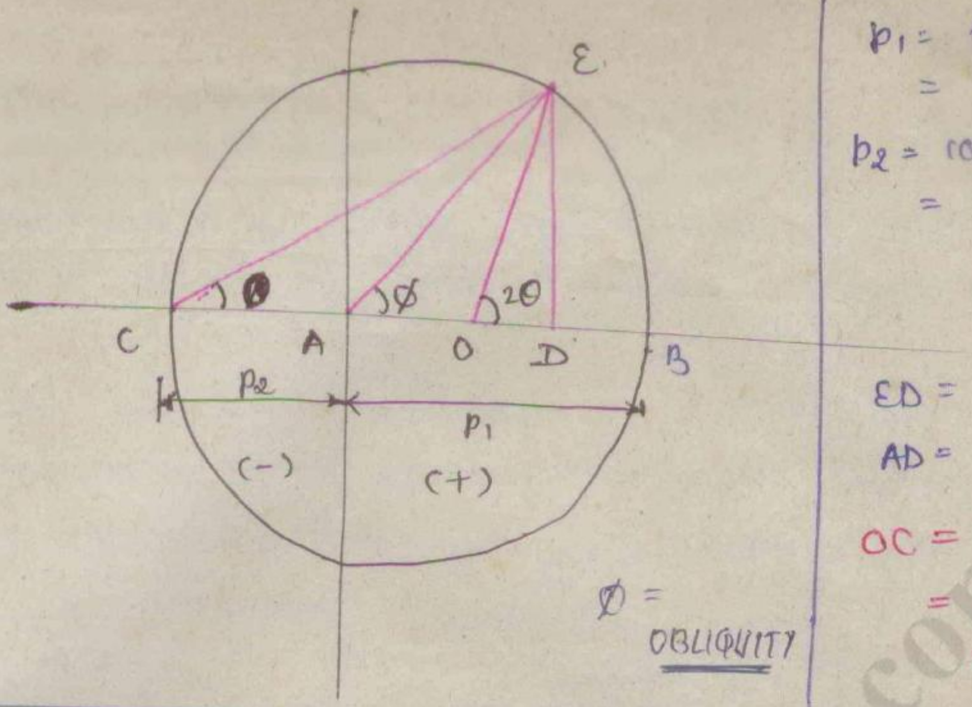
* max. normal stress = AB = p_1
* min normal stress = p_2

& $p_{t\,(max)} = \frac{p_1 - p_2}{2} = \frac{Dia}{2}$ = Radius

when pt. E is at B or C, shear stress $p_t = 0$

ϕ = Angle of obliquity & ϕ_{max} when line AE is tangent to Mohr's circle

* Mohr's circle when a body is subjected to two mutually perpendicular principal stresses which are unequal and unlike. (one is tensile (+) & one is Comp. (−)



p_1 = tensile = AB

p_2 = comp = AC

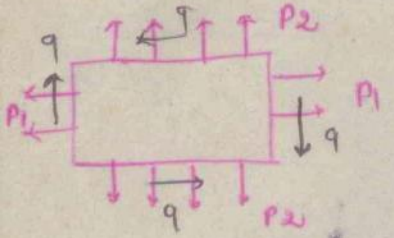
ED = p_t

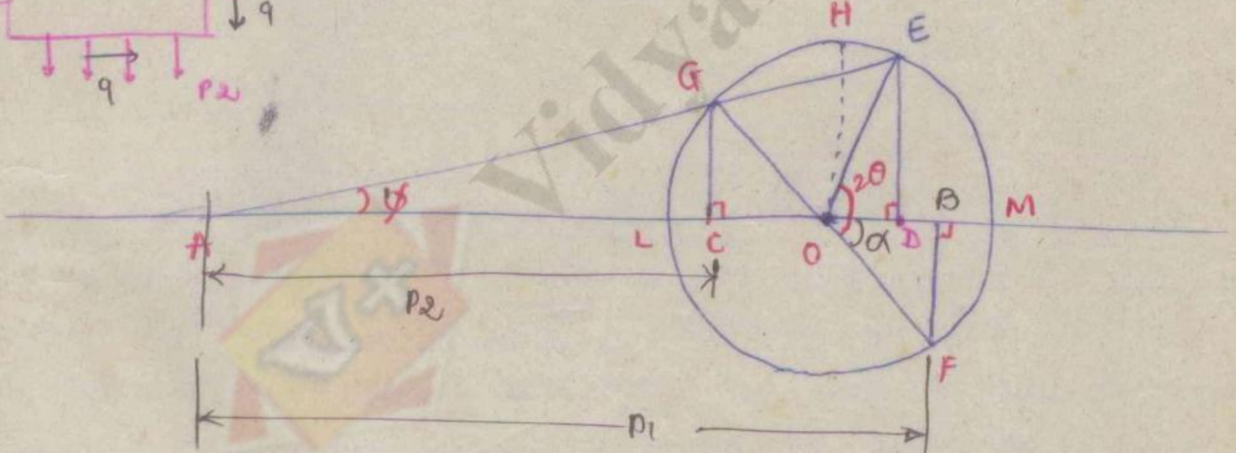
AD = p_n

OC = OB = OE = Radius of mohr's circle

ϕ = OBLIQUITY

☆ Mohr's circle when a body is subjected to two mutually perpendicular principal tensile stresses accompanied by a simple shear stress. (Graphical method for 3.4)





☆ AE = Resultant stress on the oblique plane

AD = normal stress

ED = shear (tangential) stress

OF = OH = max. shear stress.

max. shear stress = mohr's circle radius

$\therefore P_{t_{max}} = OE$

S.F & B.M DIAGRAM

Shear force:-

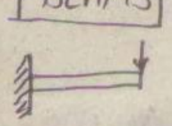
The algebraic sum of the vertical forces at any section of a beam to the right or left of the section is known as shear force

Bending moment:-

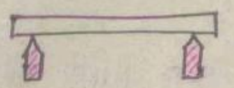
The algebraic sum of the moments of all the forces acting to the right or left of the section is called bending moment.

→ variation of the shear force (B M) along the length of the beam is known as shear force diagram. ((B.M. Dia))

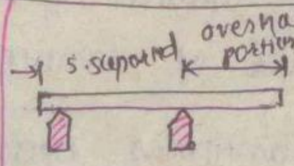
BEAMS



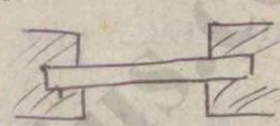
contilever Beam



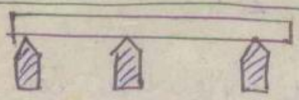
simply supported beam



overhanging beam



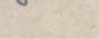
fixed beam



continuous beam

SIGN CONVENTIONS FOR S.F & B.M

S.F	B.M
★ S.F at a section will be positive when the resultant forces to the left to the section is ↑ [or] to the right of the section is ↓ forces.	★ B.M at a section is considerd positive if the B.M at that section is such that it tends to bend the beam to curvature having concavity at the top
similarly S.F at a section will be negative if the Resultant of forces to the left of section is downwards [or] to the right of the section is upwards.	& B.M at a section be negative if the B.M at that section is such that it tends to bend the beam to a curvature having convexity at the top.



Right (-ve) ⇈ forces.

or B.M is +ve for [Right] ↓ Anti clockwise

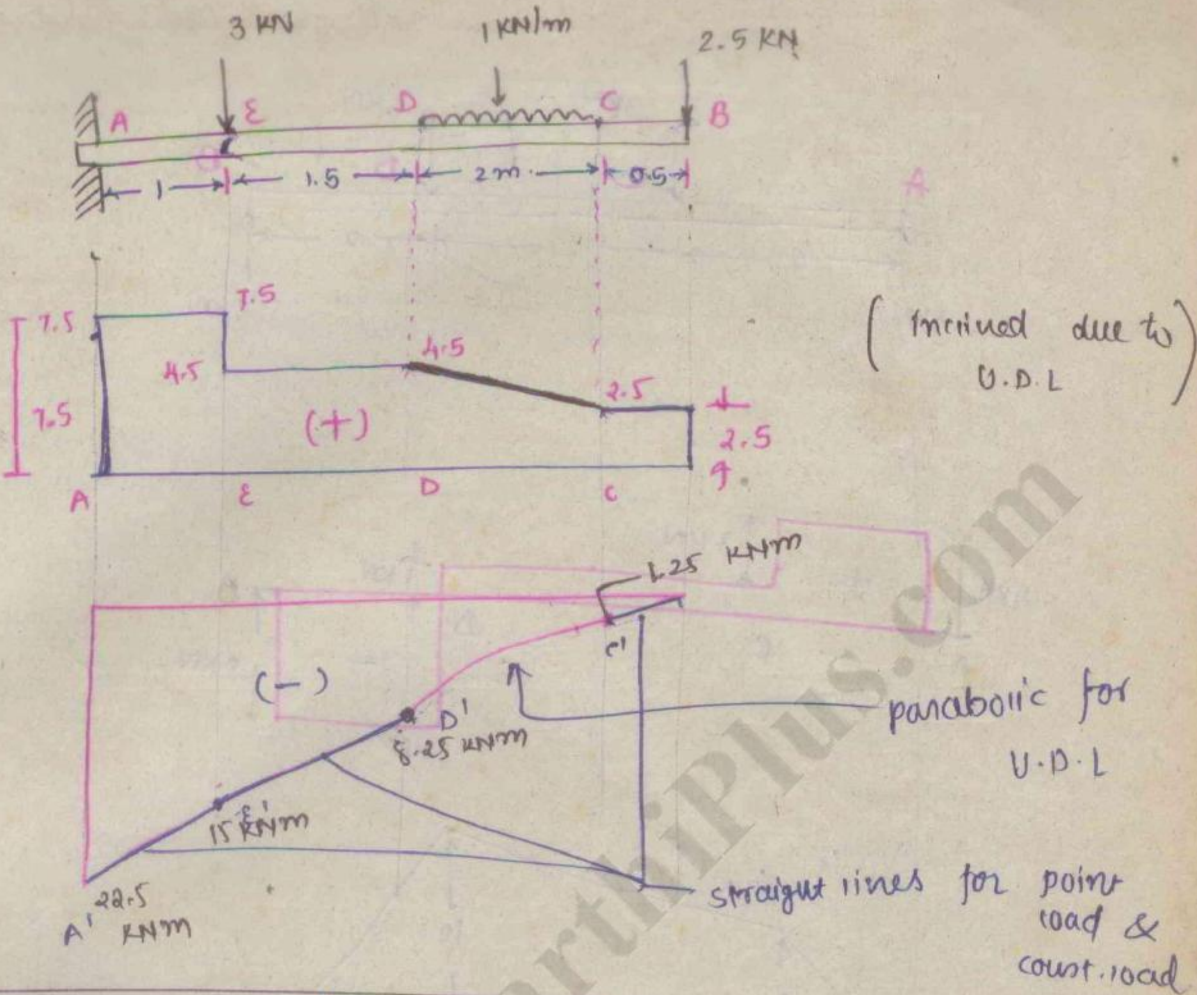
- The positive values of shear force and bending moments are plotted above the base line & negative values below the base line.
- The S.F diagram will increase or decrease suddenly i.e by a vertical straight line at section where there is a point load
- The S.F betⁿ any two vertical loads will be const. & hence S.F diagram between two vertical loads will be horizontal.
- The B.M at the two supports of a simply supported beam and at the free end of cantilever will be zero.
- In case of U.D.L on cantilever beam, S.F will be max at the fixed support & zero at free end of beam & B.M will be parabola having zero B.M at free end & $\frac{W \times L^2}{2}$ (max.) at fixed end
- S.F for a U.D.L varies according to the straight line law & B.M varies according to a parabolic curve.
- The point where B.M is zero after changing its sign, is known as $\boxed{\text{point of}}$ $\boxed{\text{contraflexure}}$ or $\boxed{\text{point of inflexion}}$
- When an inclined load is acting on a beam, then inclined load is resolved into two components. Vertical component will cause S.F & B.M whereas Hori. component will cause Axial thrust in the beam.
- When a beam is subjected to a couple at a section, then B.M changes suddenly at a section but S.F remains unaltered at the section.

S.F	B.M
Point load - straight line	inclined line
U.D.L - Inclined line	parabolic
U.V.L - parabolic	cubic.

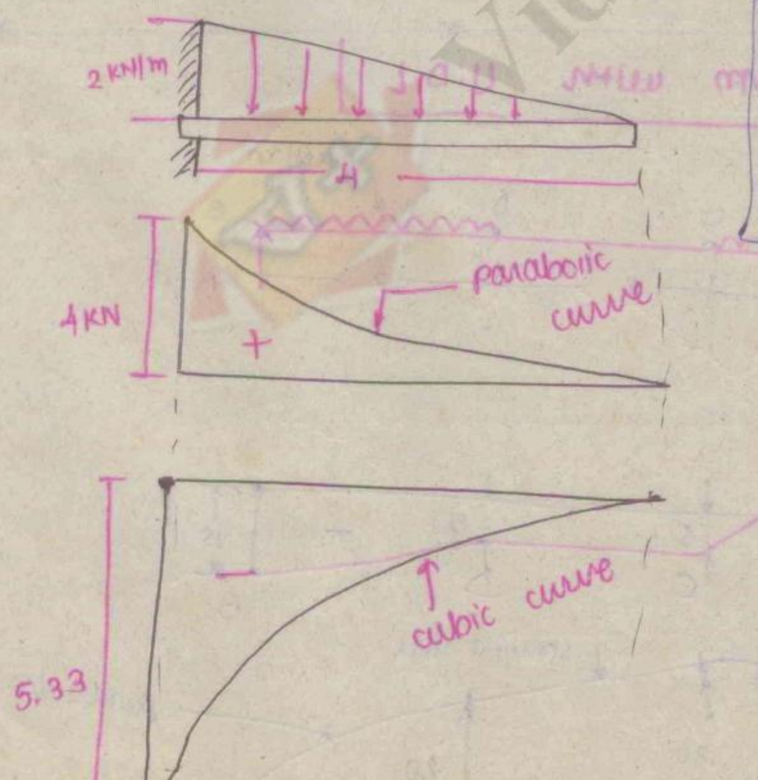
* Right to left

	S.F	B.M
(+)	↓	Anti Clockwise
(-)	↑	Clockwise

e.g.

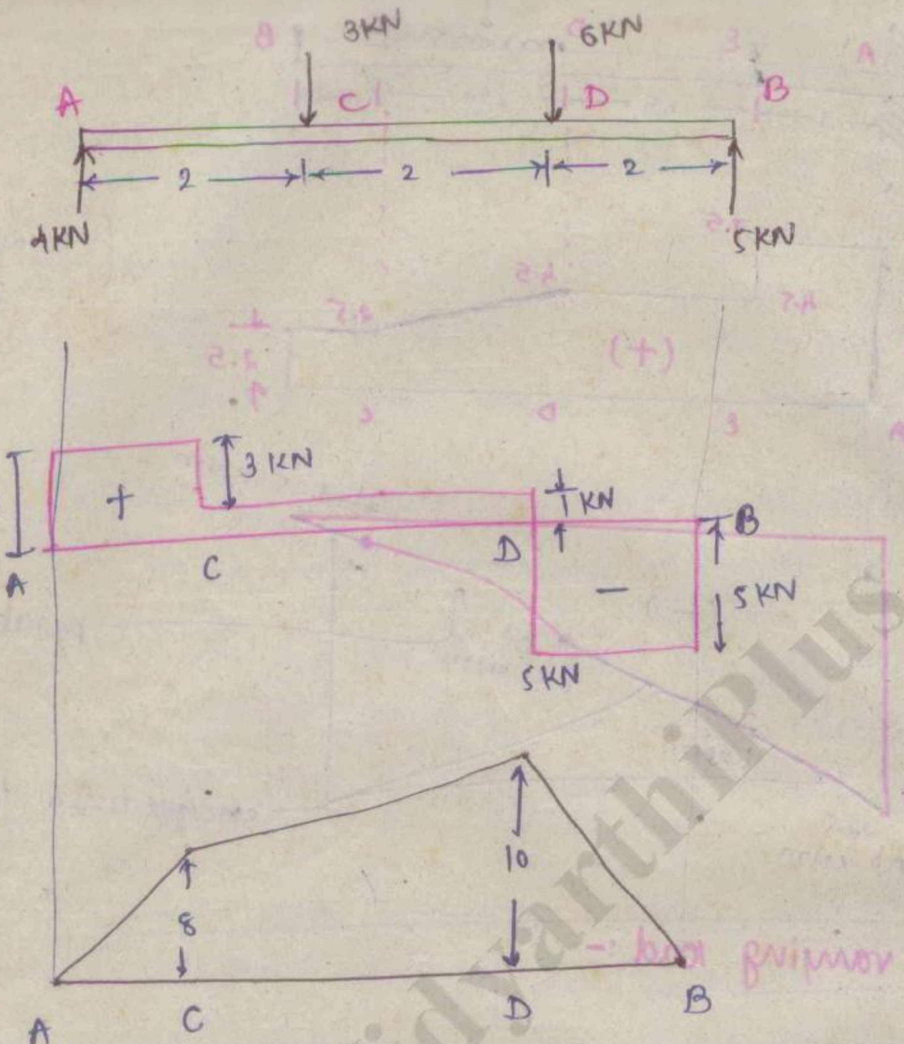


☆ Uniformly varying load :-

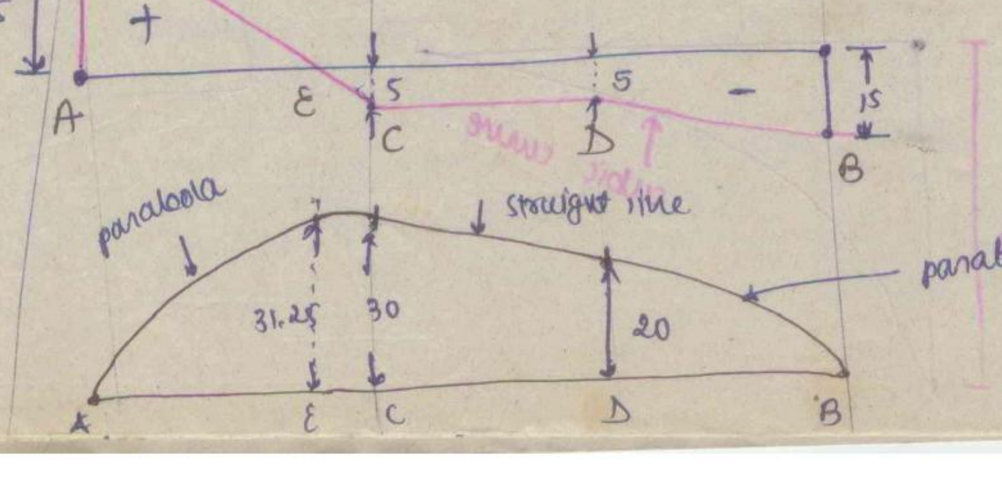


U.V.L is acting at $\frac{2}{3}$ x Total length of beam from the free end or zero intensity portion of U.V.L

✩ simply supported beam with point load.



✩ simply supported beam with U.D.L



Here at E , S.F = 0

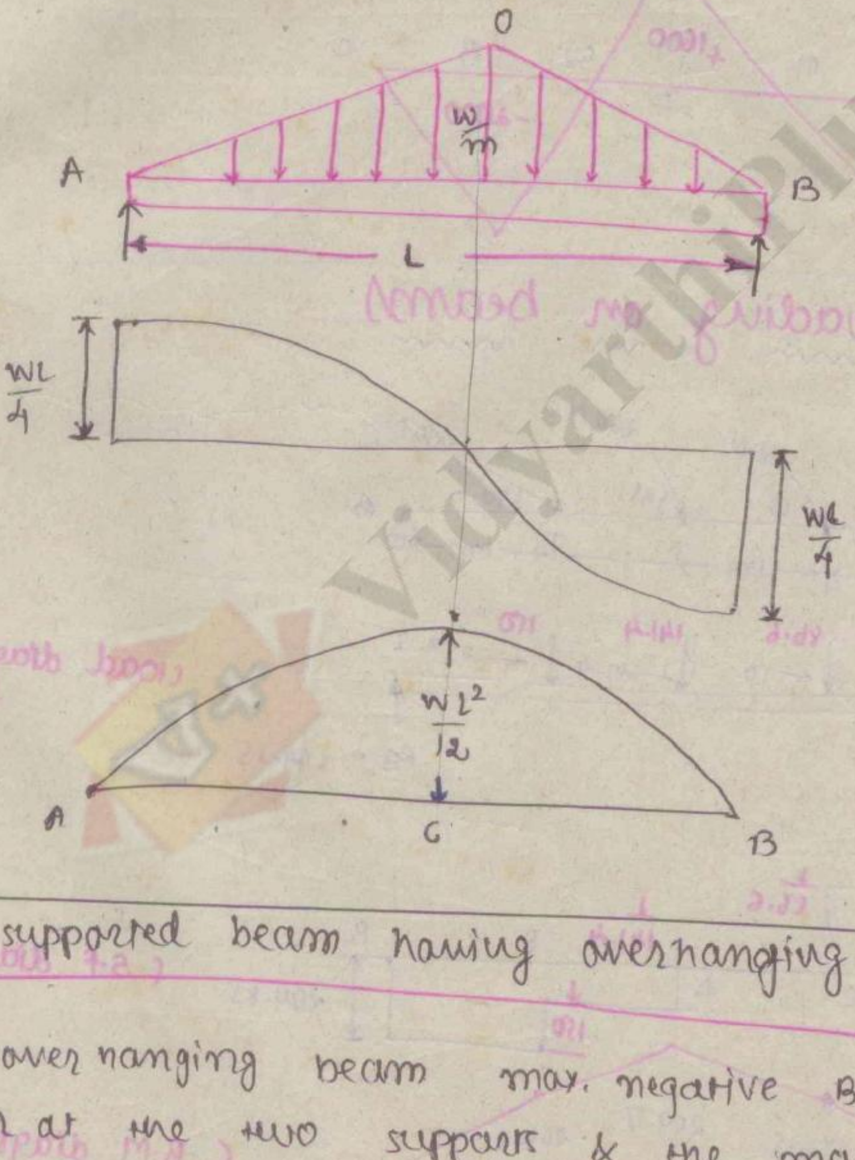
∴ from left side

$$R_A - 10 \times x = 0$$

$$25 - 10x = 0$$

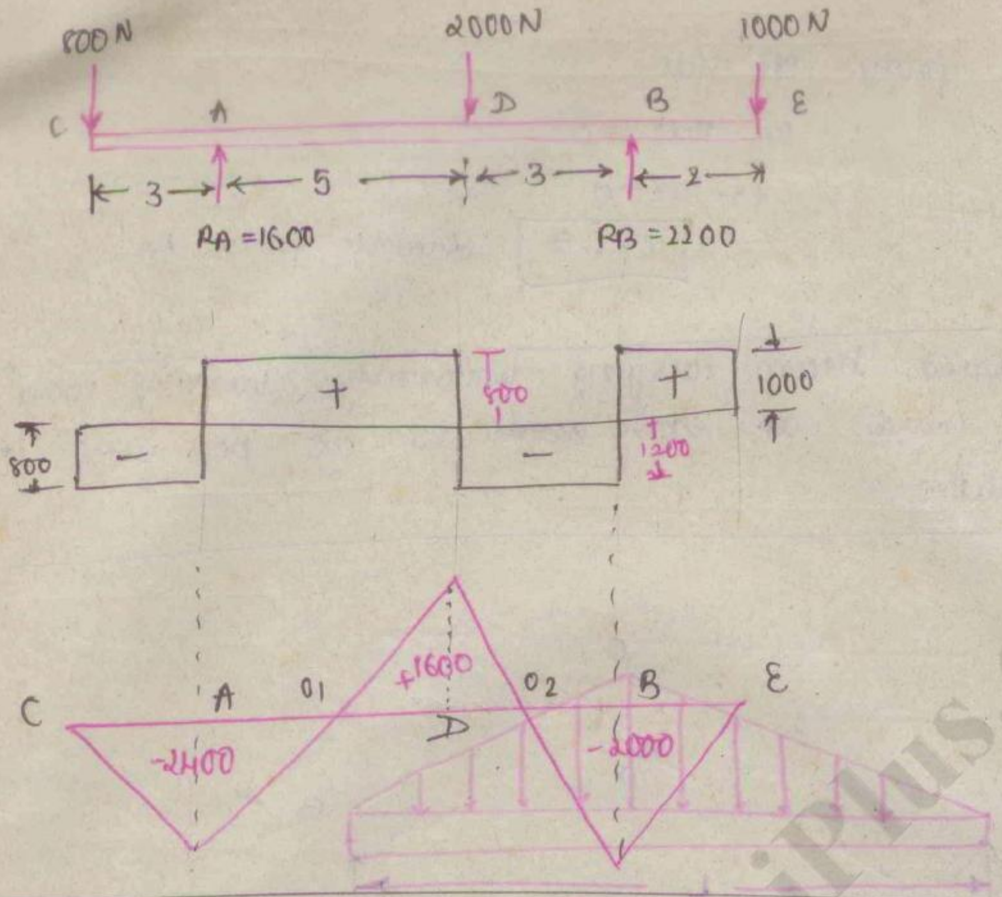
∴ $\boxed{x = 2.5}$ distance from R_A

* simply supported beam carrying uniformly varying load from zero at each end ~~at each end~~ to w per unit length at the centre

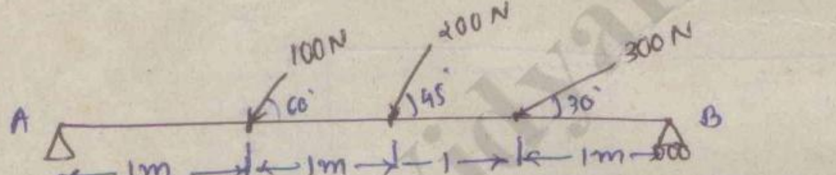


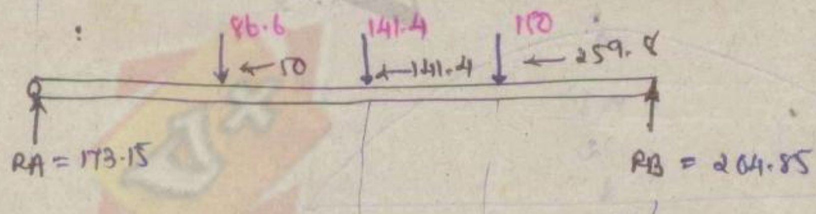
simply supported beam having overhanging portion

- In overhanging beam max. negative B.M will be either at the two supports & the maximum positive B.M will be in the span.

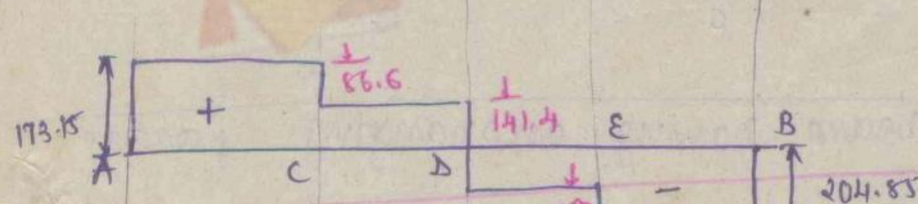


✱ inclined loading on beams

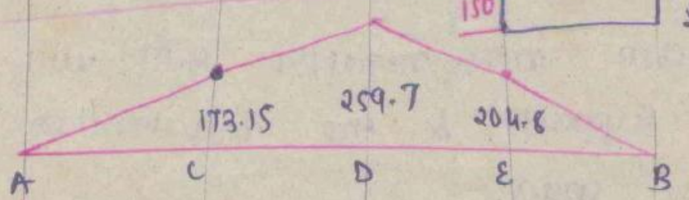




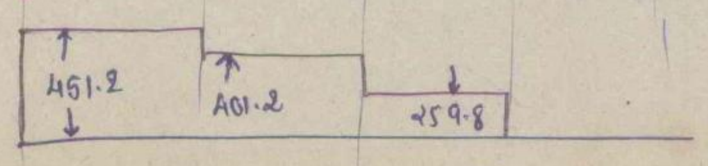
(load diagram)



(S.F diagram)



(B.M diagram)



(Thrust diagram)

• In inclined loading, inclined load is resolved into two components in which vertical components is treated as vertical pt load & Horizontal load increases Axial thrust on beam.

☆ When a beam is subjected to a couple at a section, only the bending moment at the section of the couple changes suddenly in magnitude equal to that of couple. But S.F does not change at the section of couple as there is no change in load due to couple at that section.

⟹ Relation between load, shear force & B.M

$$\boxed{\frac{dM}{dx} = F}$$

" Rate of change of moment is equal to shear force at the section ".

→ B.M is max when S.F is zero
where

Bending Stresses in Beams

☆ The stresses introduced by B.M is called bending stresses.

- pure bending occurs in section having zero S.Force.

☆ Relation

$$\boxed{\frac{f}{y} = \frac{E}{R}}$$

where

f = stress , N/mm^2

R = Radius of neutral layer ; mm

E = young's modulus, N/mm^2

y = distance of considered layer , mm

Neutral Axis of any transverse section of a beam is defined as the line of intersection of neutral layer with the transverse section.

$$\boxed{\frac{M}{I} = \frac{f}{y} = \frac{E}{R}}$$

M = Bending moment , N.mm

I = moment of inertia , mm^4

• M.O.I of hollow pipe (cylinder)	$= \frac{\pi}{64}(D^4 - d^4)$
M.O.I of plate =	$\frac{\text{width} \times th.^3}{12} = \frac{bt^3}{12}$

☆ Section modulus = ratio of M.O.I of a section about the neutral axis to the distance of the outermost layer from the neutral axis.

$$\boxed{Z = \frac{I}{y_{max}}}$$

$\therefore$ from $\frac{M}{I} = \frac{f_{max}}{y_{max}}$ $\quad \therefore \frac{M}{f_{max}} = \frac{I}{y_{max}} = Z$

$$\therefore \boxed{M = f_{max} . Z}$$

* section modulus for rectangular section : $\frac{bd^2}{6}$
* section modulus for Hollow rectangular section : $\frac{1}{6D}[BD^3 - bd^3]$
* section modulus for circular section : $\frac{\pi}{32} d^3$
* section modulus for Hollow circular section : $\frac{\pi}{32D}(D^4 - d^4)$

→ when simply supported beam is uniformly loaded with U.D.L then $B.M = \frac{WL^2}{8}$

→ when simply supported beam is acted by point load at the centre then $B.M_{max} = \frac{W}{2} \cdot \frac{l}{2}$

$$= \frac{WL}{4}$$

→ when cantilever beam is acted by U.D.L throughout then $B.M_{max} = (w \times l) \times \frac{l}{2}$

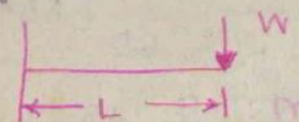
$$= \frac{wl^2}{2}$$

• Two circular beams where one is solid with dia. D and other is hollow of outer dia. D_o and inner dia D_i. of the same length, material & weight. Then

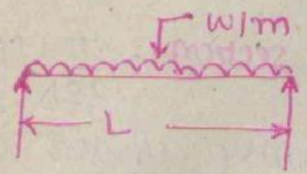
$$\frac{\text{section modulus of hollow shaft}}{\text{section modulus of solid shaft}} = \frac{2D_o}{D} - \frac{D}{D_o}$$

• The bending stress in any layer is ∝ distance of the layer from the neutral Axis (N.A)
• The bending stress on the N.A is zero
• The N.A of symmetrical section lies at a distance of $d/2$ from the outermost layer of the section where d is the depth of the section.

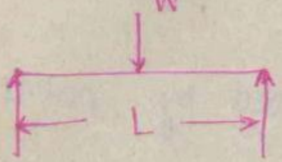
- If the top layer of the section is subjected to compressive stress then the bottom layer of the section will be subjected to tensile stress.

★ 

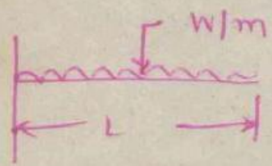
$\rightarrow \boxed{B.M_{max} = W.L}$

★ 

$\rightarrow \boxed{B.M_{max} = \frac{W \times L^2}{8}}$

★ 

$\rightarrow \boxed{B.M_{max} = \frac{W \times L}{4}}$

★ 

$\rightarrow \boxed{B.M_{max} = \frac{W \times L^2}{2}}$

DEFLECTION OF BEAMS

- When beam carries U.D.L or point load, the beam is deflected from its position in curvature manner. & This radius of curvature of deflected beam is given by

$$\boxed{\frac{M}{I} = \frac{E}{R}}$$

where
M = B.Moment
I = M.O.I
E = Young's modulus
R = Radius of curvature

- if y is the deflection of beam

$$* \quad \boxed{y = \frac{ML^2}{8EI}}$$

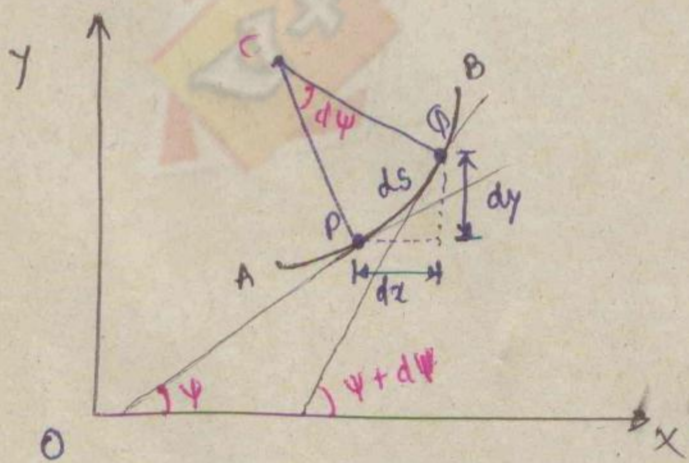
where L = length of beam

& $\boxed{y = \frac{L^2}{8R}}$

- if i is slope of beam

$$* \quad \boxed{i = \frac{M \times L}{2EI}}$$

☆ Relation betn slope, deflection & radius of curvature:

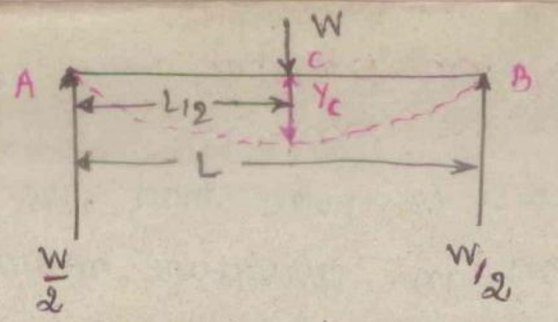


☆ Deflection	$= y$
☆ slope	$= \frac{dy}{dx}$
☆ B.M	$= EI\frac{d^2y}{dx^2}$
☆ S.F	$= EI\frac{d^3y}{dx^3}$
☆ Rate of loading	$= EI\frac{d^4y}{dx^4}$

E is in N/mm²

☆ Mom

case : 1

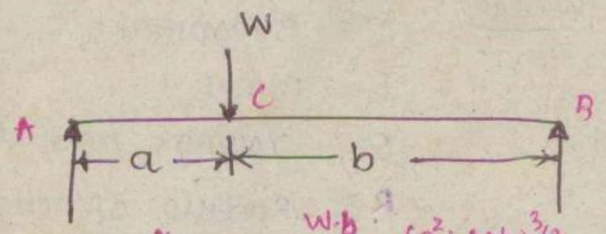


(-ve due to y is ↓)

$$y_c = -\frac{WL^3}{48EI}$$

$$i_B = i_A = \frac{WL^2}{-16EI}$$

case : 2

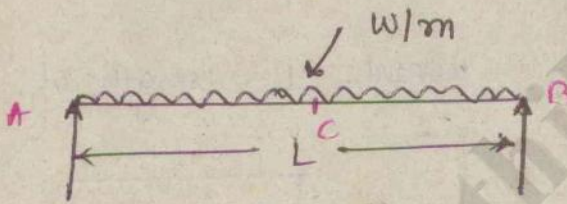


$$y_{max} = \frac{W \cdot b}{9\sqrt{3}\,EIL}(a^2+2ab)^{3/2}$$

$$y_c = \frac{Wa^2b^2}{3EIL}$$

$$i_A = -\frac{W\,ab}{6EIL}(a+2b)$$

case : 3



$$y_{c\,max} = \frac{5}{384} \times \frac{WL^3}{EI}$$

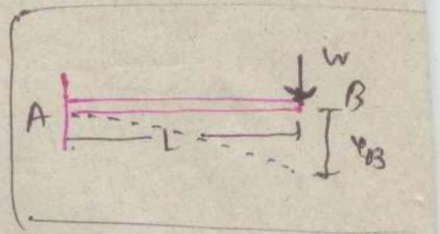
$$i_A = i_B = -\frac{WL^2}{24EI}$$

-ve due to ↓

- for max. deflection, slope $\frac{dy}{dx} = 0$

☆ DEFLECTIONS OF CANTILEVERS ☆

1. Deflection of a cantilever with a point load at the free end by double integration



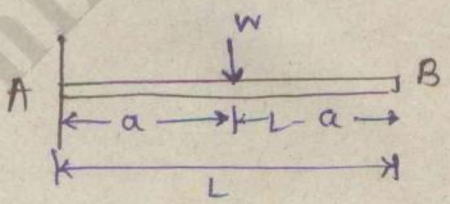
slope i_B =

$$\boxed{i_B = \frac{WL^2}{2EI}}$$

Downward deflection

$$\boxed{y_B = -\frac{WL^3}{3EI}}$$

2. " " at a distance 'a' from fixed end:



$$\boxed{i_C = \frac{Wa^2}{2EI} = i_B}$$

$$\boxed{y_C = \frac{Wa^3}{3EI}}$$

$$\boxed{y_B = y_C + i_C(L-a)}$$

3. Deflection of a cantilever with U.D.L

$$\boxed{i_B = -\frac{WL^2}{6EI}}$$

$$\boxed{y_B = \frac{WL^3}{8EI}}$$

4. Deflection of a cantilever with U.D.L for a distance 'a' from the fixed end.

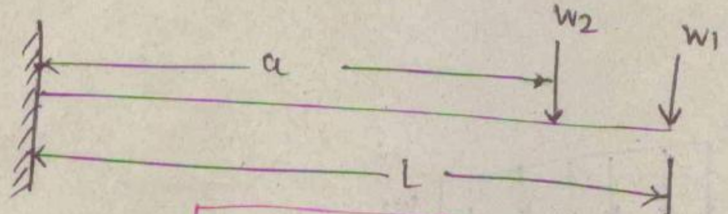
$$\boxed{i_C = \frac{wa^3}{6EI}} \qquad \boxed{y_C = \frac{wa^4}{8EI}}$$

$$\boxed{i_C = i_B} \qquad \boxed{y_B = y_C + i_C(L-a)}$$

5 Deflection of a cantilever with U.D.L for a distance 'a' from the free end.

$$\boxed{i_B = \frac{WL^3}{6EI} - \frac{W(L-a)^3}{6EI}}$$

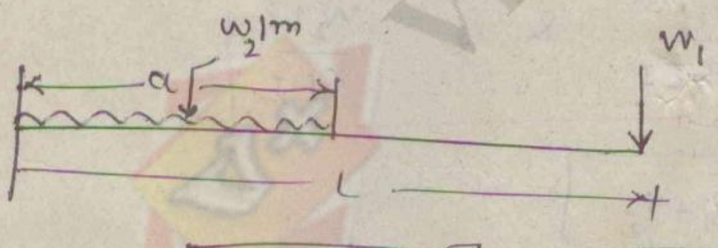
$$\boxed{y_B = \frac{WL^4}{8EI} - \left[\frac{W(L-a)^4}{8EI} + \frac{W(L-a)^3}{6EI} \times a\right]}$$

6 

$$\boxed{y_1 = \frac{W_1 L^3}{3EI}}$$

$$\boxed{y_2 = \frac{W_2 a^3}{3EI} + \frac{W_2 a^2}{2EI}(L-a)}$$

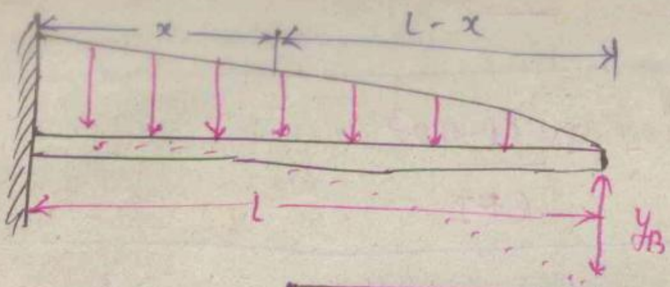
& Total deflection at free end $= y_1 + y_2$

7 

$$\boxed{y_1 = \frac{W_1 L^3}{3EI}} \quad \& \quad \boxed{y_2 = \frac{wa^4}{8EI} + \frac{wa^3}{6EI}(L-a)}$$

$\therefore$ Total deflection $= y_1 + y_2$

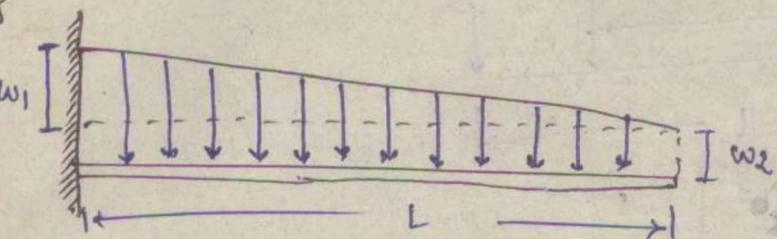
8 Deflection of a cantilever with a gradually varying load.



$$\boxed{y_B = \frac{WL^4}{30EI}}$$

$$\boxed{i_B = -\frac{WL^3}{24EI}}$$

9



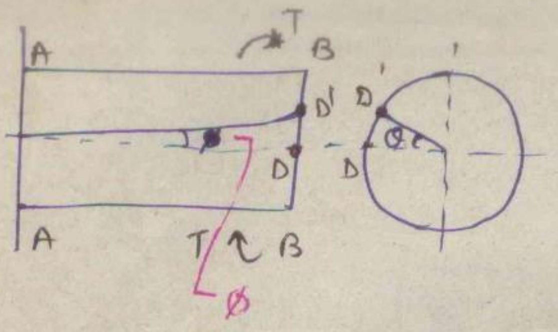
<u>NOW</u> Deflection $= y_1 + y_2$

$$\boxed{y = \frac{W_1 L^4}{30EI} + \frac{W_1 L^4}{8EI}}$$

& $$\boxed{i_1 = \frac{W_1 L^3}{24EI} \quad \& \quad i_2 = \frac{W_2 L^3}{6EI}}$$

$$\boxed{\therefore i = i_1 + i_2}$$

☆ TORSION OF SHAFTS ☆



θ = Angle of twist

ø = shear strain

R = radius of shaft

L = length of shaft

$$\boxed{\phi = \frac{R \times \theta}{L}}$$

★ BASIC TORSION EQUATION.

$$\boxed{\frac{f_s}{R} = \frac{C\theta}{L} = \frac{q}{r}}$$

f_s = shear stress

R = Radius of shaft

C = modulus of rigidity

q = shear stress induced at rad. r from centre of shaft.

☆ max. torque transmitted by solid shaft

$$\boxed{T = \frac{\pi}{16} \times f_s \times d^3}$$

d = dia. of shaft

☆ max. torque transmitted by Hollow shaft.

$$\boxed{T = \frac{\pi}{16} \times f_s \times \left[\frac{D_o^4 - D_i^4}{D_o} \right]}$$

D_o = outer dia.

D_i = inner dia.

☆ Two shafts of same material, same length & are sub. to same torque. if first is solid & second is hollow whose $d_i = \frac{2}{3} d_o$. and max f_s is same for both. Then Compare weights of shafts.

(i) $\frac{\pi}{16} \times f_s \times D^3 = \frac{\pi}{16} \times \frac{[D_o^4 - D_i^4]}{D_o}$

(ii) * weight of solid shaft = weight density × volume

$= w \times \frac{\pi}{4} D^2 \times L$

* weight of Hollow shaft $= w \times \frac{\pi}{4} [D_o^2 - D_i^2] \times L$

$\therefore \frac{W_s}{W_h} = \frac{9}{5} \frac{D^2}{D_o^2}$

$$\boxed{\frac{W_s}{W_h} = 1.55}$$

→ outside dia = inside dia + 2 × thickness

• $\boxed{\text{power} = \frac{2\pi N T}{60}}$

☆ saving in weight $= \frac{W_s - W_h}{W_s} \times 100$

$$\frac{T}{J} = \frac{f_s}{R} = \frac{C\theta}{L} = \frac{q}{r}$$

where J = Polar m.o.I

$J = \frac{\pi}{32} D^4$ (solid)

$J = \frac{\pi}{32} [D_o^4 - D_i^4]$ (hollow)

* polar moment of inertia(J) of a plane area is defined as the m.o.i of area about axis perpendicular to the plane of figure and passing through c.g of area.

* $\boxed{C \times J = \text{Torsional rigidity}}$

* $$\boxed{\text{polar modulus} = \frac{J}{R}}$$

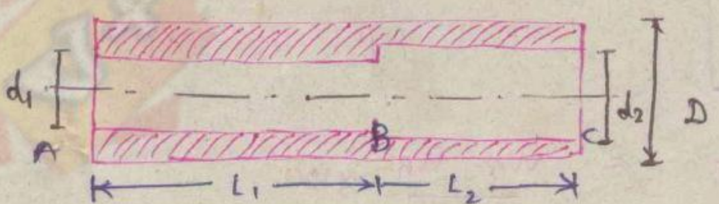
polar modulus

$$Z_p \text{ for solid shaft} = \frac{\frac{\pi}{32} D^4}{D/2} = \frac{\pi}{16} D^3$$

$$Z_p \text{ for hollow shaft} = \frac{\pi}{16} \frac{[D_0^4 - D_i^4]}{D_0}$$

$$\therefore \boxed{T = Z_p \times f_s} = \text{Torque transmitted by shaft.}$$

* STRENGTH OF SHAFT OF VARYING C/S



*

Torque transmitted by hollow shaft AB

$$T_{AB} = \frac{\pi}{16} \times f_s \times \left[\frac{D^4 - d_1^4}{D}\right] \quad \&$$

Torque transmitted by hollow shaft BC

$$T_{BC} = \frac{\pi}{16} \times f_s \times \left[\frac{D^2 - d_2^2}{D}\right]$$

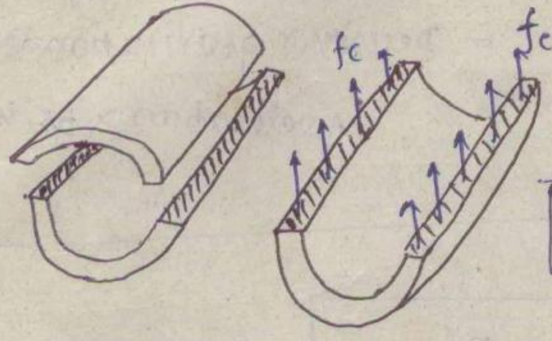
THIN CYLINDERS.

- If the thickness of the wall of the cylinder vessel is less than $\frac{1}{15}$ to $\frac{1}{20}$ of its internal diameter, the vessel is known as thin cylinder.

→ Stresses in a thin cylinder due to internal fluid pressure

(1) circumferential (hoop) stress → stress acting along the circumference

(2) longitudinal stress. → stress acting along the length of cylinder

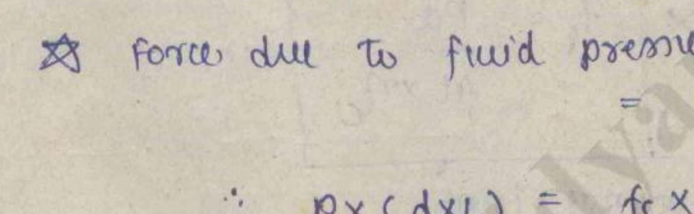


f_c = circumferential stress.

☆ force due to fluid pressure = force due to fc

$$\therefore \quad p \times (d \times L) = f_c \times (2Lt)$$

$$\therefore \boxed{f_c = \frac{pd}{2t}}$$



f_l = longitudinal stress

☆ Force due to fluid pressure = force due to fl

$$\therefore \quad p \times \frac{\pi}{4} d^2 = f_l \, (\pi d \times t)$$

$$\therefore \boxed{f_l = \frac{pd}{4t} = \frac{1}{2} f_c}$$

☆ max. shear stress

$$= \frac{f_c - f_l}{2}$$

$$= \frac{\frac{pd}{2t} - \frac{pd}{4t}}{2}$$

$$\boxed{f_{max} = \frac{pd}{8t}}$$

☆ $\boxed{p = \rho g h}$ = fluid pressure = Density x gravitational Acceleration x pr. head

---

★ efficiency of JOINT

$$\boxed{f_c = \frac{p \times d}{2t \times \eta_l}} \quad \& \quad \boxed{f_l = \frac{pd}{4t \times \eta_c}}$$

η_l = effi. of longitudinal joint

η_c = eff. of circumferential joint.

☆ In longitudinal joint f_c is developed and in circumferential joint f_l is developed.

☆ Generally, efficiency of joint = eff. of longitudinal joint

☆ if η_l is given then t is obtained by eq^n f_c as above.

⇛ if Dia. found from above eq^n, then take min. of the two because if greater is taken f_l > permissible. stress.

STRAINS

- circumferential strain $= \frac{f_c}{E} - \frac{f_l}{mE}$

$$= \frac{pd}{2tE} - \frac{pd}{4tmE}$$

$$= \frac{pd}{2tE}\left(1 - \frac{1}{2m}\right)$$

$$e_1 = \boxed{\text{circumferential strain} = \frac{f_c}{E}\left(1 - \frac{1}{2m}\right)} = \frac{\delta d}{d}$$

- longitudinal strain $= \frac{f_l}{E} - \frac{f_c}{mE}$

$$= \frac{pd}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right)$$

$$e_2 \Rightarrow \boxed{\text{longitudinal strain} = \frac{f_c}{E}\left(\frac{1}{2} - \frac{1}{m}\right)} = \frac{\delta L}{L}$$

- volumetric strain $= 2e_1 + e_2$

$$\frac{\delta V}{V} = 2e_1 + e_2$$

$$\boxed{\frac{\delta V}{V} = \frac{f_c}{E}\left(\frac{5}{2} - \frac{2}{m}\right) = 2e_1 + e_2}$$

$\boxed{\text{Max. principal stress} = f_c}$

☆ THIN CYLINDER VESSEL SUBJECTED TO INTERNAL FLUID PRESSURE AND TORQUE

☆ major principal stress $= \dfrac{f_c + f_l}{2} + \sqrt{\left(\dfrac{f_c - f_l}{2}\right)^2 + f_s^2}$

☆ minor principal stress $= \dfrac{f_c + f_l}{2} - \sqrt{\left(\dfrac{f_c - f_l}{2}\right)^2 + f_s^2}$

☆ max. shear stress $= \dfrac{1}{2}$ [major pr. stress - Minor pri. stress]

☆ There always two tensile perpendicular stresses f_c & f_l and when torque is applied, shear stress is also developed.

☆ shear force = Area x shear stress

$= (\pi d \times t) \times f_s$

Torque $= S.F \times \dfrac{d}{2}$

☆ THIN SPHERICAL SHELLS

$$f_c = \frac{p \times \frac{\pi}{4} d^2}{\pi . d . t}$$

$$\boxed{f_c = \frac{p.d}{4t}}$$ ← tensile

when η is given

$$f_c = \frac{pd}{4t\eta}$$

STRAINS FOR SPERICAL SHELL

- in sperical shell $f_l = f_c = \frac{pd}{4t}$

$\therefore$ $\boxed{\text{max. shear stress} = \frac{f_c - f_l}{2} = \frac{f_c - f_c}{2} = 0 \text{ at any pt.}}$

(1) $e = \frac{f_c}{E} - \frac{f_l}{mE}$

$$\boxed{\frac{\delta d}{d} = \frac{f_c}{E}\left(1 - \frac{1}{m}\right)}$$

(2) volumetric strain =

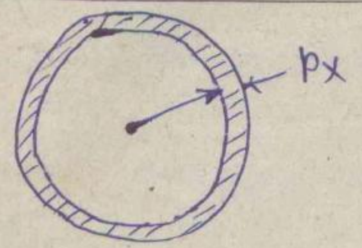
$$\frac{dv}{v} = \frac{3 \times f_c}{E}\left(1 - \frac{1}{m}\right)$$

$$\frac{\delta v}{v} = 3 \times \frac{\delta d}{d} = e_1 + 2e$$
$$= 3e$$

THICK CYLINDERS

→ If the ratio of $\frac{t}{d_i} > \frac{1}{20}$ is known as thick cylinder.

→ f_c in case of thick cylinder will not be uniform across the thickness. Actually f_c will vary from max. value at inner circumference to a min. value at outer circumference.

→ 

p_x = Radial pressure

f_x = Hoop stress.

Now,

$$\boxed{p_x = \frac{b}{x^2} - a}$$

$$\boxed{f_x = \frac{b}{x^2} + a}$$

These two equations are called <u>Lame's equations</u> and constant (a) & (b) are obtained from the boundary conditions.

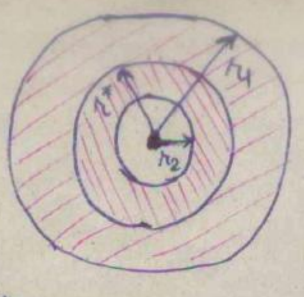
(i) $x = r_2$, p_x = pressure of fluid inside the cylinder

(ii) $x = r_1$, $p_x = 0$, atmospheric pressure.

r_1 = External & r_2 = internal radius

x = at any radius.

Stresses on compound thick cylinders



- The hoop stresses in a compound thick cylinder is the algebraic sum of the hoop stresses caused due to shrinking & those due to internal fluid pressure.
- The hoop stresses in case of thin cylinder are reduced by wire winding on the cylinders.
- The hoop stresses in case of thick cylinder are reduced by shrinking one cylinder over another cylinder.

$\Longrightarrow$ the original difference of radii at the junction of compound cylinder for shrinkage is given by

$$\boxed{dr = \frac{2r^*}{E}(a_1 - a_2)}$$

r = radius of junction after shrinking

a_1 & a_2 are consts. obtained from boundary conditions

(i) $x = r_1$, $p_x = 0$

(ii) $x = r^*$, $p_x = p^*$ = Radial pressure at jun.

$\boxed{\text{max. } f_c \text{ at } d_i > \text{fluid pressure}}$

for a thick - spherical shell

$$p_x = \frac{2b}{x^3} - a$$

$$f_x = \frac{b}{x^3} + a$$

STRAIN ENERGY & IMPACT LOADING

- whenever a body is strained, the energy is absorbed in the body, this energy due to straining effect is called strain energy.
- The strain energy stored in the body is equal to the work done by the applied load in stretching the body.

Resilience:- The total energy stored in a body is known as Resilience

Proof Resilience :- The max. energy stored in a body is known as proof resilience.

$$\boxed{\text{modulus of Resilience :- } \frac{\text{Proof Resilience}}{\text{Volume of body}}}$$

★ Energy stored in body when load is gradually applied.

$$\boxed{U = \frac{p^2}{2E} \times V}$$

p = stress
E = modulus of elasticity
V = volume of body

★ $\boxed{\text{proof resilience :- } \frac{p^{*2}}{2E} \times V}$

p^* = stress at elastic limit

★ Modulus of resilience $= \dfrac{\frac{p^2}{2E} \times V}{V}$

$$\boxed{\text{Modulus of resilience} = \frac{p^2}{2E}}$$

stretch or extension :
$$x = \frac{p}{E} \times L$$

when load is suddenly applied:

- max. instantaneous stress

$$\boxed{p = 2 \times \frac{P}{A}}$$

- $\boxed{x = \frac{p}{E} \times L}$ = Extension
- $\boxed{U = \frac{p^2}{2E} \times V}$

strain energy in stepped bar $>$ strain energy in uniform bar

A bar of uniform c/s 'A' and length hangs vertically, subjected to its own weight. Then strain energy

$$\boxed{U = \frac{A \times \rho^2 \times L^3}{6E}}$$

ρ = weight / unit vol

E = modulus of elasticity

when impact load is applied

p = max. stress induced

$$\boxed{p = \frac{P}{A}\left(1 + \sqrt{1 + \frac{2AEh}{P \cdot L}}\right)}$$

h = height through which load falls

L = length of body

P = Applied impact load

- when δL is so small compared to h

The work done by load $= P \times h = \frac{p^2}{2E} A \times L$

$$\boxed{\frac{\delta L}{L} = \frac{p}{E}}$$

$$\therefore \boxed{p = \sqrt{\frac{2EPh}{A \cdot L}}}$$

if $h=0$

$$p = \frac{P}{A}\left(1+\sqrt{1+0}\right)$$

$$\boxed{p = \frac{2P}{A}}$$

- Instantaneous stress = E X Instantaneous strain

$$\boxed{p = E \times \frac{\delta L}{L}}$$

→ Total distance through which load is falling = $h + \delta L$

⟹ workdone = Total distance x P = $\frac{p^2}{2E} \times V$

Strain energy due to shear stresses

strain energy stored = $\frac{q^2}{2C} \times V$

|| C.G & M.I ||

- **Centre of Gravity**

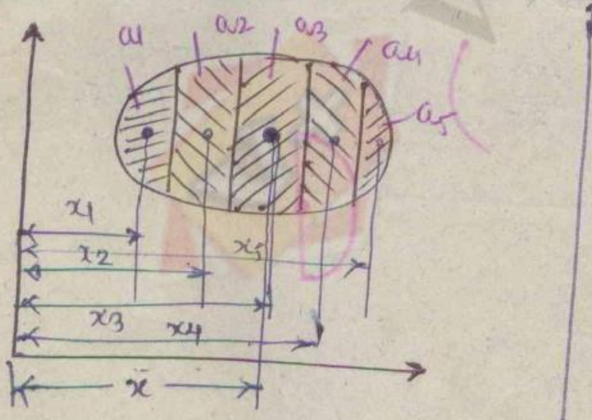
C.G of a body is the point through which the whole weight of the body acts.

- **Centroid**

The point at which the total area of a plane figure (rectangle, square, triangle) is assummed to be concentrated is known as centroid of that area.

☆ Centroid & C.G are at same point.

* C.G of uniform rod lies at its middle point.
* C.G of rectangle or parallelogram lies at point where its diagonal meet each other.
* C.G of triangle lies at the point where the three median of triangle meet.
* C.G of circle is at its centre.

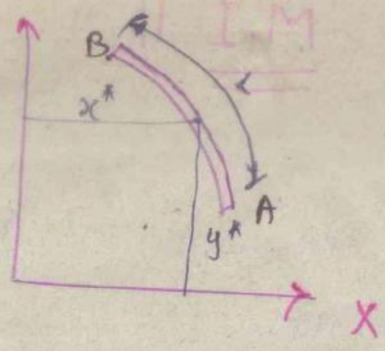


$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + \dots}{a_1 + a_2 + a_3 + \dots}$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3 + a_4y_4 + \dots}{a_1 + a_2 + a_3 + \dots}$$

$$\boxed{\bar{x} = \frac{\Sigma a_i x_i}{\Sigma a_i}} \qquad \boxed{\bar{y} = \frac{\Sigma a_i y_i}{\Sigma a_i}}$$

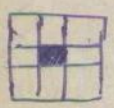
★ C.G of line



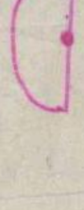
$$\boxed{\bar{x} = \frac{\int x^* \, dL}{\int dL}}$$

$$\boxed{\bar{y} = \frac{\int y^* \, dL}{\int dL}}$$

- The axis about which moments of areas are taken, is known as axis of reference.
- The axis of reference of plane figures, is generally taken as the lowest line of figure for determining $\bar{y}$ & left line of the figure for $\bar{x}$
- If the given section is symmetrical about X-X Axis or Y-Y Axis, then C.G of the section will lie on the axis of symmetry



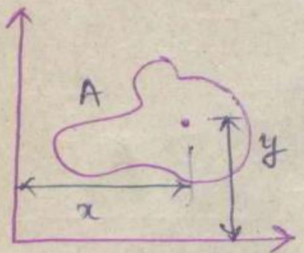
for h.circle, C.G is at $\left(r - \frac{4r}{3\pi} \right)$ from curvature part.



M.I

Moment of Inertia :

" Product of area (or mass) and square of the distance of the centre of gravity of area (or mass) from an axis is known as moment of inertia of area (or mass) about that axis. "



$\therefore$ moment of area

$= A \times x$

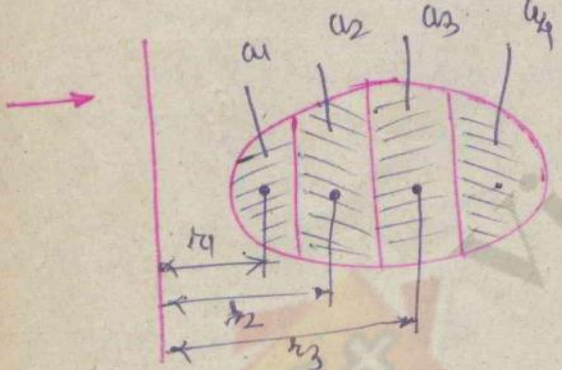
$=$ First moment of area

& $= Ax^2$

$=$ Second moment of area

$=$ M.O. area

$=$ M.O. Inertia.



$\therefore$ M.O.I $= I$

$= a_1 r_1^2 + a_2 r_2^2 + a_3 r_3^2 + \ldots$

$$\boxed{I = \Sigma a r^2}$$

⇒ RADIUS OF GYRATION

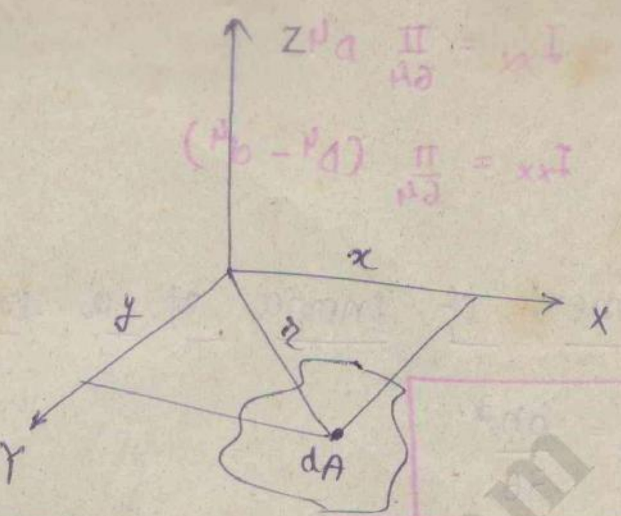
" Radius of Gyration of a body is defined as the distance from an axis of reference where the whole mass of the body is assumed to be concentrated. so as not to alter the m.o.i about given Axis.

$I = a_1 r_1^2 + a_2 r_2^2 + a_3 r_3^2$

$\boxed{I = AK^2}$ $\boxed{I = mK^2}$

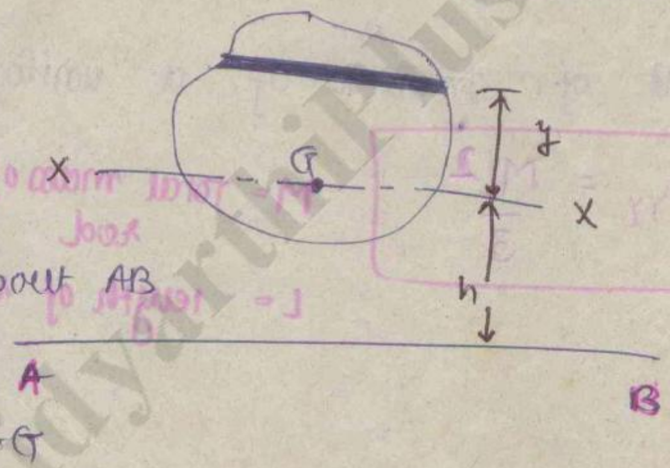
THEOREM OF PERPENDICULAR AXIS

$$\boxed{I_{zz} = I_{xx} + I_{yy}}$$



THEOREM OF PARALLEL AXIS :

$$\boxed{I_{AB} = I_G + Ah^2}$$

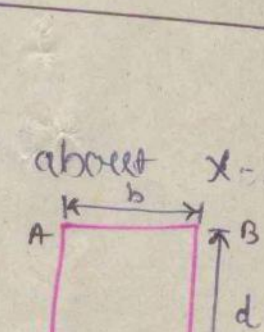


I_{AB} = m.o.i of section about AB

I_G = m.o.i of section about C.G

✲ M.O.I of Rectangle :

- moment of Inertia of Rectangle about X-X axis passing through the C.G of the section



$$I_{YY} = \frac{db^3}{12}$$

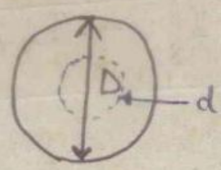
$$I_{XX} = bd^3/12$$

- moment of Inertia of Rectangle about a line passing through the base

$$I_{base} = \frac{bd^3}{3}$$

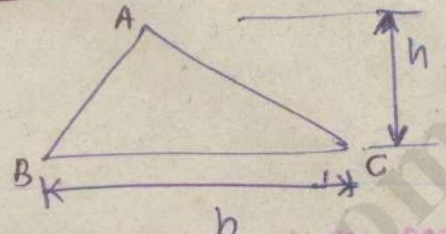
Moment of Inertia of a circular section:-

$$I_{xx} = \frac{\pi}{64} D^4$$



$$I_{xx} = \frac{\pi}{64} (D^4 - d^4)$$

Moment of Inertia of a triangle:-



$$\boxed{I_G = \frac{bh^3}{36}}$$

$$\boxed{I_{BC} = \frac{bh^3}{12}}$$

Moment of Inertia of a uniform thin rod

$$\boxed{I_{YY} = \frac{ML^2}{3}}$$

M = Total mass of rod

L = length of rod.