UNIT IV
CO-ORDINATE GEOMETRY

Learning Objectives

- To understand the basic concepts of two dimensional coordinate geometry with points and straight lines.
- To learn different forms of straight lines with different methods to understand them.

4.1 POINTS

Cartesian Plane: Let XOX' and YOY' be two perpendicular lines. 'O' be their intersecting point called origin. XOX' is horizontal line called X-axis and YOY' is vertical line called Y-axis. The plane made by these axes is called Cartesian plane or coordinate plane.

The axes divide the plane into four parts called quadrant: 1^{st} quadrant, 2^{nd} quadrant, 3^{rd} quadrant and 4^{th} quadrant as shown in the Fig. 4.1. OX is known as positive direction of X-axis and OX' is known as negative direction of X-axis. Similarly, OY is known as positive direction of Y-axis and OY' is known as negative direction of Y-axis.

The axes XOX' and YOY' are together known as rectangular axes or coordinate axes.

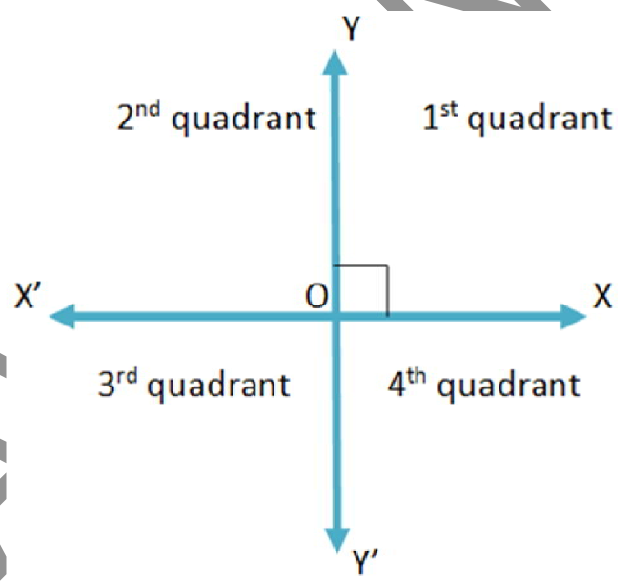

Fig. 4.1

Point: A point is a mark of location on a plane. It has no dimension i.e. no length, no breadth and no height. For example, tip of pencil, toothpick etc. A point in a plane is represented as an ordered pair of real numbers called coordinates of point.

The perpendicular distance of a point from the Y-axis is $P(x, y)$ called abscissa or x-coordinate and the perpendicular distance of a point from the X-axis is called ordinate or y-coordinate. If $P(x, y)$ be any point in the plane (see Fig. 4.2) then x is the abscissa of the point P and y is the ordinate of the point P.

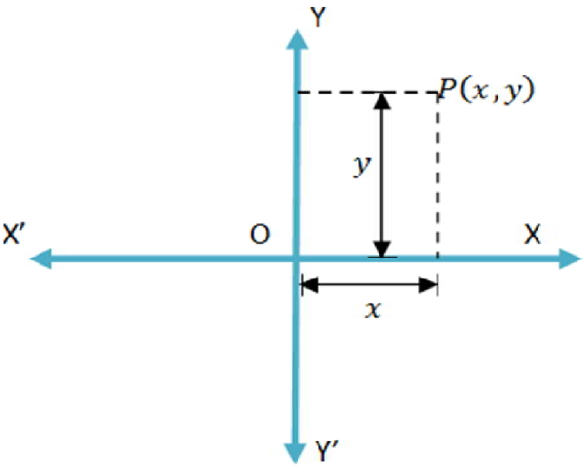

Fig. 4.2

Note: **(i)** If distance along X-axis is measured to the right of Y-axis then it is positive and if it is measured to the left of Y-axis then it is negative.

(ii) If distance along Y-axis is measured to the above of X-axis then it is positive and if it is measured to the below of X-axis then it is negative.

(iii) The coordinates of origin 'O' are $(0,0)$.

(iv) A point on X-axis is represented as $(x, 0)$ i.e. ordinate is zero.

(v) A point on Y-axis is represented as $(0, y)$ i.e. abscissa is zero.

(vi) In the 1st quadrant $x > 0$ and $y > 0$

In the 2nd quadrant $x < 0$ and $y > 0$

In the 3rd quadrant $x < 0$ and $y < 0$

In the 4th quadrant $x > 0$ and $y < 0$.

Distance between Two Points in a Plane: Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be any two points in a plane then the distance between these points is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 1. Plot the following points and find the quadrant in which they lie:

(i) $A(2,2)$ (ii) $B(-3,-1)$ (iii) $C(-1,3)$ (iv) $D(3,-2)$

Sol.

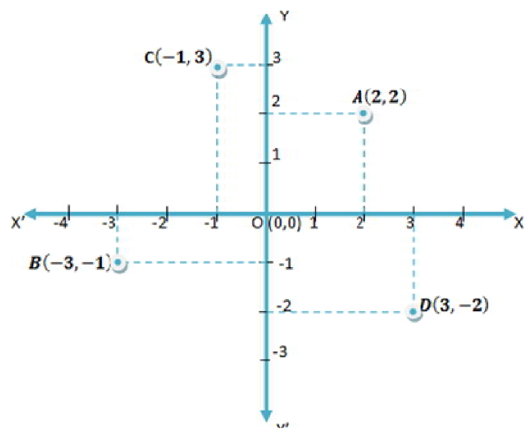

Fig. 4.3

In the Fig. 4.3, it is clear that

(i) Point $\boldsymbol{A}(\mathbf{2},\mathbf{2})$ lies in the 1st quadrant.
(ii) Point $\boldsymbol{B}(-\mathbf{3},-\mathbf{1})$ lies in the 3rd quadrant.
(iii) Point $\boldsymbol{C}(-\mathbf{1},\mathbf{3})$ lies in the 2nd quadrant.
(iv) Point $\boldsymbol{D}(\mathbf{3},-\mathbf{2})$ lies in the 4th quadrant.

Example 2. Without plotting, find the quadrant in which the following points lie:

(i) $A(2,-3)$	(ii) $B(-5,-6)$	(iii) C$(4,3)$	(iv) $D(-1,5)$
(v) $E(0,9)$	(vi) $F(-3,0)$	(vii) G$(0,-7)$	(viii) $H(1,0)$

Sol.

(i) The given point is $A(2,-3)$

Here X-coordinate $= 2$, which is positive and Y-coordinate $= -3$, which is negative.

Hence the point $A(2,-3)$ lies in 4th quadrant.

(ii) The given point is $B(-5,-6)$

Here X-coordinate $= -5$, which is negative and Y-coordinate $= -6$, which is also negative.

Hence the point $B(-5,-6)$ lies in 3rd quadrant.

(iii) The given point is C$(4,3)$

Here X-coordinate = 4 > 0 and Y-coordinate = 3 > 0.

Hence the point C$(4,3)$ lies in 1st quadrant.

(iv) The given point is $D(-1,5)$

Here X-coordinate $= -1$ < 0 and Y-coordinate = 5 > 0.

Hence the point $D(-1,5)$ lies in 2nd quadrant.

(v) The given point is $E(0,9)$

Here X-coordinate = 0 and Y-coordinate = 9 > 0.

Hence the point $E(0,9)$ lies on Y-axis above the origin.

(vi) The given point is $F(-3,0)$

Here X-coordinate $= -3 < 0$ and Y-coordinate = 0.

Hence the point $F(-3,0)$ lies on X-axis left to origin.

(vii) The given point is $G(0,-7)$

Here X-coordinate = 0 and Y-coordinate $= -7 < 0$.

Hence the point $G(0,-7)$ lies on Y-axis below the origin.

(viii) The given point is $H(1,0)$

Here X-coordinate = $1 > 0$ and Y-coordinate = 0.

Hence the point $H(1,0)$ lies on X-axis right to origin.

Example 3. Find the distance between the following pairs of points:

(i) $(0,5),(3,6)$ (ii) $(-1,2),(4,3)$ (iii) $(2,0),(-3,-2)$

(iv) $(1,2),(4,5)$ (v) $(-2,3),(-5,7)$ (vi) $(-1,-3),(-2,-4)$

(vii) $(a-b,c-d),(-b+c,c+d)$

(viii) $(sin\theta, cos\theta),(-sin\theta, cos\theta)$

Sol.

(i) Let **A** represents the point $(0,5)$ and **B** represents the point $(3,6)$.

So, the distance between **A** and **B** is:

$$AB=\sqrt{(3-0)^2+(6-5)^2}$$
$$=\sqrt{(3)^2+(1)^2}=\sqrt{9+1}=\sqrt{10}\ units$$

(ii) Let **A** represents the point $(-1,2)$ and **B** represents the point $(4,3)$.

So, the distance between **A** and **B** is:

$$AB=\sqrt{(4-(-1))^2+(3-2)^2}$$
$$=\sqrt{(5)^2+(1)^2}=\sqrt{25+1}=\sqrt{26}\ units$$

(iii) Let **A** represents the point $(2,0)$ and **B** represents the point $(-3,-2)$.

So, the distance between **A** and **B** is:

$$AB=\sqrt{(-3-2)^2+(-2-0)^2}$$
$$=\sqrt{(-5)^2+(-2)^2}=\sqrt{25+4}=\sqrt{29}\ units$$

(iv) Let **A** represents the point $(1,2)$ and **B** represents the point $(4,5)$.

So, the distance between **A** and **B** is:

$$AB=\sqrt{(4-1)^2+(5-2)^2}$$
$$=\sqrt{(3)^2+(3)^2}=\sqrt{9+9}$$
$$=\sqrt{18}=\sqrt{9\times 2}=3\sqrt{2}\ units$$

(v) Let **A** represents the point $(-2,3)$ and **B** represents the point $(-5,7)$.

So, the distance between **A** and **B** is:

$$AB=\sqrt{(-5-(-2))^2+(7-3)^2}$$
$$=\sqrt{(-5+2)^2+(4)^2}=\sqrt{(-3)^2+(4)^2}$$

$$=\sqrt{9+16}=\sqrt{25}=5\,units$$

(vi) Let **A** represents the point $(-1,-3)$ and **B** represents the point $(-2,-4)$.

So, the distance between **A** and **B** is:

$$AB=\sqrt{(-2-(-1))^2+(-4-(-3))^2}$$
$$=\sqrt{(-2+1)^2+(-4+3)^2}=\sqrt{(-1)^2+(-1)^2}$$
$$=\sqrt{1+1}=\sqrt{2}\,units$$

(vii) Let **A** represents the point $(a-b,c-d)$ and **B** represents the point $(-b+c,c+d)$.

So, the distance between **A** and **B** is:

$$AB=\sqrt{(-b+c-(a-b))^2+(c+d-(c-d))^2}$$
$$=\sqrt{(-b+c-a+b)^2+(c+d-c+d)^2}$$
$$=\sqrt{(c-a)^2+(d+d)^2}=\sqrt{c^2+a^2-2ac+(2d)^2}$$
$$=\sqrt{c^2+a^2+4d^2-2ac}\;\;units$$

(viii) Let **A** represents the point $(sin\theta, cos\theta)$ and **B** represents the point $(-sin\theta, cos\theta)$.

So, the distance between **A** and **B** is:

$$AB=\sqrt{(-\sin\theta-\sin\theta)^2+(\cos\theta-\cos\theta)^2}$$
$$=\sqrt{(-2\sin\theta)^2+(0)^2}=\sqrt{4\sin^2\theta}$$
$$=2\sin\theta\,units$$

Example 4. Using distance formula, prove that the triangle formed by the points $A(4,0)$, $B(-1,-1)$ and $C(3,5)$ is an isosceles triangle.

Sol. Given that vertices of the triangle are $A(4,0)$, $B(-1,-1)$ and $C(3,5)$.

To find the length of edges of the triangle, we will use the distance formula:

Distance between **A** and **B** is

$$AB=\sqrt{(4-(-1))^2+(0-(-1))^2}$$
$$=\sqrt{(5)^2+(1)^2}=\sqrt{25+1}=\sqrt{26}\,units$$

Distance between **B** and **C** is

$$BC=\sqrt{(-1-3)^2+(-1-5)^2}$$
$$=\sqrt{(-4)^2+(-6)^2}=\sqrt{16+36}=\sqrt{52}\,units$$

Distance between **A** and **C** is

$$AC=\sqrt{(4-3)^2+(0-5)^2}$$
$$=\sqrt{(1)^2+(-5)^2}=\sqrt{1+25}=\sqrt{26}\,units$$

We can see that $AB=AC\neq BC$

Hence the triangle formed by the points $A(4,0)$, $B(-1,-1)$ and $C(3,5)$ is an isosceles triangle.

Example 5. Using distance formula, prove that the triangle formed by the points $A(0,0)$, $B(0,2)$ and $C(\sqrt{3},1)$ is an equilateral triangle.

Sol. Given that vertices of the triangle are $A(0,0)$, $B(0,2)$ and $C(\sqrt{3},1)$.

To find the length of edges of the triangle, we will use the distance formula:

Distance between **A** and **B** is

$$AB=\sqrt{(0-0)^2+(0-2)^2}$$
$$=\sqrt{(0)^2+(-2)^2}=\sqrt{0+4}=\sqrt{4}=2\,units$$

Distance between **B** and **C** is

$$BC=\sqrt{\left(0-\sqrt{3}\right)^2+(2-1)^2}$$
$$=\sqrt{\left(-\sqrt{3}\right)^2+(1)^2}=\sqrt{3+1}=\sqrt{4}=2\,units$$

Distance between **A** and **C** is

$$AC=\sqrt{\left(0-\sqrt{3}\right)^2+(0-1)^2}$$
$$=\sqrt{\left(-\sqrt{3}\right)^2+(-1)^2}=\sqrt{3+1}=\sqrt{4}=2\,units$$

We can see that $AB=BC=AC$

Hence the triangle formed by the points $A(0,0)$, $B(0,2)$ and $C(\sqrt{3},1)$ is an equilateral triangle.

<u>Mid-point between two points</u>: If $A(x_1,y_1)$ and $B(x_2,y_2)$ be any two points then the mid-point between these points is given by:

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

Example 6. Find the mid points between the following pairs of points:

(i) $(2,3)$, $(8,5)$ (ii) $(6,3)$, $(6,-9)$ (iii) $(-2,-4)$, $(3,-6)$

(iv) $(0,8)$, $(6,0)$ (v) $(0,0)$, $(-12,10)$ (vi) (a,b) , (c,d)

(vii) $(a+b,c-d)$, $(-b+3a,c+d)$

Sol.

(i) The given points are $(2,3)$ and $(8,5)$.

So, the mid-point between these points is given by:

$$\left(\frac{2+8}{2},\frac{3+5}{2}\right)=\left(\frac{10}{2},\frac{8}{2}\right)=(5,4)$$

(ii) The given points are $(6,3)$ and $(6,-9)$.

So, the mid-point between these points is given by:

$$\left(\frac{6+6}{2},\frac{3+(-9)}{2}\right)=\left(\frac{12}{2},\frac{3-9}{2}\right)=\left(\frac{12}{2},\frac{-6}{2}\right)=(6,-3)$$

(iii) The given points are $(-2,-4)$ and $(3,-6)$.

So, the mid-point between these points is given by:

$$\left(\frac{-2+3}{2},\frac{-4+(-6)}{2}\right)=\left(\frac{1}{2},\frac{-4-6}{2}\right)=\left(\frac{1}{2},\frac{-10}{2}\right)=\left(\frac{1}{2},-5\right)$$

(iv) The given points are $(0,8)$ and $(6,0)$.

So, the mid-point between these points is given by:

$$\left(\frac{0+6}{2},\frac{8+0}{2}\right)=\left(\frac{6}{2},\frac{8}{2}\right)=(3,4)$$

(v) The given points are $(0,0)$ and $(-12,10)$.

So, the mid-point between these points is given by:

$$\left(\frac{0+(-12)}{2},\frac{0+10}{2}\right)=\left(\frac{-12}{2},\frac{10}{2}\right)=(-6,5)$$

(vi) The given points are (a,b) and (c,d).

So, the mid-point between these points is given by:

$$\left(\frac{a+c}{2},\frac{b+d}{2}\right)$$

(vii) The given points are $(a+b,c-d)$ and $(-b+3a,c+d)$.

So, the mid-point between these points is given by:

$$\left(\frac{a+b-b+3a}{2},\frac{c-d+c+d}{2}\right)=\left(\frac{4a}{2},\frac{2c}{2}\right)=(2a,c)$$

Example 7. If the mid-point between two points is $(3,5)$ and one point between them is $(-1,2)$, find the other point.

Sol. Let the required point is (a,b).

So, according to given statement $(3,5)$ is the mid-point of $(-1,2)$ and (a,b).

$$\Rightarrow \quad (3,5)=\left(\frac{-1+a}{2},\frac{2+b}{2}\right)$$

$$\Rightarrow \quad \frac{-1+a}{2}=3 \quad \& \quad \frac{2+b}{2}=5$$

$$\Rightarrow \quad -1+a=6 \quad \& \quad 2+b=10$$

$$\Rightarrow \quad a=7 \quad \& \quad b=8$$

Hence the required point is $(7,8)$.

Example 8. If the mid-point between two points is $(-7,6)$ and one point between them is $(3,-9)$, find the other point.

Sol. Let the required point is (a,b).

So, according to given statement $(-7,6)$ is the mid-point of $(3,-9)$ and (a,b).

$$\Rightarrow \quad (-7,6)=\left(\frac{3+a}{2},\frac{-9+b}{2}\right)$$

$$\Rightarrow \quad \frac{3+a}{2}=-7 \quad \& \quad \frac{-9+b}{2}=6$$

$$\Rightarrow \quad 3+a=-14 \quad \& \quad -9+b=12$$

$$\Rightarrow \quad a=-17 \quad \& \quad b=21$$

Hence the required point is $(-17,21)$.

Centroid of a Triangle: The centroid of a triangle is the intersection point of the three medians of the triangle. In other words, the **average** of the three vertices of the triangle is called the centroid of the triangle.

i.e. If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are three vertices of a triangle then the centroid of the triangle is given by:

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

In the Fig. 4.4, the point G is the centroid of the Triangle.

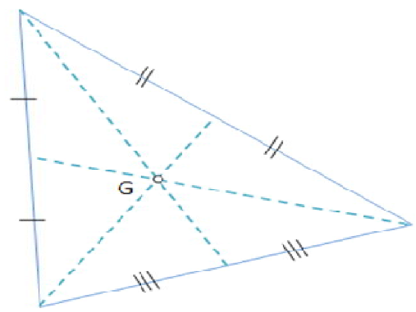

Fig. 4.4

Example 9. Vertices of the triangles are given below, find the centroid of the triangles:

(i) $(5,2)$, $(5,4)$, $(8,6)$ (ii) $(4,-3)$, $(-4,8)$, $(5,7)$

(iii) $(2,-4)$, $(0,-10)$, $(4,5)$ (iv) $(9,-9)$, $(5,8)$, $(-7,-2)$

Sol.

(i) The given vertices of the triangle are $(5,2)$, $(5,4)$ and $(8,6)$.

So, the centroid of the triangle is

$$\left(\frac{5+5+8}{3}, \frac{2+4+6}{3}\right) = \left(\frac{18}{3}, \frac{12}{3}\right) = (6,4)$$

(ii) The given vertices of the triangle are $(4,-3)$, $(-4,8)$ and $(5,7)$.

So, the centroid of the triangle is

$$\left(\frac{4-4+5}{3}, \frac{-3+8+7}{3}\right) = \left(\frac{5}{3}, \frac{12}{3}\right) = \left(\frac{5}{3}, 4\right)$$

(iii) The given vertices of the triangle are $(2,-4)$, $(0,-10)$ and $(4,5)$.

So, the centroid of the triangle is

$$\left(\frac{2+0+4}{3}, \frac{-4-10+5}{3}\right) = \left(\frac{6}{3}, \frac{-9}{3}\right) = (2,-3)$$

(iv) The given vertices of the triangle are $(9,-9)$, $(5,8)$ and $(-7,-2)$.

So, the centroid of the triangle is

$$\left(\frac{9+5-7}{3}, \frac{-9+8-2}{3}\right) = \left(\frac{7}{3}, \frac{-3}{3}\right) = \left(\frac{7}{3}, -1\right)$$

Example 10. If centroid of the triangle is $(10,18)$ and two vertices of the triangle are $(1,-5)$ and $(3,7)$, find the third vertex of the triangle.

Sol. Let the required vertex of the triangle is (a, b).

So, according to given statement and definition of centroid, we get

$$\Rightarrow \quad (10,18)=\left(\frac{1+3+a}{3},\frac{-5+7+b}{3}\right)$$

$$\Rightarrow \quad \frac{1+3+a}{3}=10 \quad \& \quad \frac{-5+7+b}{3}=18$$

$$\Rightarrow \quad 1+3+a=30 \quad \& \quad -5+7+b=54$$

$$\Rightarrow \quad a=26 \quad \& \quad b=52$$

Hence the required vertex of triangle is $(26,52)$.

Example 11. If centroid of the triangle is $(-5,-7)$ and two vertices of the triangle are $(0,6)$ and $(-3,2)$, find the third vertex of the triangle.

Sol. Let the required vertex of the triangle is (a,b).

So, according to given statement and definition of centroid, we get

$$\Rightarrow \quad (-5,-7)=\left(\frac{0-3+a}{3},\frac{6+2+b}{3}\right)$$

$$\Rightarrow \quad \frac{0-3+a}{3}=-5 \quad \& \quad \frac{6+2+b}{3}=-7$$

$$\Rightarrow \quad 0-3+a=-15 \quad \& \quad 6+2+b=-21$$

$$\Rightarrow \quad a=-12 \quad \& \quad b=-29$$

Hence the required vertex of triangle is $(-12,-29)$.

Example 12. If centroid of a triangle formed by the points $(1,a)$, $(9,b)$ and $(c^2,-5)$ lies on the X-axis, prove that $a+b=5$.

Sol. Given that vertices of the triangle are $(1,a)$, $(9,b)$ and $(c^2,-5)$.

Centroid of the triangle is given by

$$\left(\frac{1+9+c^2}{3},\frac{a+b-5}{3}\right)$$

By given statement, the centroid of the triangle lies on the X-axis.

Therefore, Y-coordinate of centroid is zero.

$$\Rightarrow \quad \frac{a+b-5}{3}=0$$

$$\Rightarrow \quad a+b-5=0$$

$$\Rightarrow \quad a+b=5$$

Example 13. If centroid of a triangle formed by the points $(-a,a)$, (c^2,b) and $(d,5)$ lies on the Y-axis, prove that $c^2=a-d$.

Sol. Given that vertices of the triangle are $(-a,a)$, (c^2,b) and $(d,5)$.

Centroid of the triangle is given by

$$\left(\frac{-a+c^2+d}{3},\frac{a+b+5}{3}\right)$$

By given statement, the centroid of the triangle lies on the Y-axis.

Therefore, X-coordinate of centroid is zero.

$\Rightarrow \quad \dfrac{-a+c^2+d}{3}=0$

$\Rightarrow \quad -a+c^2+d=0$

$\Rightarrow \quad c^2=a-d$

Hence proved.

EXERCISE-I

1. The point $(-3,-4)$ lies in the quadrant:
   (a) First (b) Second (c) Third (d) Fourth
2. The point $(7,4)$ lies in the quadrant
   (a) First (b) Second (c) Third (d) Fourth
3. Find the distance between the following pairs of points:
   (i) $(-1,2)$, $(4,3)$ (ii) $(a-b, c-d)$, $(-b+c, c+d)$
4. Find the mid points between the following pairs of points:
   (i) $(0,8)$, $(6,0)$ (ii) $(a+b, c-d)$, $(-b+3a, c+d)$
5. The mid-point between two points is $(3,5)$ and one point between them is $(-1,2)$. Find the other point.
6. Find the centroids of triangles whose vertices are:
   (i) $(4,-3)$, $(-4,8)$, $(5,7)$ (ii) $(9,-9)$, $(5,8)$, $(-7,-2)$

ANSWERS

1. (c)
2. (a)
3. (i) $\sqrt{26}$, (ii) $\sqrt{c^2+a^2+4d^2-2ac}$
4. (i) $(3,4)$, (ii) $(2a, c)$
5. $(7,8)$
6. (i) $\left(\frac{5}{3}, 4\right)$, (ii) $\left(\frac{7}{3}, -1\right)$

4.2 STRAIGHT LINES

Definition of Straight Line: A path traced by a moving point travelling in a constant direction is called a straight line.

OR

The shortest distance between two points in a plane is called a straight line.

General Equation of Straight Line: A straight line in XY plane has general form

$$ax+by+c=0$$

where a is the coefficient of x, b is the coefficient of y and c is the constant term.

Note: **(i)** Any point (x_1, y_1) lies on the line $ax + by + c = 0$ if it satisfies the equations of the line **i.e.** if we substitute the values x_1 at the place of x and y_1 at the place of y in the equation of line, the result $ax_1 + by_1 + c$ becomes zero.

(ii) X-axis is usually represented horizontally and its equation is $y = 0$.

(iii) Y-axis is usually represented vertically and its equation is $x = 0$.

(vi) $x = k$ represents the line parallel to Y-axis, where k is some constant .

(v) $y = k$ represents the line parallel to X-axis, where k is some constant .

Slope of a Straight Line: Slope of straight line measures how slanted the line is relative to the horizontal (see Fig. 4.5). It is usually represented by $\boldsymbol{m}$.

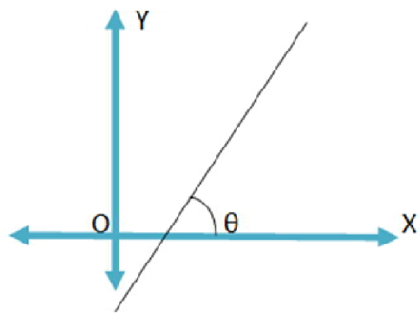

Fig. 4.5

To find Slope of a Straight Line:

(i) If a line making an angle θ with positive X-axis then the slope m of the line is given by $m = tan\theta$.

(ii) If a line passes through two points (x_1, y_1) and (x_2, y_2) then the slope m of the line is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$.

(iii) If equation of a straight line is $ax + by + c = 0$, then its slope m is given by $m = -\frac{a}{b}$.

Note: **(i)** Slope of a horizontal line is always zero **i.e.** slope of a line parallel to X-axis is zero as $m = tan0° = 0$.

(ii) Slope of a vertical line is always infinity **i.e.** slope of a line perpendicular to X-axis is infinity as $= tan90° = \infty$.

(iii) Let L_1 and L_2 represents two straight lines. Let m_1 and m_2 be slopes of L_1 and L_2 respectively. We say that L_1 and L_2 are parallel lines iff $m_1 = m_2$ **i.e.** slopes are equal. We say that L_1 and L_2 are perpendicular iff $m_1 . m_2 = -1$ **i.e.** product of slopes is equal to -1 (*except the cases of axes and lines parallel to axes*).

Example 14. Find the slope of the straight lines which make following angles:

(i) 45° (ii) 120° (iii) 30° (iv) 150° (v) 210°

with the positive direction of X-axis.

Sol.

(i) Let m be the slope of the straight line and θ be the angle which the straight line makes with the positive direction of X-axis.

Therefore $\theta = 45°$ and $m = tan\theta$

$\Rightarrow \quad m = \tan 45^\circ$

$\Rightarrow \quad m = 1$

which is the required slope.

(ii) Let m be the slope of the straight line and θ be the angle which the straight line makes with the positive direction of X-axis.

Therefore $\theta = 120°$ and $m = tan\theta$

$\Rightarrow \quad m = \tan 120^\circ$

$\Rightarrow \quad m = \tan\left(180^\circ - 60^\circ\right)$

$\Rightarrow \quad m = -\tan\left(60^\circ\right)$

$\Rightarrow \quad m = -\sqrt{3}$

which is the required slope.

(iii) Let m be the slope of the straight line and θ be the angle which the straight line makes with the positive direction of X-axis.

Therefore $\theta = 30°$ and $m = tan\theta$

$\Rightarrow \quad m = \tan 30^\circ$

$\Rightarrow \quad m = \dfrac{1}{\sqrt{3}}$

which is the required slope.

(iv) Let m be the slope of the straight line and θ be the angle which the straight line makes with the positive direction of X-axis.

Therefore $\theta = 150°$ and $m = tan\theta$

$\Rightarrow \quad m = \tan 150^\circ$

$\Rightarrow \quad m = \tan\left(180^\circ - 30^\circ\right)$

$\Rightarrow \quad m = -\tan\left(30^\circ\right)$

$\Rightarrow \quad m = -\dfrac{1}{\sqrt{3}}$

which is the required slope.

(v) Let m be the slope of the straight line and θ be the angle which the straight line makes with the positive direction of X-axis.

Therefore $\theta = 210°$ and $m = tan\theta$

$\Rightarrow \quad m = \tan 210^\circ$

$\Rightarrow \quad m = \tan\left(180^\circ + 30^\circ\right)$

$\Rightarrow \quad m = \tan\left(30^\circ\right)$

$\Rightarrow \quad m = \frac{1}{\sqrt{3}}$

which is the required slope.

Example 15. Find the slope of the straight lines which pass through the following pairs of points:

(i) $(2,5)$, $(6,17)$ (ii) $(-8,7)$, $(3,-5)$

(iii) $(0,-6)$, $(7,9)$ (iv) $(-11,-5)$, $(-3,-10)$

(v) $(0,0)$, $(10,-12)$.

Sol.

(i) Given that the straight line passes through the points $(2,5)$ and $(6,17)$.
Comparing these points with (x_1, y_1) and (x_2, y_2) respectively, we get
$x_1 = 2$, $y_1 = 5$, $x_2 = 6$ and $y_2 = 17$
Let m be the slope of the straight line.

Therefore $m = \frac{y_2 - y_1}{x_2 - x_1}$

$\Rightarrow \quad m = \frac{17-5}{6-2} = \frac{12}{4}$

$\Rightarrow \quad m = 3$

which is the required slope.

(ii) Given that the straight line passes through the points $(-8,7)$ and $(3,-5)$.

Comparing these points with (x_1, y_1) and (x_2, y_2) respectively, we get

$x_1 = -8$, $y_1 = 7$, $x_2 = 3$ and $y_2 = -5$

Let m be the slope of the straight line.

Therefore $m = \frac{y_2 - y_1}{x_2 - x_1}$

$\Rightarrow \quad m = \frac{-5-7}{3-(-8)} = \frac{-12}{3+8}$

$\Rightarrow \quad m = -\frac{12}{11}$

which is the required slope.

(iii) Given that the straight line passes through the points $(0,-6)$ and $(7,9)$.

Comparing these points with (x_1, y_1) and (x_2, y_2) respectively, we get

$x_1 = 0$, $y_1 = -6$, $x_2 = 7$ and $y_2 = 9$

Let m be the slope of the straight line.

Therefore $m = \frac{y_2 - y_1}{x_2 - x_1}$

$\Rightarrow \quad m = \frac{9-(-6)}{7-0} = \frac{9+6}{7}$

$\Rightarrow \quad m = \frac{15}{7}$

which is the required slope.

(iv) Given that the straight line passes through the points $(-11,-5)$ and $(-3,-10)$.

Comparing these points with (x_1, y_1) and (x_2, y_2) respectively, we get

$x_1 = -11$, $y_1 = -5$, $x_2 = -3$ and $y_2 = -10$

Let m be the slope of the straight line.

Therefore $m = \frac{y_2 - y_1}{x_2 - x_1}$

$\Rightarrow \quad m = \frac{-10-(-5)}{-3-(-11)} = \frac{-10+5}{-3+11}$

$\Rightarrow \quad m = -\frac{5}{8}$

which is the required slope.

(v) Given that the straight line passes through the points $(0,0)$ and $(10,-12)$.

Comparing these points with (x_1, y_1) and (x_2, y_2) respectively, we get

$x_1 = 0$, $y_1 = 0$, $x_2 = 10$ and $y_2 = -12$

Let m be the slope of the straight line.

Therefore $m = \frac{y_2 - y_1}{x_2 - x_1}$

$\Rightarrow \quad m = \frac{-12-0}{10-0} = \frac{-12}{10}$

$\Rightarrow \quad m = -\frac{6}{5}$

which is the required slope.

Example 16. Find the slopes of the following straight lines:

(i) $2x + 4y + 5 = 0$ **(ii)** $x - 3y + 9 = 0$

(iii) $5y - 10x + 1 = 0$ **(iv)** $-2x - 6y = 0$

(v) $x = 5$ **(vi)** $y = -6$

Sol.

(i) Given that equation of the straight line is $2x + 4y + 5 = 0$.

Comparing this equation with $ax + by + c = 0$, we get

$a = 2$, $b = 4$ and $c = 5$

Let m be the slope of given straight line.

Therefore, $m = -\frac{a}{b}$

$\Rightarrow \quad m = -\frac{2}{4}$

$\Rightarrow \quad m = -\frac{1}{2}$

which is the required slope.

(ii) Given that equation of the straight line is $-3y + 9 = 0$.
Comparing this equation with $ax + by + c = 0$, we get
$a = 1$, $b = -3$ and $c = 9$
Let m be the slope of given straight line.
Therefore, $m = -\left(\frac{1}{-3}\right)$

$\Rightarrow \quad m = \frac{1}{3}$

which is the required slope.

(iii) Given that equation of the straight line is $5y - 10x + 1 = 0$.
Comparing this equation with $ax + by + c = 0$, we get
$a = -10$, $b = 5$ and $c = 1$
Let m be the slope of given straight line.
Therefore, $m = -\left(\frac{-10}{5}\right)$

$\Rightarrow \quad m = 2$

which is the required slope.

(iv) Given that equation of the straight line is $-2x - 6y = 0$.
Comparing this equation with $ax + by + c = 0$, we get
$a = -2$, $b = -6$ and $c = 0$
Let m be the slope of given straight line.
Therefore, $m = -\frac{a}{b}$

$\Rightarrow \quad m = -\left(\frac{-2}{-6}\right)$

$\Rightarrow \quad m = -\frac{1}{3}$

which is the required slope.

(v) Given that equation of the straight line is $x = 5$.
This equation is parallel to Y-axis.
Hence the slope of the line is infinity.

(vi) Given that equation of the straight line is $y = -6$.
This equation is parallel to X-axis.
Hence the slope of the line is zero.

Example 17. Find the equation of straight line which is parallel to X-axis passes through $(1,5)$.

Sol. Equation of straight line parallel to X-axis is given by

$$y = k \qquad (1)$$

Given that the straight line passes through the point $(1,5)$.

Put $x = 1$ and $y = 5$ in equation (1), we get

$$5 = k$$

So, $y = 5$ be the required equation of straight line.

Example 18. Find the equation of straight line which is parallel to Y-axis passes through $(-3, -7)$.

Sol. Equation of straight line parallel to Y-axis is given by

$$x = k \qquad (1)$$

Given that the straight line passes through the point $(-3, -7)$.

Put $x = -3$ and $y = -7$ in equation (1), we get

$$-3 = k$$

So, $x = -3$ be the required equation of straight line.

<u>Equation of Straight Line Passing Through Origin</u>:

If a line passes through origin and m be its slope. $P(x, y)$ be any point on the line (see Fig. 4.6), then equation of straight line is $\boldsymbol{y = mx}$.

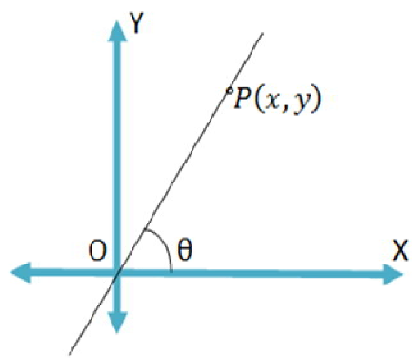

Fig. 4.6

(*except the cases of Y-axis and parallel to Y-axis*)

Example 19. Find the equation of straight line having slope equal to 5 and passes through origin.

Sol. Let m be the slope of required line. Therefore $m = 5$.

Also it is given that the required line passes through the origin.

We know that equation of straight line passes through origin is $y = mx$, where m be the slope of the line.

So, $y = 5x$ be the required equation of straight line.

Example 20. Find the equation of straight line having slope equal to -10 and passes through origin.

Sol. Let m be the slope of required line. Therefore $m = -10$.

Also it is given that the required line passes through the origin.

We know that equation of straight line passes through origin is $y = mx$, where m be the slope of the line.

So, $y = -10x$ be the required equation of straight line.

Example 21. Find the equation of straight line which passes through origin and makes an angle $60°$ with the positive direction of X-axis.

Sol. Let m be the slope of required line.

Therefore $m = \tan 60^\circ$

$\Rightarrow \quad m = \sqrt{3}$

Also it is given that the required line passes through the origin.

We know that equation of straight line passes through origin is $y = mx$, where m be the slope of the line.

So, $y = \sqrt{3}\,x$ be the required equation of straight line.

Example 22. Find the equation of straight line which passes through origin and makes an angle $135°$ with the positive direction of X-axis.

Sol. Let m be the slope of required line.

Therefore $m = \tan 135^\circ$

$\Rightarrow \quad m = \tan\left(180^\circ - 45^\circ\right)$

$\Rightarrow \quad m = -\tan 45^\circ$

$\Rightarrow \quad m = -1$

Also it is given that the required line passes through the origin.

We know that equation of straight line passes through origin is $y = mx$, where m be the slope of the line.

So, $y = -x$ be the required equation of straight line.

<u>**Equation of Straight Line in Point-Slope form**</u>:

If a line passes through a point (x_1, y_1) , m be its slope and $P(x, y)$ be any point on the line (see Fig. 4.7), then equation of straight line is $\boldsymbol{y - y_1 = m(x - x_1)}$.

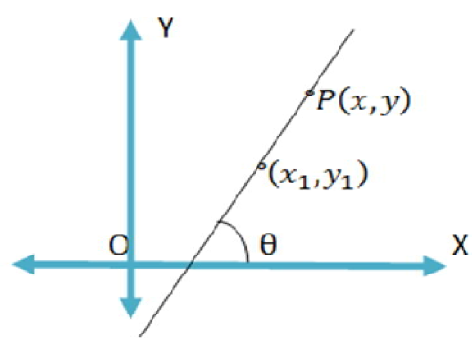

Fig. 4.7

(*except the cases of Y-axis and parallel to Y-axis*)

Example 23. Find the equation of straight line having slope equal to 9 and passes through the point $(1,5)$.

Sol. Let m be the slope of required line. Therefore $m = 9$.

Also it is given that the required line passes through the point $(1,5)$.

We know that equation of straight line in point slope form is $y - y_1 = m(x - x_1)$.

$\Rightarrow \quad y - 5 = 9(x - 1)$

$\Rightarrow \quad y - 5 = 9x - 9$

$\Rightarrow \quad 9x - y - 9 + 5 = 0$

$\Rightarrow \quad 9x - y - 4 = 0$

which is the required equation of straight line.

Example 24. Find the equation of straight line passes through $(-4, -2)$ and having slope -8.

Sol. Let m be the slope of required line. Therefore $m = -8$.

Also it is given that the required line passes through the point $(-4, -2)$.

We know that equation of straight line in point slope form is $y - y_1 = m(x - x_1)$.

$\Rightarrow \quad y - (-2) = -8(x - (-4))$

$\Rightarrow \quad y + 2 = -8(x + 4)$

$\Rightarrow \quad y + 2 = -8x - 32$

$\Rightarrow \quad 8x + y + 2 + 32 = 0$

$\Rightarrow \quad 8x + y + 34 = 0$

which is the required equation of straight line.

Example 25. Find the equation of straight line passes through $(0, -8)$ and makes an angle $30°$ with positive direction of X-axis.

Sol. Let m be the slope of required line.

Therefore $m = \tan 30^\circ$

$\Rightarrow \quad m = \frac{1}{\sqrt{3}}$

Also it is given that the required line passes through the point $(0, -8)$.

We know that equation of straight line in point slope form is $y - y_1 = m(x - x_1)$.

$\Rightarrow \quad y - (-8) = \frac{1}{\sqrt{3}}(x - 0)$

$\Rightarrow \quad y + 8 = \frac{x}{\sqrt{3}}$

$\Rightarrow \quad \sqrt{3}\,y + 8\sqrt{3} = x$

$\Rightarrow \quad x - \sqrt{3}\,y - 8\sqrt{3} = 0$

which is the required equation of straight line.

Example 26. Find the equation of straight line passes through $(-9,0)$ and makes an angle $150°$ with positive direction of X-axis.

Sol. Let m be the slope of required line.

Therefore $m = \tan 150^\circ$

$\Rightarrow \quad m = \tan\left(180^\circ - 30^\circ\right)$

$\Rightarrow \quad m = -\tan(30^\circ)$

$\Rightarrow \quad m = -\frac{1}{\sqrt{3}}$

Also it is given that the required line passes through the point $(-9,0)$.

We know that equation of straight line in point slope form is $y - y_1 = m(x - x_1)$.

$\Rightarrow \quad y - 0 = -\frac{1}{\sqrt{3}}(x - (-9))$

$\Rightarrow \quad -\sqrt{3}\,y = x + 9$

$\Rightarrow \quad x + \sqrt{3}\,y + 9 = 0$

which is the required equation of straight line.

<u>Equation of Straight Line in Two Points form</u>:

If a line passes through two points (x_1, y_1) and (x_2, y_2) and $P(x, y)$ be any point on the line (see Fig. 4.8), then the equation of straight line is

$$\boldsymbol{y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)} \qquad \text{where } \boldsymbol{x_1 \neq x_2}$$

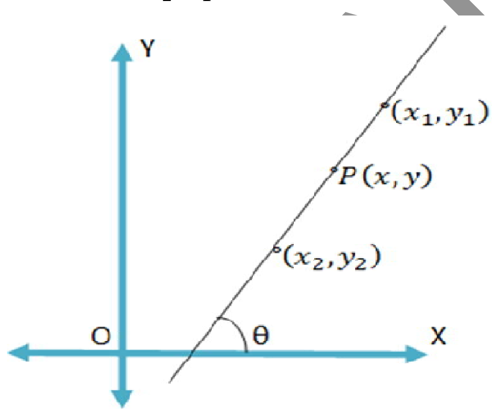

Fig. 4.8

Note: (i) In above case, if $x_1 = x_2$ then the equation of straight line is $x = x_1$.

Example 27. Find the equation of straight line passes through the points $(2, -2)$ and $(0,6)$.

Sol. Given that the straight line passes through the points $(2, -2)$ and $(0,6)$.

Comparing these points with (x_1, y_1) and (x_2, y_2) respectively, we get

$x_1 = 2, y_1 = -2, x_2 = 0$ and $y_2 = 6$.

We know that equation of straight line in two points slope form is

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1).$$

$\Rightarrow \quad y - (-2) = \left(\frac{6 - (-2)}{0 - 2}\right)(x - 2)$

$\Rightarrow \quad y+2=\left(\frac{6+2}{-2}\right)(x-2)$

$\Rightarrow \quad y+2=-4(x-2)$

$\Rightarrow \quad y+2=-4x+8$

$\Rightarrow \quad 4x+y-6=0$

which is the required equation of straight line.

Example 28. Find the equation of straight line passes through the points $(0,8)$ and $(5,0)$.

Sol. Given that the straight line passes through the points $(0,8)$ and $(5,0)$.

Comparing these points with (x_1, y_1) and (x_2, y_2) respectively, we get

$x_1 = 0, y_1 = 8, x_2 = 5$ and $y_2 = 0$.

We know that equation of straight line in two points slope form is

$$y-y_1=\left(\frac{y_2-y_1}{x_2-x_1}\right)(x-x_1).$$

$\Rightarrow \quad y-8=\left(\frac{0-8}{5-0}\right)(x-0)$

$\Rightarrow \quad y-8=\frac{-8x}{5}$

$\Rightarrow \quad 5y-40=-8x$

$\Rightarrow \quad 8x+5y-40=0$

which is the required equation of straight line.

Example 29. Find the equation of straight line passes through the points $(7,-4)$ and $(-1,5)$.

Sol. Given that the straight line passes through the points $(7,-4)$ and $(-1,5)$.

Comparing these points with (x_1, y_1) and (x_2, y_2) respectively, we get

$x_1 = 7, y_1 = -4, x_2 = -1$ and $y_2 = 5$.

We know that equation of straight line in two points slope form is

$$y-y_1=\left(\frac{y_2-y_1}{x_2-x_1}\right)(x-x_1).$$

$\Rightarrow \quad y-(-4)=\left(\frac{5-(-4)}{-1-7}\right)(x-7)$

$\Rightarrow \quad y+4=-\frac{9}{8}(x-7)$

$\Rightarrow \quad 8y+32=-9x+63$

$\Rightarrow \quad 9x+8y-31=0$

which is the required equation of straight line.

<u>**Equation of Straight Line in Slope-Intercept form**</u>:

If a line having slope m, its y-intercept is equal to c and $P(x, y)$ be any point on the line (see Fig. 4.9), then equation of straight line is $\boldsymbol{y = mx + c}$.

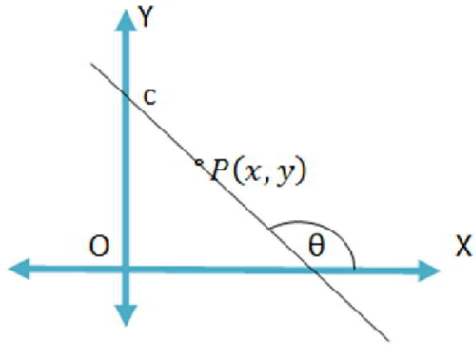

Fig. 4.9

Note: **(i)** If intercept c is given above the X-axis or above the origin then it is positive.

(ii) If intercept c is given below the X-axis or below the origin then it is negative.

Example 30. Find the equation of straight line having slope 3 and cuts of an intercept -2 on Y-axis.

Sol. Given that the slope m of straight line is 3 and Y-intercept is -2 i.e. $c = -2$.

We know that equation of straight line in slope-intercept form is

$$y = mx + c$$
$$\Rightarrow \quad y = 3x - 2$$
$$\Rightarrow \quad 3x - y - 2 = 0$$

which is the required equation of straight line.

Example 31. Find the equation of straight line having slope -6 and cuts of an intercept 5 on Y-axis above the origin.

Sol. Given that the slope m of straight line is -6 and Y-intercept is 5 i.e. $c = 5$.

c is taken positive as Y-intercept is above the origin.

We know that equation of straight line in slope-intercept form is

$$y = mx + c$$
$$\Rightarrow \quad y = -6x + 5$$
$$\Rightarrow \quad 6x + y - 5 = 0$$

which is the required equation of straight line.

Example 32. Find the equation of straight line having slope 2 and cuts of an intercept 9 on Y-axis below the origin.

Sol. Given that the slope m of straight line is 2 and Y-intercept is -9 i.e. $c = -9$.

c is taken negative as Y-intercept is below the origin.

We know that equation of straight line in slope-intercept form is

$$y = mx + c$$
$$\Rightarrow \quad y = 2x - 9$$
$$\Rightarrow \quad 2x - y - 9 = 0$$

which is the required equation of straight line.

Example 33. Find the equation of straight line which makes an angle $45°$ with X-axis and cuts of an intercept 8 on Y-axis below the X-axis.

Sol. Given that the required line makes an angle $45°$ with X-axis.

Therefore slope m of straight line is given by $m = \tan 45^\circ$ *i.e.* $m = 1$.

Also Y-intercept is -8 i.e. $c = -8$.

c is taken negative as Y-intercept is below the X-axis.

We know that equation of straight line in slope-intercept form is

$$y = mx + c$$

$$\Rightarrow \quad y = 1x - 8$$

$$\Rightarrow \quad x - y - 8 = 0$$

which is the required equation of straight line.

Example 34. Find the equation of straight line which makes an angle $60°$ with X-axis and cuts of an intercept 5 on Y-axis above the X-axis.

Sol. Given that the required line makes an angle $60°$ with X-axis.

Therefore slope m of straight line is given by $m = \tan 60^\circ$ *i.e.* $m = \sqrt{3}$.

Also Y-intercept is 5 i.e. $c = 5$.

c is taken positive as Y-intercept is above the X-axis.

We know that equation of straight line in slope-intercept form is

$$y = mx + c$$

$$\Rightarrow \quad y = \sqrt{3}\,x + 5$$

$$\Rightarrow \quad \sqrt{3}\,x - y + 5 = 0$$

which is the required equation of straight line.

Example 35. Find the equation of straight line which is parallel to the line passes through the points $(0,3)$ and $(2,0)$ and cuts of an intercept 12 on Y-axis below the origin.

Sol. Given that the required line is parallel to the line passes through the points $(0,3)$ and $(2,0)$.

Let m be the slope of the required line.

Therefore, $m = \dfrac{0-3}{2-0}$ *i.e.* $m = -\dfrac{3}{2}$ (as the slopes of parallel lines are same)

Also Y-intercept is -12 i.e. $c = -12$.

c is taken negative as Y-intercept is below the origin.

We know that equation of straight line in slope-intercept form is

$$y = mx + c$$

$$\Rightarrow \quad y = -\frac{3}{2}x - 12$$

$$\Rightarrow \quad 2y = -3x - 24$$

$$\Rightarrow \quad 3x + 2y + 24 = 0$$

which is the required equation of straight line.

<u>Equation of Straight Line in Intercept form</u>:

If a line having intercepts a and b on X-axis and Y-axis respectively and $P(x, y)$ be any point on the line (see figure 4.10), then equation of straight line

$$\frac{x}{a} + \frac{y}{b} = 1$$

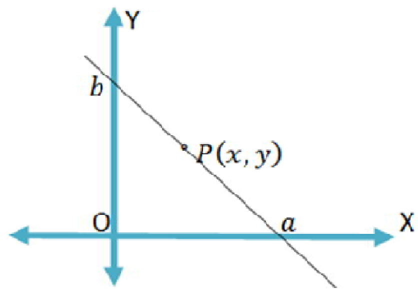

Fig. 4.10

Example 36. Find the equation of straight line which makes intercepts 2 and 5 on X-axis and Y-axis respectively.

Sol. Given that X-intercept is 2 and Y-intercept is 5

i.e. $a = 2$ and $b = 5$

We know that equation of straight line in Intercept form is

$$\frac{x}{a}+\frac{y}{b}=1$$

$$\Rightarrow \quad \frac{x}{2}+\frac{y}{5}=1$$

$$\Rightarrow \quad \frac{5x+2y}{10}=1$$

$$\Rightarrow \quad 5x+2y=10$$

$$\Rightarrow \quad 5x+2y-10=0$$

which is the required equation of straight line.

Example 37. Find the equation of straight line which makes intercepts 3 and -15 on the axes.

Sol. Given that X-intercept is 3 and Y-intercept is -15

i.e. $a = 3$ and $b = -15$

We know that equation of straight line in Intercept form is

$$\frac{x}{a}+\frac{y}{b}=1$$

$$\Rightarrow \quad \frac{x}{3}+\frac{y}{-15}=1$$

$$\Rightarrow \quad \frac{x}{3}-\frac{y}{15}=1$$

$$\Rightarrow \quad \frac{5x-y}{15}=1$$

$$\Rightarrow \quad 5x-y=15$$

$$\Rightarrow \quad 5x-y-15=0$$

which is the required equation of straight line.

Example 38. Find the equation of straight line which passes through $(1,-4)$ and makes intercepts on axes which are equal in magnitude and opposite in sign.

Sol. Let the intercepts on the axes are p and $-p$

i.e. $a = p$ and $b = -p$

We know that equation of straight line in Intercept form is

$$\frac{x}{a}+\frac{y}{b}=1$$

$$\Rightarrow \quad \frac{x}{p}+\frac{y}{-p}=1$$

$$\Rightarrow \quad \frac{x}{p}-\frac{y}{p}=1$$

$$\Rightarrow \quad \frac{x-y}{p}=1$$

$$\Rightarrow \quad x-y=p \qquad (1)$$

Given that this line passes through $(1,-4)$.

Therefore put $x = 1$ and $y = -4$ in equation (1), we get

$$1-(-4)=p$$

$$\Rightarrow \quad p=5$$

Using this value in equation (1), we get

$$x-y=5$$

$$\Rightarrow \quad x-y-5=0$$

which is the required equation of straight line.

Example 39. Find the equation of straight line which passes through $(1,4)$ and sum of whose intercepts on axes is 10.

Sol. Let the intercepts on the axes are p and $10-p$

i.e. $a = p$ and $b = 10-p$

We know that equation of straight line in Intercept form is

$$\frac{x}{a}+\frac{y}{b}=1$$

$$\Rightarrow \quad \frac{x}{p}+\frac{y}{10-p}=1$$

$$\Rightarrow \quad \frac{(10-p)x+p\,y}{p(10-p)}=1$$

$$\Rightarrow \quad (10-p)x+p\,y=p(10-p) \qquad (1)$$

Given that this line passes through $(1,4)$.

Therefore put $x = 1$ and $y = 4$ in equation (1), we get

$$(10-p)(1)+p(4)=p(10-p)$$

$$\Rightarrow \quad 10-p+4p=10p-p^2$$

$$\Rightarrow \quad p^2-7p+10=0$$

$\Rightarrow \quad p^2 - 5p - 2p + 10 = 0$

$\Rightarrow \quad p(p-5) - 2(p-5) = 0$

$\Rightarrow \quad (p-2)(p-5) = 0$

either $\quad p-2=0 \quad$ *or* $\quad p-5=0$

either $\quad p=2 \quad$ *or* $\quad p=5$

Put $p=2$ in equation (1), we get

$\Rightarrow \quad (10-2)x + 2y = 2(10-2)$

$\Rightarrow \quad 8x + 2y = 16$

$\Rightarrow \quad 4x + y = 8 \qquad (2)$

Put $p=5$ in equation (1), we get

$\Rightarrow \quad (10-5)x + 5y = 5(10-5)$

$\Rightarrow \quad 5x + 5y = 25$

$\Rightarrow \quad x + y = 5 \qquad (3)$

Equations (2) and (3) are the required equations of straight lines.

Symmetric Form of a Straight Line

If α is the inclination of a straight line L passing through the point (x_1, y_1), then the equation of the straight line is

$$\frac{x - x_1}{\cos\alpha} = \frac{y - y_1}{\sin\alpha}.$$

Example 40. Find the equation of the straight line with inclination 45° and passing through the point $(\sqrt{3}, 1)$ by Symmetric form.

Sol. Given that the straight line passing through the point $(\sqrt{3}, 1)$ with inclination 45°.

Here $x_1 = \sqrt{3}$, $y_1 = 1$ and $\alpha = 45°$. So, by Symmetric form, equation of straight line is

$$\frac{x - x_1}{\cos\alpha} = \frac{y - y_1}{\sin\alpha}$$

$$\Rightarrow \quad \frac{x - \sqrt{3}}{\cos 45°} = \frac{y - 1}{\sin 45°}$$

$$\Rightarrow \quad \frac{x - \sqrt{3}}{\left(\frac{1}{\sqrt{2}}\right)} = \frac{y - 1}{\left(\frac{1}{\sqrt{2}}\right)}$$

$$\Rightarrow \quad \frac{x - \sqrt{3}}{1} = \frac{y - 1}{1}$$

$$\Rightarrow \quad x - \sqrt{3} = y - 1$$

$$\Rightarrow \quad x - y - \sqrt{3} + 1 = 0$$

which is required equation of straight line.

Example 41. Find the equation of the straight line with inclination 30° and passing through the point $(2, 5)$ by Symmetric form.

Sol. Given that the straight line passing through the point $(2, 5)$ with inclination 30°.

Here $x_1 = 2, y_1 = 5$ and $\alpha = 30°$. So, by Symmetric form, equation of straight line is

$$\frac{x - x_1}{\cos\alpha} = \frac{y - y_1}{\sin\alpha}$$

$$\Rightarrow \quad \frac{x - 2}{\cos 30°} = \frac{y - 5}{\sin 30°}$$

$$\Rightarrow \quad \frac{x - 2}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{y - 5}{\left(\frac{1}{2}\right)}$$

$$\Rightarrow \quad \frac{x - 2}{\sqrt{3}} = \frac{y - 5}{1}$$

$$\Rightarrow \quad x - 2 = \sqrt{3}\,y - 5\sqrt{3}$$

$$\Rightarrow \quad x - \sqrt{3}\,y - 2 + 5\sqrt{3} = 0$$

which is required equation of straight line.

Equation of Straight Line in Normal form:

Let p be the length of perpendicular from the origin to the straight line and α be the angle which this perpendicular makes with the positive direction of X-axis (see Fig. 4.11). If (x, y) be moving point on the line, then equation of straight line is

$$\boldsymbol{p = x\,cos\alpha + y\,sin\alpha}$$

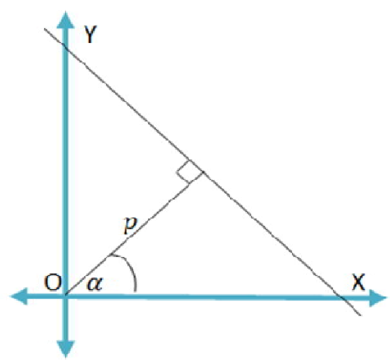

Fig. 4.11

Example 42. Find the equation of straight line such that the length of perpendicular from the origin to the straight line is 2 and the inclination of this perpendicular to the X-axis is $120°$.

Sol. We know that equation of straight line in Normal form is

$$x\cos\alpha + y\sin\alpha = p \qquad (1)$$

where p be the length of perpendicular from the origin to the straight line and α be the angle which this perpendicular makes with the positive direction of X-axis.

Here $p = 2$ and $\alpha = 120^\circ$. Put these values in (1), we get

$$x\cos 120^\circ + y\sin 120^\circ = 2$$

$\Rightarrow \quad x\cos(180^\circ - 60^\circ) + y\sin(180^\circ - 60^\circ) = 2$

$\Rightarrow \quad -x\cos(60^\circ) + y\sin(60^\circ) = 2$

$$\Rightarrow \quad -x\left(\frac{1}{2}\right) + y\left(\frac{\sqrt{3}}{2}\right) = 2$$

$$\Rightarrow \quad \frac{-x+\sqrt{3}\,y}{2} = 2$$

$\Rightarrow \quad -x+\sqrt{3}\,y = 4$

$\Rightarrow \quad -x+\sqrt{3}\,y - 4 = 0$

which is the required equation of straight line.

Example 43. Find the equation of straight line such that the length of perpendicular from the origin to the straight line is 7 and the inclination of this perpendicular to the X-axis is $45°$.

Sol. We know that equation of straight line in Normal form is

$$x\cos\alpha + y\sin\alpha = p \qquad (1)$$

where p be the length of perpendicular from the origin to the straight line and α be the angle which this perpendicular makes with the positive direction of X-axis.

Here $p = 7$ and $\alpha = 45^\circ$. Put these values in (1), we get

$$x\cos 45^\circ + y\sin 45^\circ = 7$$

$$\Rightarrow \quad x\left(\frac{1}{\sqrt{2}}\right) + y\left(\frac{1}{\sqrt{2}}\right) = 7$$

$\Rightarrow \quad x + y = 7\sqrt{2}$

$\Rightarrow \quad x + y - 7\sqrt{2} = 0$

which is the required equation of straight line.

<u>Intersection of Two Lines</u>:

Let $a_1x + b_1y + c_1 = 0 \qquad (1)$

and $a_2x + b_2y + c_2 = 0 \qquad (2)$

be the two straight lines.

Their point of intersection can be obtained by:

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$(provided\ that\ a_1b_2 - a_2b_1 \neq 0)$$

$$\Rightarrow \quad x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \quad and \quad y = \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}$$

Hence the point of intersection of given straight lines is

$$\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}\right)$$

In case when $a_1b_2 - a_2b_1 = 0$, then the above coordinates have no meaning. In this case the lines do not intersect, but are parallel.

Example 44. Check whether the following straight lines are intersecting lines and if they are intersecting lines find their point of intersection:

(i) $2x + 5y = -4$ and $x + 6y = 5$

(ii) $3x - 4y = -5$ and $6x - 8y = -10$

Sol. (i) Given equations are $2x + 5y = -4$ and $x + 6y = 5$

$\Rightarrow$ $2x + 5y + 4 = 0$ and $x + 6y - 5 = 0$

Comparing these equations with

$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

We have, $a_1 = 2,\ b_1 = 5,\ c_1 = 4,\ a_2 = 1,\ b_2 = 6\ and\ c_2 = -5$.

Now, $a_1b_2 - a_2b_1 = 2(6) - 1(5) = 12 - 5 = 7 \neq 0$.

Therefore the given lines are intersecting lines.

Also their point of intersection is given by

$$\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}\right)$$

$$= \left(\frac{5(-5) - 6(4)}{2(6) - 1(5)}, \frac{1(4) - 2(-5)}{2(6) - 1(5)}\right)$$

$$= \left(\frac{-25 - 24}{12 - 5}, \frac{4 + 10}{12 - 5}\right)$$

$$= \left(\frac{-49}{7}, \frac{14}{7}\right)$$

$$= (-7, 2)$$

which is required point.

(ii) Given equations are $3x - 4y = -5$ and $6x - 8y = -10$

$\Rightarrow$ $3x - 4y + 5 = 0$ and $6x - 8y + 10 = 0$

Comparing these equations with

$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

We have, $a_1 = 3,\ b_1 = -4,\ c_1 = 5,\ a_2 = 6,\ b_2 = -8\ and\ c_2 = 10$.

Now, $a_1b_2 - a_2b_1 = 3(-8) - 6(-4) = -24 + 24 = 0$.

Therefore the given lines are not intersecting lines.

<u>Concurrency of Lines</u>:

Three or more than three straight lines are said to be concurrent if these are intersecting at the same point. The point of intersection of these lines is called point of concurrency.

<u>Condition of Concurrency of Three Lines</u>:

Let $a_1x + b_1y + c_1 = 0$ (1)

$a_2x + b_2y + c_2 = 0$ (2)

and $a_3x + b_3y + c_3 = 0$ (3)

+be the three straight lines.

These three straight lines will be concurrent if the point of intersection of any two lines satisfies the third line.

The point of intersection of (1) and (2) is

$$\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}\right)$$

Using this point in equation (3), we have

$$a_3\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}\right) + b_3\left(\frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}\right) + c_3 = 0$$

$\Rightarrow \quad a_3(b_1c_2 - b_2c_1) + b_3(a_2c_1 - a_1c_2) + c_3(a_1b_2 - a_2b_1) = 0$

This is same as $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0.$

Example 45. Show that the following lines are concurrent and also find their find of concurrency:

$$x - y + 6 = 0$$
$$2x + y - 5 = 0$$

and $\quad -x - 2y + 11 = 0$

Sol. Given equations are

$$x - y + 6 = 0\,, \tag{1}$$
$$2x + y - 5 = 0 \tag{2}$$

and $\quad -x - 2y + 11 = 0 \qquad (3)$

Comparing these equations with

$a_1x + b_1y + c_1 = 0\,,\ a_2x + b_2y + c_2 = 0$ and $a_3x + b_3y + c_3 = 0$

We have, $a_1 = 1,\ b_1 = -1,\ c_1 = 6,\ a_2 = 2,\ b_2 = 1,\ c_2 = -5,$

$a_3 = -1,\ b_3 = -2\ and\ c_3 = 11$

Now, $\quad a_1(b_2c_1 - b_1c_2) + b_1(a_1c_2 - a_2c_1) + c_1(a_2b_1 - a_1b_2)$

$= 1\,[1(6) - (-1)(-5)] - 1\,[1(-5) - 2(6)] + 6\,[2(-1) - 1(1)]$

$= 1\,[6 - 5] - 1\,[-5 - 12] + 6\,[-2 - 1]$

$= 1 + 17 - 18 = 0$

Hence the given lines are concurrent.

To find the point of concurrency, let us solve equation (1) and equation (2).

Intersection point of equation (1) and equation (2) is given by

$$\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}\right)$$

$$= \left(\frac{-1(-5) - 1(6)}{1(1) - 2(-1)}, \frac{2(6) - 1(-5)}{1(1) - 2(-1)}\right)$$

$= \left(\frac{5-6}{1+2}, \frac{12+5}{1+2}\right)$

$= \left(\frac{-1}{3}, \frac{17}{3}\right)$

which is the required point of concurrency.

Example 46. Find the value of k, if the following lines are concurrent:

$x - 2y + 1 = 0$

$2x - 5y + 3 = 0$

and $5x + 9y + k = 0$

Sol. Given equations are

$x - 2y + 1 = 0$, (1)

$2x - 5y + 3 = 0$ (2)

and $5x + 9y + k = 0$ (3)

Comparing these equations with

$a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ and $a_3x + b_3y + c_3 = 0$

We have, $a_1 = 1,\ b_1 = -2,\ c_1 = 1,\ a_2 = 2,\ b_2 = -5,\ c_2 = 3,$

$a_3 = 5,\ b_3 = 9\ and\ c_3 = k$

It is given that the given lines are concurrent. Therefore by condition of concurrency, we have

$a_3(b_1c_2 - b_2c_1) + b_3(a_2c_1 - a_1c_2) + c_3(a_1b_2 - a_2b_1) = 0$

$\Rightarrow\ 5[-2(3) - (-5)(1)] + 9[2(1) - 1(3)] + k[1(-5) - 2(-2)] = 0$

$\Rightarrow\ 5[-6 + 5] + 9[2 - 3] + k[-5 + 4] = 0$

$\Rightarrow\ -5 - 9 - k = 0$

$\Rightarrow\ k = -14$

which is required value of k.

Angle Between Two Straight Lines: Two intersecting lines always intersects at two angles in which one angle is acute angle and another angle is obtuse angle. The sum of both the angles is 180° i.e. they are supplementary to each other. For example, if one angle between intersecting lines is 60° then another angle is $180^\circ - 60^\circ = 120^\circ$.

Generally, we take acute angle as the angle between the lines (see Fig. 4.12).

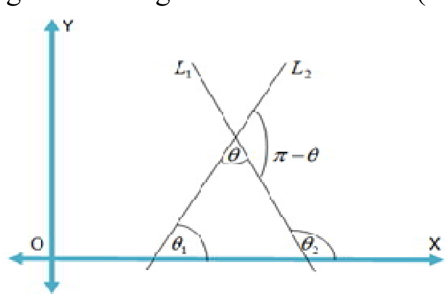

Fig. 4.12

Let L_1 & L_2 be straight lines and m_1 & m_2 be their slopes respectively.

Let θ_1 & θ_2 be the angles which L_1 & L_2 make with positive X-axis respectively.

Therefore $m_1 = \tan(\theta_1)$ & $m_2 = \tan(\theta_2)$.

Let θ be the acute angle between lines, then

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \quad or \quad \tan\theta = \left|\frac{m_2 - m_1}{1 + m_2 m_1}\right|$$

Example 47. Find the acute angle between the lines whose slopes are 1 and 0.

Sol. Given that slopes of lines are 1 and 0.

Let $m_1 = 1$ and $m_2 = 0$.

Also let θ be the acute angle between lines.

Therefore, $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\Rightarrow \quad \tan\theta = \left|\frac{1-0}{1+(1)(0)}\right|$

$\Rightarrow \quad \tan\theta = \left|\frac{1}{1+0}\right|$

$\Rightarrow \quad \tan\theta = 1$

$\Rightarrow \quad \tan\theta = \tan\left(\frac{\pi}{4}\right)$

$\Rightarrow \quad \theta = \frac{\pi}{4}$

which is the required acute angle.

Example 48. Find the acute angle between the lines whose slopes are $2+\sqrt{3}$ and $2-\sqrt{3}$.

Sol. Given that slopes of lines are $2+\sqrt{3}$ and $2-\sqrt{3}$.

Let $m_1 = 2+\sqrt{3}$ and $m_2 = 2-\sqrt{3}$.

Also let θ be the acute angle between lines.

Therefore, $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\Rightarrow \quad \tan\theta = \left|\frac{(2+\sqrt{3})-(2-\sqrt{3})}{1+(2+\sqrt{3})(2-\sqrt{3})}\right|$

$\Rightarrow \quad \tan\theta = \left|\frac{2+\sqrt{3}-2+\sqrt{3}}{1+(4-3)}\right|$

$\Rightarrow \quad \tan\theta = \left|\frac{2\sqrt{3}}{2}\right|$

$\Rightarrow \quad \tan\theta = \sqrt{3}$

$\Rightarrow \quad \tan\theta = \tan\left(\frac{\pi}{3}\right)$

$\Rightarrow \quad \theta = \frac{\pi}{3}$

which is the required acute angle.

Example 49. Find the obtuse angle between the lines whose slopes are $\sqrt{3}$ and $\frac{1}{\sqrt{3}}$.

Sol. Given that slopes of lines are $\sqrt{3}$ and $\frac{1}{\sqrt{3}}$.

Let $m_1 = \sqrt{3}$ and $m_2 = \frac{1}{\sqrt{3}}$.

Also let θ be the acute angle between lines.

Therefore, $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\Rightarrow \quad \tan\theta = \left|\frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \left(\sqrt{3}\right)\left(\frac{1}{\sqrt{3}}\right)}\right|$

$\Rightarrow \quad \tan\theta = \left|\frac{\frac{3-1}{\sqrt{3}}}{1+1}\right|$

$\Rightarrow \quad \tan\theta = \left|\frac{2}{2\sqrt{3}}\right|$

$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \tan\theta = \tan\left(30^\circ\right)$

$\Rightarrow \quad \theta = 30^\circ$

Therefore, $180^\circ - \theta$ is the obtuse angle between the lines.

i.e. $180^\circ - 30^\circ = 150^\circ$ is the obtuse angle between the lines.

Example 50. Find the angle between the lines whose slopes are -3 and 5.

Sol. Given that slopes of lines are -3 and 5.

Let $m_1 = -3$ and $m_2 = 5$.

Also let θ be the angle between lines.

Therefore, $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\Rightarrow \quad \tan\theta = \left|\frac{-3-5}{1+(-3)(5)}\right|$

$\Rightarrow \quad \tan\theta = \left|\frac{-8}{1-15}\right|$

$\Rightarrow \quad \tan\theta = \left|\frac{-8}{-14}\right|$

$\Rightarrow \quad \tan\theta = \frac{4}{7}$

$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{4}{7}\right)$

which is the required angle.

Example 51. Find the angle between the lines joining the points $(0,0)$, $(2,3)$ and $(2,-2)$, $(3,5)$.

Sol. Let m_1 be the slope of the line joining $(0,0)$ and $(2,3)$ and m_2 be the slope of the line joining $(2,-2)$ and $(3,5)$.

$\Rightarrow \quad m_1 = \frac{3-0}{2-0} = \frac{3}{2}$

and $\quad m_2 = \frac{5-(-2)}{3-2} = \frac{7}{1} = 7$.

Also let θ be the angle between lines.

Therefore, $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\Rightarrow \quad \tan\theta = \left|\frac{\frac{3}{2} - 7}{1 + \left(\frac{3}{2}\right)(7)}\right|$

$\Rightarrow \quad \tan\theta = \left|\frac{\frac{3-14}{2}}{\frac{2+21}{2}}\right|$

$\Rightarrow \quad \tan\theta = \left|\frac{\frac{-11}{2}}{\frac{23}{2}}\right|$

$\Rightarrow \quad \tan\theta = \frac{11}{23}$

$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{11}{23}\right)$

which is the required angle.

Example 52. Find the angle between the lines joining the points $(6,-5)$, $(-2,1)$ and $(0,3)$, $(-8,6)$.

Sol. Let m_1 be the slope of the line joining $(6,-5)$ and $(-2,1)$ and m_2 be the slope of the line joining $(0,3)$ and $(-8,6)$.

$\Rightarrow \quad m_1 = \frac{1-(-5)}{-2-6} = -\frac{6}{8} = -\frac{3}{4}$

and $\quad m_2 = \frac{6-3}{-8-0} = -\frac{3}{8}$.

Also let θ be the angle between lines.

Therefore, $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

$$\Rightarrow \quad \tan\theta = \left|\frac{-\frac{3}{4} - \left(-\frac{3}{8}\right)}{1 + \left(-\frac{3}{4}\right)\left(-\frac{3}{8}\right)}\right|$$

$$\Rightarrow \quad \tan\theta = \left|\frac{-\frac{3}{4} + \frac{3}{8}}{1 + \frac{9}{32}}\right|$$

$$\Rightarrow \quad \tan\theta = \left|\frac{\frac{-6+3}{8}}{\frac{32+9}{32}}\right|$$

$$\Rightarrow \quad \tan\theta = \left|\frac{\frac{-3}{8}}{\frac{41}{32}}\right|$$

$$\Rightarrow \quad \tan\theta = \left|\frac{-3}{8} \times \frac{32}{41}\right|$$

$$\Rightarrow \quad \tan\theta = \frac{12}{41}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{12}{41}\right)$$

which is the required angle.

Example 53. Find the angle between the pair of straight lines

$(-2 + \sqrt{3})x + y + 9 = 0$ and $(2 + \sqrt{3})x - y + 20 = 0$.

Sol. Given that equations of lines are

$$(-2 + \sqrt{3})x + y + 9 = 0 \qquad (1)$$

and $$(2 + \sqrt{3})x - y + 20 = 0 \qquad (2)$$

Let m_1 be the slope of the line (1) and m_2 be the slope of the line (2).

$$\Rightarrow \quad m_1 = -\frac{-2 + \sqrt{3}}{1} = 2 - \sqrt{3}$$

and $$m_2 = -\frac{2 + \sqrt{3}}{-1} = 2 + \sqrt{3}.$$

Also let θ be the angle between lines.

Therefore, $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\Rightarrow \quad \tan\theta = \left|\dfrac{(2-\sqrt{3})-(2+\sqrt{3})}{1+(2-\sqrt{3})(2+\sqrt{3})}\right|$

$\Rightarrow \quad \tan\theta = \left|\dfrac{2-\sqrt{3}-2-\sqrt{3}}{1+4-3}\right|$

$\Rightarrow \quad \tan\theta = \left|\dfrac{-2\sqrt{3}}{2}\right|$

$\Rightarrow \quad \tan\theta = \left|-\sqrt{3}\right|$

$\Rightarrow \quad \tan\theta = \sqrt{3}$

$\Rightarrow \quad \tan\theta = \tan\left(\dfrac{\pi}{3}\right)$

$\Rightarrow \quad \theta = \dfrac{\pi}{3}$

which is the required angle.

Example 54. Find the angle between the pair of straight lines

$x + \sqrt{3}\,y - 8 = 0$ and $x - \sqrt{3}\,y + 2 = 0$.

Sol. Given that equations of lines are

$$x + \sqrt{3}\,y - 8 = 0 \qquad (1)$$

and $$x - \sqrt{3}\,y + 2 = 0 \qquad (2)$$

Let m_1 be the slope of the line (1) and m_2 be the slope of the line (2).

$\Rightarrow \quad m_1 = -\dfrac{1}{\sqrt{3}}$

and $\quad m_2 = -\dfrac{1}{-\sqrt{3}} = \dfrac{1}{\sqrt{3}}$.

Also let θ be the angle between lines.

Therefore, $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\Rightarrow \quad \tan\theta = \left|\dfrac{-\dfrac{1}{\sqrt{3}} - \dfrac{1}{\sqrt{3}}}{1 + \left(-\dfrac{1}{\sqrt{3}}\right)\left(\dfrac{1}{\sqrt{3}}\right)}\right|$

$\Rightarrow \quad \tan\theta = \left|\dfrac{-\dfrac{2}{\sqrt{3}}}{1 - \dfrac{1}{3}}\right|$

$$\Rightarrow \qquad \tan\theta = \left| \frac{-\frac{2}{\sqrt{3}}}{\frac{3-1}{3}} \right|$$

$$\Rightarrow \qquad \tan\theta = \left| \frac{-\frac{2}{\sqrt{3}}}{\frac{2}{3}} \right|$$

$$\Rightarrow \qquad \tan\theta = \left| -\frac{3}{\sqrt{3}} \right|$$

$$\Rightarrow \qquad \tan\theta = \frac{3}{\sqrt{3}} = \frac{3\sqrt{3}}{\sqrt{3}\times\sqrt{3}} = \frac{3\sqrt{3}}{3}$$

$$\Rightarrow \qquad \tan\theta = \sqrt{3}$$

$$\Rightarrow \qquad \tan\theta = \tan\left(\frac{\pi}{3}\right)$$

$$\Rightarrow \qquad \theta = \frac{\pi}{3}$$

which is the required angle.

Example 55. Find the equations of straight lines making an angle $45°$ with the line $6x + 5y - 1 = 0$ and passing through the point $(2, -1)$.

Sol. Given that equations of line is

$$6x + 5y - 1 = 0 \qquad (1)$$

Let m_1 be the slope of the line (1).

$$\Rightarrow \qquad m_1 = -\frac{6}{5}.$$

Let m_2 be the slope of required line.

Therefore, $\tan 45^\circ = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

$$\Rightarrow \qquad 1 = \left| \frac{-\frac{6}{5} - m_2}{1 + \left(-\frac{6}{5}\right)(m_2)} \right|$$

$$\Rightarrow \qquad 1 = \left| \frac{\frac{-6-5m_2}{5}}{\frac{5-6m_2}{5}} \right|$$

$$\Rightarrow \qquad 1 = \left| \frac{-6-5m_2}{5-6m_2} \right|$$

$\Rightarrow \quad 1=\pm\left(\frac{-6-5m_2}{5-6m_2}\right)$

$\Rightarrow \quad 5-6m_2=\pm(-6-5m_2)$

Taking positive sign, we get

$5-6m_2=+(-6-5m_2)$

$\Rightarrow \quad 5-6m_2=-6-5m_2$

$\Rightarrow \quad -m_2=-11$

$\Rightarrow \quad m_2=11$

So, equation of line passing through $(2,-1)$ with slope 11 is

$y+1=11(x-2)$

$\Rightarrow \quad y+1=11x-22$

$\Rightarrow \quad 11x-y-23=0 \qquad (2)$

Now taking negative sign, we get

$5-6m_2=-(-6-5m_2)$

$\Rightarrow \quad 5-6m_2=6+5m_2$

$\Rightarrow \quad -11m_2=1$

$\Rightarrow \quad m_2=-\frac{1}{11}$

So, equation of line passing through $(2,-1)$ with slope $-\frac{1}{11}$ is

$y+1=-\frac{1}{11}(x-2)$

$\Rightarrow \quad 11y+11=-x+2$

$\Rightarrow \quad x+11y+9=0 \qquad (3)$

Equations (2) and (3) are required equations of straight lines.

Parallel And Perpendicular Lines:

Parallel Lines: Two lines are said to be parallel if they never intersect.

Perpendicular Lines: Two lines are said to be perpendicular if they intersect at right angle($i.e.\ 90°$).

Note I: Let L_1 and L_2 be two straight lines with slopes m_1 and m_2 respectively, then

(i) L_1 is parallel to L_2 ($i.e. L_1||L_2$) if and only if $m_1=m_2$.

(ii) L_1 is perpendicular to L_2 ($i.e. L_1 \perp L_2$) if and only if their slopes are negative-reciprocals to each other $i.e.$ $m_1=-\frac{1}{m_2}$ or $m_2=-\frac{1}{m_1}$.

Note II: Let $a_1x+b_1y+c_1=0 \qquad (1)$

and $a_2x+b_2y+c_2=0 \qquad (2)$

be the two straight lines.

(i) If $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$, then the straight lines (1) and (2) are said to be parallel lines.

(ii) If $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$, then the straight lines (1) and (2) are said to be coincident lines.

(iii) If $a_1 a_2 + b_1 b_2 = 0$, then the straight lines (1) and (2) are said to be perpendicular lines.

Example 56. Check that the following pair of straight lines are parallel or perpendicular or neither:

(i) $2x + 3y = 9$ and $4x + 6y = 12$

(ii) $3x - 5y = 7$ and $10x + 6y = 12$

(iii) $4x + 4y = 18$ and $3x - 2y = 4$

(iv) $y = 2x - 3$ and $y = 2x + 1$

(v) $3y = 7x + 2$ and $7y = -3x - 5$

Sol. (i) The given equations of straight lines are

$$2x + 3y = 9 \quad \text{and} \quad 4x + 6y = 12$$

Comparing these equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$$a_1 = 2, \quad b_1 = 3, \quad c_1 = -9, \quad a_2 = 4, \quad b_2 = 6 \ and \ c_2 = -12$$

Now

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2},$$

$$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$$

and $$\frac{c_1}{c_2} = \frac{-1}{-9} = \frac{4}{3}$$

From above it is clear that $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

Hence the given straight lines are parallel lines.

Alternate Method: The given equations of straight lines are

$$2x + 3y = 9 \tag{1}$$

and $$4x + 6y = 12 \tag{2}$$

Let m_1 and m_2 are slopes of lines (1) and (2) respectively, then

$$m_1 = -\frac{2}{3} \quad \text{and} \quad m_2 = -\frac{4}{6} = -\frac{2}{3}$$

From above it is clear that $m_1 = m_2$.

Hence the given straight lines are parallel lines.

(ii) The given equations of straight lines are

$$3x - 5y = 7 \quad \text{and} \quad 10x + 6y = 12$$

Comparing these equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$$a_1 = 3, \quad b_1 = -5, \quad c_1 = -7, \quad a_2 = 10, \quad b_2 = 6 \ and \ c_2 = -12$$

Now

$$\frac{a_1}{a_2} = \frac{3}{10},$$

$\frac{b_1}{b_2} = \frac{-5}{6}$

and $\frac{c_1}{c_2} = \frac{-7}{-12} = \frac{7}{12}$

From above it is clear that $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

Now, $a_1\, a_2 + b_1\, b_2 = 3(10) - 5(6) = 30 - 30 = 0$

Hence the given straight lines are perpendicular lines.

Alternate Method: The given equations of straight lines are

$3x - 5y = 7$ (1)

and $10x + 6y = 12$ (2)

Let m_1 and m_2 are slopes of lines (1) and (2) respectively, then

$m_1 = -\frac{3}{-5} = \frac{3}{5}$ and $m_2 = -\frac{10}{6} = -\frac{5}{3}$

From above it is clear that $m_1 . m_2 = \frac{3}{5} . \left(-\frac{5}{3}\right) = -1$.

Hence the given straight lines are perpendicular lines.

(iii) The given equations of straight lines are

$4x + 4y = 18$ and $3x - 2y = 4$

Comparing these equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 4,\quad b_1 = 4,\quad c_1 = -18,\quad a_2 = 3,\quad b_2 = -2$ *and* $c_2 = -4$

Now

$\frac{a_1}{a_2} = \frac{4}{3}$,

$\frac{b_1}{b_2} = \frac{4}{-2} = -2$

and $\frac{c_1}{c_2} = \frac{-18}{-4} = \frac{9}{2}$

From above it is clear that $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

Now, $a_1\, a_2 + b_1\, b_2 = 4(3) + 4(-2) = 12 - 8 = 4$

Hence the given straight lines are neither parallel nor perpendicular.

(iv) The given equations of straight lines are

$y = 2x - 3$ (1)

and $y = 2x + 1$ (2)

Let m_1 and m_2 are slopes of lines (1) and (2) respectively, then

$m_1 = 2$ and $m_2 = 2$ (by slope-intercept form)

From above it is clear that $m_1 = m_2$.

Hence the given straight lines are parallel lines.

(v) The given equations of straight lines are

$3y = 7x + 2$ and $7y = -3x - 5$

i.e. $7x - 3y + 2 = 0$ and $-3x - 7y - 5 = 0$

Comparing these equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 7, \quad b_1 = -3, \quad c_1 = 2, \quad a_2 = -3, \quad b_2 = -7 \text{ and } c_2 = -5$

Now

$$\frac{a_1}{a_2} = \frac{7}{-3} = -\frac{7}{3},$$

$$\frac{b_1}{b_2} = \frac{-3}{-7} = \frac{3}{7}$$

and
$$\frac{c_1}{c_2} = \frac{2}{-5} = -\frac{2}{5}$$

From above it is clear that $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

Now, $a_1\, a_2 + b_1\, b_2 = 7(-3) - 3(-7) = -21 + 21 = 0$

Hence the given straight lines are perpendicular lines.

The Perpendicular Distance from a Point to a Straight Line

Let $P_1(x_1\,,\ y_1)$ be any point and $ax + by + c = 0$ be any straight line. To find the distance from the point P_1 to the given straight line, draw the perpendicular P_1P_2 from P_1 to the given straight line (as shown in Fig. 4.13). Let the coordinates of P_2 are $(x_2\,,\ y_2)$.

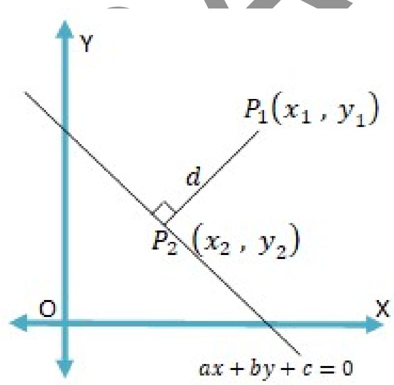

Fig. 4.13

Let d is the distance between the points P_1 and P_2. So by distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad (1)$$

There is requirement to find the coordinates of point P_2.

Since $P_2(x_2\,,\ y_2)$ lies on $ax + by + c = 0$, therefore we have

$$ax_2 + by_2 + c = 0 \qquad (2)$$

The slope of $ax + by + c = 0$ is $-\frac{a}{b}$.

Let m is the slope of perpendicular $\overline{P_1P_2}$, so $m = \frac{-1}{\left(-\frac{a}{b}\right)} = \frac{b}{a}$.

Also the slope of a straight line passing through $(x_1\,,\ y_1)$ and $(x_2\,,\ y_2)$ is $\frac{y_2 - y_1}{x_2 - x_1}$.

Therefore, $\frac{y_2 - y_1}{x_2 - x_1} = \frac{b}{a}$

$\Rightarrow$ $ay_2 - ay_1 = bx_2 - bx_1$

$\Rightarrow$ $bx_2 - ay_2 + ay_1 - bx_1 = 0$ (3)

Solving (2) and (3), we have

$$\frac{x_2}{b(ay_1 - bx_1) + ac} = \frac{-y_2}{a(ay_1 - bx_1) - bc} = \frac{1}{-a^2 - b^2}$$

$\Rightarrow$ $x_2 = \dfrac{b^2x_1 - aby_1 - ac}{a^2 + b^2}$

and $y_2 = \dfrac{a^2y_1 - abx_1 - bc}{a^2 + b^2}$

Using the values of x_2 and y_2 in equation (1), we have

$$d = \sqrt{\left(\frac{b^2x_1 - aby_1 - ac}{a^2+b^2} - x_1\right)^2 + \left(\frac{a^2y_1 - abx_1 - bc}{a^2+b^2} - y_1\right)^2}$$

After simplifying, we have

$$d = \frac{|\ ax_1 + by_1 + c\ |}{\sqrt{a^2+b^2}}$$

which is the required distance from point $P_1(x_1\ ,\ y_1)$ to the straight line $ax + by + c = 0$.

Example 57. Find the distance from the point $(-5\ ,\ 2)$ to the straight line $4x - 2y - 3 = 0$.

Sol. Let d is the distance from the point $(-5\ ,\ 2)$ to the straight line $4x - 2y - 3 = 0$.

Therefore, $d = \frac{|\ 4\,(-5) - 2\,(2) - 3\ |}{\sqrt{(4)^2+(-2)^2}}$

$\Rightarrow$ $d = \frac{|-20 - 4 - 3|}{\sqrt{16+4}}$

$\Rightarrow$ $d = \frac{|-27\ |}{\sqrt{20}}$

$\Rightarrow$ $d = \frac{27}{2\sqrt{5}}$

which is the required distance.

Example 58. Find the distance from the point $(3\ ,\ 5)$ to the straight line $3x + 4y + 6 = 0$.

Sol. Let d is the distance from the point $(3\ ,\ 5)$ to the straight line $3x + 4y + 6 = 0$.

Therefore, $d = \frac{|\ 3\,(3) + 4\,(5) + 6\ |}{\sqrt{(3)^2+(4)^2}}$

$\Rightarrow$ $d = \frac{|\ 9+20+\quad |}{\sqrt{9+16}}$

$\Rightarrow$ $d = \frac{|\ 35\ |}{\sqrt{25}}$

$\Rightarrow$ $d = \frac{35}{5}$

$\Rightarrow$ $d = 7$

which is the required distance.

Example 59. If p is the perpendicular distance from origin to the straight line whose intercepts are a and b on x-axis and y-axis respectively, prove that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

Sol. The equation of straight line whose intercepts are a and b on is given by

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\Rightarrow \quad bx + ay - ab = 0 \tag{1}$$

Since p is the perpendicular distance from origin to the straight line (1).

Therefore, $p = \frac{|\, b\,(0) + a\,(0) - ab\,|}{\sqrt{(b)^2 + (a)^2}}$

$$\Rightarrow \quad p = \frac{|-ab\,|}{\sqrt{(b)^2 + (a)^2}}$$

$$\Rightarrow \quad p^2 = \frac{a^2\,b^2}{a^2 + b^2}$$

$$\Rightarrow \quad \frac{1}{p^2} = \frac{a^2 + b^2}{a^2 b^2}$$

$$\Rightarrow \quad \frac{1}{p^2} = \frac{a^2}{a^2 b^2} + \frac{b^2}{a^2 b^2}$$

$$\Rightarrow \quad \frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{a^2}$$

$$\Rightarrow \quad \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

Hence proved.

General Equation of Straight Line:

The general equation of straight line in variables x and y is an equation of the form

$$ax + by + c = 0 \tag{1}$$

where a, b and c are given real numbers and a and b are not both zero simultaneously.

Reduction of General form $ax + by + c = 0$ to other forms:

(i) Reduction to Slope–Intercept Form:

The general form of straight line is $ax + by + c = 0$

$$\Rightarrow \quad by = -ax - c$$

$$\Rightarrow \quad y = -\frac{a}{b}x - \frac{c}{b}$$

which is of the form $y = mx + c$.

(ii) Reduction to Intercept Form:

The general form of straight line is $ax + by + c = 0$

$$\Rightarrow \quad ax + by = -c$$

$$\Rightarrow \quad -\frac{a}{c}x - \frac{b}{c}y = 1$$

$$\Rightarrow \quad \frac{x}{\left(-\frac{c}{a}\right)} + \frac{y}{\left(-\frac{c}{b}\right)} = 1$$

which is of the form $\frac{x}{A} + \frac{y}{B} = 1$.

(iii) Reduction to Normal Form:

The general form of straight line is

$$ax + by + c = 0 \tag{1}$$

Comparing this equation with the Normal form

$$\cos\alpha\, x + \sin\alpha\, y - p = 0 \tag{2}$$

we have, $\frac{a}{\cos\alpha} = \frac{b}{\sin\alpha} = \frac{c}{-p} = k$

because the coefficient of (1) and (2) are proportional.

So $a = k\cos\alpha$, $b = k\sin\alpha$ and $c = -pk$

$\Rightarrow \quad a^2 + b^2 = k^2(cos^2\alpha + sin^2\alpha)$

$\Rightarrow \quad a^2 + b^2 = k^2$

$\Rightarrow \quad a^2 + b^2 = k^2$

$\Rightarrow \quad k = \pm\sqrt{a^2 + b^2}$

Therefore, $\quad p = -\frac{c}{k}$

$$\Rightarrow \quad p = -\left(\pm\frac{c}{\sqrt{a^2 + b^2}}\right)$$

As p must be positive, so the signs of c and $\sqrt{a^2 + b^2}$ must be opposite.

Case I: If c is positive, then $k = -\sqrt{a^2 + b^2}$

Then from $\cos\alpha = \frac{a}{k}$, $\sin\alpha = \frac{b}{k}$ and $p = -\frac{c}{k}$, we have

$$\cos\alpha = -\frac{a}{\sqrt{a^2 + b^2}}, \qquad \sin\alpha = -\frac{b}{\sqrt{a^2 + b^2}} \quad and \quad p = \frac{c}{\sqrt{a^2 + b^2}}$$

Using these values in equation (2), we have

$$-\frac{a}{\sqrt{a^2 + b^2}}\,x - \frac{b}{\sqrt{a^2 + b^2}}\,y - \frac{c}{\sqrt{a^2 + b^2}} = 0$$

$$\Rightarrow \quad \frac{a}{\sqrt{a^2 + b^2}}\,x + \frac{b}{\sqrt{a^2 + b^2}}\,y + \frac{c}{\sqrt{a^2 + b^2}} = 0$$

which is the required Normal form.

Case II: If c is negative, then $k = \sqrt{a^2 + b^2}$

Then from $\cos\alpha = \frac{a}{k}$, $\sin\alpha = \frac{b}{k}$ and $p = -\frac{c}{k}$, we have

$$\cos\alpha = \frac{a}{\sqrt{a^2+b^2}}\,, \qquad \sin\alpha = \frac{b}{\sqrt{a^2+b^2}} \quad and \quad p = -\frac{c}{\sqrt{a^2+b^2}}$$

Using these values in equation (2), we have

$$\frac{a}{\sqrt{a^2+b^2}}\,x + \frac{b}{\sqrt{a^2+b^2}}\,y + \frac{c}{\sqrt{a^2+b^2}} = 0$$

which is the required Normal form.

Example 60. Reduce the equation $3x + 4y = 10$ to the

(i) Slope-intercept from

(ii) Intercept form

(iii) Normal form

Sol. (i) Given equation is $3x + 4y = 10$

$\Rightarrow \quad 4y = -3x + 10$

$\Rightarrow \quad y = -\frac{3}{4}x + \frac{10}{4}$

$\Rightarrow \quad y = -\frac{3}{4}x + \frac{5}{2}$

which is the slope-intercept form with slope $-\frac{3}{4}$ and $y - intercept = \frac{5}{2}$.

(ii) Given equation is $3x + 4y = 10$

$\Rightarrow \quad \frac{3}{10}x + \frac{4}{10}y = 1$

$\Rightarrow \quad \frac{3}{10}x + \frac{2}{5}y = 1$

$\Rightarrow \quad \frac{x}{\left(\frac{10}{3}\right)} + \frac{y}{\left(\frac{5}{2}\right)} = 1$

which is the Intercept form with $x - intercept = \frac{10}{3}$ and $y - intercept = \frac{5}{2}$.

(iii) Given equation is $3x + 4y = 10$

$\Rightarrow \quad 3x + 4y - 10 = 0$

Here $a = 3,\ b = 4$ and $c = -10$.

Therefore, $k = \sqrt{a^2+b^2}$

$\Rightarrow \quad k = \sqrt{3^2+4^2}$

$\Rightarrow \quad k = \sqrt{9+16}$

$\Rightarrow \quad k = \sqrt{25}$

$\Rightarrow \quad k = 5$

Dividing the given equation by k, i.e., 5 , we have

$\Rightarrow \quad \frac{3}{5}x + \frac{4}{5}y - \frac{10}{5} = 0$

$\Rightarrow \quad \frac{3}{5}x + \frac{4}{5}y = 2$

which is the required Normal form with $\cos\alpha = \frac{3}{5}$, $\sin\alpha = \frac{4}{5}$ and $p = 2$.

To find α: $\frac{\sin\alpha}{\cos} = \frac{4}{3}$

$\Rightarrow \qquad \tan\alpha = \frac{4}{3}$

$\Rightarrow \qquad \alpha = tan^{-1}\left(\frac{4}{3}\right)$

Hence the given straight line is at a distance of 2 unit from the origin and is perpendicular from the origin on the line with angle of inclination $= tan^{-1}\left(\frac{4}{3}\right)$.

EXERCISE-II

1. The slope of Y-axis is:
   (a) infinity (b) 0 (c) ½ (d) 1
2. If two lines are intersecting at an angle of 60°, then the other angle between these two lines is:
   (a) 120° (b) 60° (c) 90° (d) 180°
3. If the equation of straight line is $ax + by + c = 0$, then slope of straight line is:
   (a) $-\frac{b}{a}$ (b) $-\frac{a}{b}$ (c) $\frac{b}{a}$ (d) c
4. The equation of a straight line passing through (x_1, y_1) and having slope m is:
   (a) $y - y_1 = m(x - x_1)$ (b) $x - x_1 = m(y - y_1)$
   (c) $y - y_1 = -m(x - x_1)$ (d) None of these
5. Find the equation of straight line passing through the point $(-5, 4)$ and having slope equal to -2.
6. Find the equations of straight lines which passing through the following pairs of points:
   (i) $(-11,-5),(-3,10)$ (ii) $(0,0),(10,-12)$
7. Find the equation of straight line which makes an angle 60° with x-axis and cuts on intercepts 5 on y-axis above the x-axis.
8. Find the equation of straight line whose intercepts on X-axis and Y-axis are 3 and -9 respectively.
9. Find the equation of straight line which passes through $(2,3)$ and makes intercepts on axes which are equal in magnitude and are of same sign.
10. Find the equation of straight line which passes through $(2,4)$ and sum of whose intercepts on axes is 15.
11. Find the equation of straight line with inclination 45° and passing through the point $(2, \sqrt{2})$ by Symmetric form.
12. Find the equation of straight line such that the length of perpendicular from the origin to the straight line is 10 and the inclination of this perpendicular to the x-axis is 60°.
13. Find the angle between the lines joining the points $(0,0),(4,6)$ and $(1,-1)$, $(6,10)$.
14. Find the angle between the pair of straight lines
    $(-4+\sqrt{3})x + y + 9 = 0$ and $(4+\sqrt{3})x - y + 10 = 0$
15. Find the equation of straight line making an angle 60° with the line

$6x + 5y - 1 = 0$ and passing through the point $(1, -1)$.

16. Check whether the following straight lines are intersecting lines and if they are intersecting lines find their point of intersection:
    (i) $-x + 2y = 4$ and $2x + 6y = -1$
    (ii) $2x - y = -3$ and $4x - 2y = -6$

17. Show that the following lines are concurrent and also find their find of concurrency:
$$2x + 5y - 1 = 0$$
$$x - 3y - 6 = 0$$
and $$x + 5y + 2 = 0$$

18. Check that the following pair of straight lines are parallel or perpendicular or neither:
    (i) $x + 2y = -5$ and $2x + 4y = 12$
    (ii) $3x + 4y = 2$ and $8x - 6y = 5$
    (iii) $y = -5x + 1$ and $5y = x + 10$
    (iv) $y = -5x - 3$ and $y = -5x + 7$
    (v) $4x - 3y = 10$ and $5x - y = 3$

19. Find the distance from the point $(1, 1)$ to the straight line $12x + 5y + 9 = 0$.

20. Find the distance from the point $(2, 3)$ to the straight line $4y = 3x + 1$.

21. Reduce the equation $4x - 3y = 5$ to the
    (i) Slope-intercept from
    (ii) Intercept form
    (iii) Normal form

ANSWERS

1. (a) 2. (a)

3. (b) 4.(a)

5. $2x + y + 6 = 0$

6. (i) $15x - 8y + 125 = 0$ (ii) $6x + 5y = 0$

7. $y = \sqrt{3}x + 5$ 8. $3x - y - 9 = 0$

9. $x + y = 5$ 10. $\frac{x}{10} + \frac{y}{5} = 1$ and $\frac{x}{3} + \frac{y}{12} = 1$

11. $x - y - 2 + \sqrt{2} = 0$ 12. $\frac{x}{2} + \frac{\sqrt{3}}{2}y = 10$

13. $tan^{-1}\left(\frac{7}{43}\right)$ 14. $tan^{-1}\left(\frac{\sqrt{3}}{7}\right)$

15. $y + 1 = \left(\frac{5\sqrt{3}-6}{6\sqrt{3}+5}\right)(x - 1)$ and $y + 1 = \left(\frac{5\sqrt{3}+6}{6\sqrt{3}-5}\right)(x - 1)$

16. (i) The given lines are intersecting lines and their point of intersection is $\left(-\frac{13}{5}, \frac{7}{10}\right)$.

(ii) The given lines are not intersecting lines.

17. Point of concurrency is $(3, -1)$.

18. (i) Parallel Lines (ii) Perpendicular Lines

(iii) Perpendicular Lines (iv) Parallel Lines

(v) Neither Parallel nor Perpendicular

19. 2 units 20. 1 unit

21. (i) $y = \frac{4}{3}x + \frac{5}{3}$, having slope $\frac{4}{3}$ and $y - intercept = \frac{5}{3}$

(ii) $\frac{x}{\left(\frac{5}{4}\right)} + \frac{y}{\left(-\frac{5}{3}\right)} = 1$, having $x - intercept = \frac{5}{4}$ and $y - intercept = -\frac{5}{3}$

(iii) $\frac{4}{5}x - \frac{3}{5}y = 1$, $\cos\alpha = \frac{4}{5}$, $\sin\alpha = -\frac{3}{5}$ and $p = 1$

UNIT V
Geometry of Circle and Software

Learning Objectives

- Students will be able to illustrate a circle and the terms related to it: center, radius and diameter.
- Students will be able to determine the equations of circles in various forms.
- Students will be able to learn about the basic fundamentals of MATLAB/ Scilab and to use these languages for mathematical calculations with MATLAB/ Scilab software.

5.1 CIRCLE

Circle: Circle is the locus of a point which moves in a plane such that its distance from a fixed point always remains constant. The fixed point is called the centre of the circle and the constant distance is called the radius of the circle.

In Fig. 5.1, $C(h,k)$ be the centre of the circle, r be the radius of the circle and $P(x,y)$ be the moving point on the circumference of the circle.

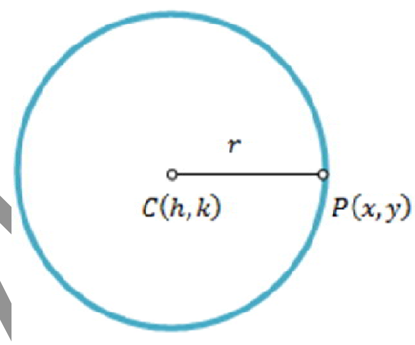

Fig. 5.1

Standard Equation of the Circle: Let $C(h,k)$ be the centre of the circle, r be the radius of the circle and $P(x,y)$ be any point on the circle, then equation of circle is

$$(x-h)^2+(y-k)^2=r^2 \quad (1)$$

which is known as standard equation of circle. This is also known as central form of equation of circle.

Some Particular Cases:

Let $C(h,k)$ be the centre of the circle, r be the radius of the circle and $P(x,y)$ be any point on the circle:

(i) When the centre of the circle coincides with the origin (see Fig. 5.2) i.e. $h=k=0$:

Thus equation (1) becomes:

$\Rightarrow \quad (x-0)^2+(y-0)^2=r^2$

$\Rightarrow \quad x^2+y^2=r^2$

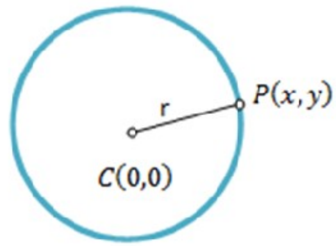

Fig. 5.2

(ii) When the circle passes through the origin (see Fig. 5.3):
Let CR be the perpendicular from the centre on X-axis. Therefore,

$$OR^2+CR^2=OC^2$$

$\Rightarrow \quad (h-0)^2+(k-0)^2=r^2$

$\Rightarrow \quad h^2+k^2=r^2$

Thus equation (1) becomes:

$\Rightarrow \quad (x-h)^2+(y-k)^2=h^2+k^2$

$\Rightarrow \quad x^2+h^2-2hx+y^2+k^2-2ky=h^2+k^2$

$\Rightarrow \quad x^2+y^2-2hx-2ky=0$

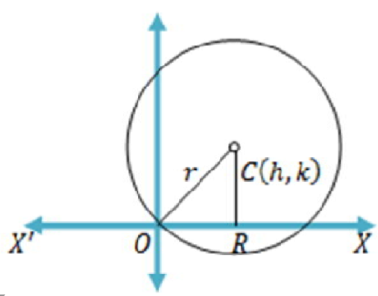

Fig. 5.3

(iii) When the circle passes through the origin and centre lies on the X-axis (see Fig. 5.4) i.e. $k=0$:

In this case radius $r=|h|$
Thus equation (1) becomes:

$\Rightarrow \quad (x-h)^2+(y-0)^2=h^2$

$\Rightarrow \quad x^2+h^2-2hx+y^2=h^2$

$\Rightarrow \quad x^2+y^2-2hx=0$

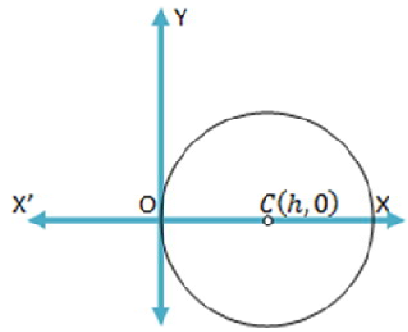

Fig. 5.4

(iv) When the circle passes through the origin and centre lies on the Y-axis(see Fig. 5.5) i.e. $h = 0$:

In this case radius $r = |k|$
Thus equation (1) becomes:

$\Rightarrow \quad (x-0)^2 + (y-k)^2 = k^2$

$\Rightarrow \quad x^2 + y^2 + k^2 - 2ky = k^2$

$\Rightarrow \quad x^2 + y^2 - 2ky = 0$

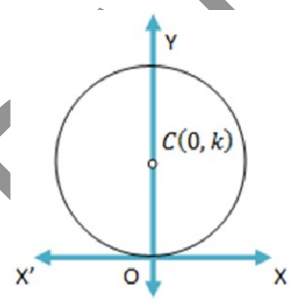

Fig. 5.5

(v) When the circle touches the X-axis (see Fig. 5.6):
In this case radius $r = |k|$
Thus equation (1) becomes:

$\Rightarrow \quad (x-h)^2 + (y-k)^2 = k^2$

$\Rightarrow \quad x^2 + h^2 - 2hx + y^2 + k^2 - 2ky = k^2$

$\Rightarrow \quad x^2 + y^2 - 2hx - 2ky + h^2 = 0$

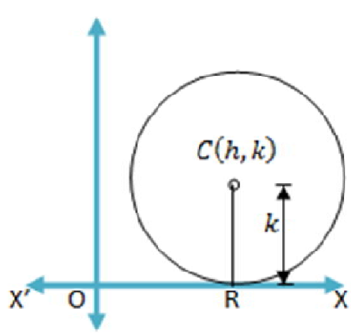

Fig. 5.6

(vi) When the circle touches the Y-axis (see Fig. 5.7):
In this case radius $r = |h|$
Thus equation (1) becomes:

$\Rightarrow \quad (x-h)^2 + (y-k)^2 = h^2$

$\Rightarrow \quad x^2 + h^2 - 2hx + y^2 + k^2 - 2ky = h^2$

$\Rightarrow \quad x^2 + y^2 - 2hx - 2ky + k^2 = 0$

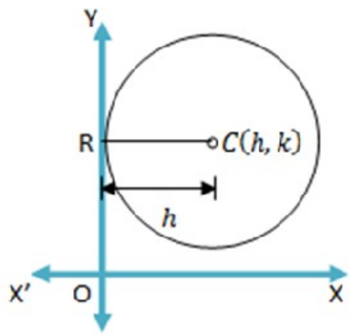

Fig. 5.7

(vii) When the circle touches both the axes (see Fig. 5.8):
In this case radius $r = |h| = |k|$
Thus equation (1) becomes:

$\Rightarrow \quad (x-h)^2 + (y-h)^2 = h^2$

$\Rightarrow \quad x^2 + h^2 - 2hx + y^2 + h^2 - 2hy = h^2$

$\Rightarrow \quad x^2 + y^2 - 2hx - 2hy + h^2 = 0$

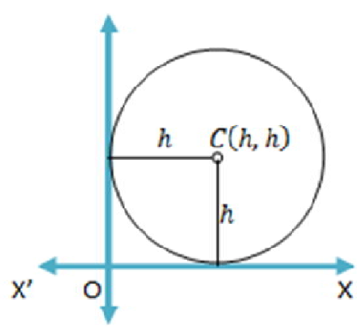

Fig. 5.8

General Equation of Circle: An equation of the form $x^2 + y^2 + 2gx + 2fy + c = 0$ is known as general equation of circle, where g , f and c are arbitrary constants.

To Convert General Equation into Standard Equation: Let the general equation of circle is

$$x^2 + y^2 + 2gx + 2fy + c = 0 \qquad (2)$$

$$\Rightarrow \quad (x^2 + 2gx) + (y^2 + 2fy) + c = 0$$

$$\Rightarrow \quad (x^2 + 2gx + g^2 - g^2) + (y^2 + 2fy + f^2 - f^2) + c = 0$$

$\Rightarrow \quad (x+g)^2 - g^2 + (y+f)^2 - f^2 + c = 0$

$\Rightarrow \quad (x+g)^2 + (y+f)^2 = f^2 + g^2 - c$

$\Rightarrow \quad (x+g)^2 + (y+f)^2 = \left(\sqrt{g^2+f^2-c}\right)^2$

which is the required standard form.

Comparing it with $(x-h)^2 + (y-k)^2 = r^2$, we get

$h = -g$, $k = -f$ and $r = \sqrt{g^2+f^2-c}$.

Hence, centre of given circle (2) is $(-g, -f)$ and radius is $\sqrt{g^2+f^2-c}$.

We observe that the centre of circle (2) is $\left(-\frac{1}{2} \times \mathit{Coefficient\ of}\ x, -\frac{1}{2} \times \mathit{Coefficient\ of}\ y\right)$.

Example 1. Find the centre and radius of the following circles:

(i) $x^2 + y^2 + 2x + 4y - 4 = 0$ (ii) $x^2 + y^2 - 6x + 10y + 3 = 0$

(iii) $x^2 + y^2 - 3x - 5y - 1 = 0$ (iv) $2x^2 + 2y^2 + 5x - 6y + 2 = 0$

(v) $3x^2 + 3y^2 - 6x - 15y + 12 = 0$ (vi) $x^2 + y^2 - 12y + 6 = 0$

(vii) $x^2 + y^2 + 10x - 3 = 0$ (viii) $x^2 + y^2 + 7x - 9y = 0$

Sol.

(i) Given that equation of circle is

$x^2 + y^2 + 2x + 4y - 4 = 0 \qquad (1)$

Compare equation (1) with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get

$2g = 2$, $2f = 4$ and $c = -4$

i.e. $g = 1$, $f = 2$ and $c = -4$

We know that centre of circle is given by $(-g, -f)$

and radius r is given by $\sqrt{g^2+f^2-c}$.

Therefore, centre of circle (1) is $(-1, -2)$

and radius r of circle (1) is

$r = \sqrt{1^2 + 2^2 - (-4)}$

$\Rightarrow \quad r = \sqrt{1+4+4}$

$\Rightarrow \quad r = \sqrt{9} = 3$

(ii) Given that equation of circle is

$x^2 + y^2 - 6x + 10y + 3 = 0 \qquad (2)$

Compare equation (2) with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get

$2g = -6$, $2f = 10$ and $c = 3$

i.e. $g = -3$, $f = 5$ and $c = 3$

We know that centre of circle is given by $(-g, -f)$

and radius r is given by $\sqrt{g^2+f^2-c}$.

Therefore, centre of circle (2) is $(3, -5)$

and radius r of circle (2) is

$$r=\sqrt{(-3)^2+5^2-3}$$

$\Rightarrow$ $r=\sqrt{9+25-3}$

$\Rightarrow$ $r=\sqrt{31}$

(iii) Given that equation of circle is

$x^2+y^2-3x-5y-1=0$ (3)

Compare equation (3) with $x^2+y^2+2gx+2fy+c=0$, we get

$2g=-3$, $2f=-5$ and $c=-1$

i.e. $g=-\frac{3}{2}$, $f=-\frac{5}{2}$ and $c=-1$

We know that centre of circle is given by $(-g,-f)$

and radius r is given by $\sqrt{g^2+f^2-c}$.

Therefore, centre of circle (3) is $\left(\frac{3}{2},\frac{5}{2}\right)$

and radius r of circle (3) is

$$r=\sqrt{\left(-\frac{3}{2}\right)^2+\left(-\frac{5}{2}\right)^2-(-1)}$$

$\Rightarrow$ $r=\sqrt{\frac{9}{4}+\frac{25}{4}+1}$

$\Rightarrow$ $r=\sqrt{\frac{9+25+4}{4}}=\sqrt{\frac{38}{4}}$

$\Rightarrow$ $r=\sqrt{\frac{19}{2}}$

(iv) Given that equation of circle is

$2x^2+2y^2+5x-6y+2=0$

Diving this equation by 2, we get

$x^2+y^2+\frac{5}{2}x-3y+1=0$ (4)

Compare equation (4) with $x^2+y^2+2gx+2fy+c=0$, we get

$2g=\frac{5}{2}$, $2f=-3$ and $c=1$

i.e. $g=\frac{5}{4}$, $f=-\frac{3}{2}$ and $c=1$

We know that centre of circle is given by $(-g,-f)$

and radius r is given by $\sqrt{g^2+f^2-c}$.

Therefore, centre of circle (4) is $\left(-\frac{5}{4},\frac{3}{2}\right)$

and radius r of circle (4) is

$$r=\sqrt{\left(\frac{5}{4}\right)^2+\left(-\frac{3}{2}\right)^2-1}$$

$\Rightarrow \quad r=\sqrt{\frac{25}{16}+\frac{9}{4}-1}$

$\Rightarrow \quad r=\sqrt{\frac{25+36-16}{16}}=\sqrt{\frac{45}{16}}$

$\Rightarrow \quad r=\frac{3\sqrt{5}}{4}$

(v) Given that equation of circle is

$3x^2+3y^2-6x-15y+12=0$

Diving this equation by 3, we get

$x^2+\ y^2-2x-5y+4=0$ (5)

Compare equation (5) with $x^2+y^2+2gx+2fy+c=0$, we get

$2g=-2$, $2f=-5$ and $c=4$

i.e. $g=-1$, $f=-\frac{5}{2}$ and $c=4$

We know that centre of circle is given by $(-g,-f)$

and radius r is given by $\sqrt{g^2+f^2-c}$

Therefore, centre of circle (5) is $\left(1,\frac{5}{2}\right)$

and radius r of circle (5) is

$r=\sqrt{(-1)^2+\left(-\frac{5}{2}\right)^2-4}$

$\Rightarrow \quad r=\sqrt{1+\frac{25}{4}-4}$

$\Rightarrow \quad r=\sqrt{\frac{4+25-16}{4}}=\sqrt{\frac{13}{4}}$

$\Rightarrow \quad r=\frac{\sqrt{13}}{2}$

(vi) Given that equation of circle is

$x^2+y^2-12y+6=0$ (6)

Compare equation (6) with $x^2+y^2+2gx+2fy+c=0$, we get

$2g=0$, $2f=-12$ and $c=6$

i.e. $g=0$, $f=-6$ and $c=6$

We know that centre of circle is given by $(-g,-f)$

and radius r is given by $\sqrt{g^2+f^2-c}$.

Therefore, centre of circle (6) is $(0,6)$

and radius r of circle (6) is

$r=\sqrt{(0)^2+(-6)^2-6}$

$\Rightarrow \quad r=\sqrt{0+36-6}$

$\Rightarrow \quad r = \sqrt{30}$

(vii) Given that equation of circle is

$$x^2 + y^2 + 10x - 3 = 0 \qquad (7)$$

Compare equation (7) with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get

$2g = 10$, $2f = 0$ and $c = -3$

i.e. $g = 5$, $f = 0$ and $c = -3$

We know that centre of circle is given by $(-g, -f)$

and radius r is given by $\sqrt{g^2 + f^2 - c}$.

Therefore, centre of circle (7) is $(-5, 0)$

and radius r of circle (7) is

$$r = \sqrt{(5)^2 + (0)^2 - (-3)}$$

$\Rightarrow \quad r = \sqrt{25 + 0 + 3} = \sqrt{28}$

$\Rightarrow \quad r = 2\sqrt{7}$

(viii) Given that equation of circle is

$$x^2 + y^2 + 7x - 9y = 0 \qquad (8)$$

Compare equation (8) with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get

$2g = 7$, $2f = -9$ and $c = 0$

i.e. $g = \frac{7}{2}$, $f = -\frac{9}{2}$ and $c = 0$

We know that centre of circle is given by $(-g, -f)$

and radius r is given by $\sqrt{g^2 + f^2 - c}$.

Therefore, centre of circle (8) is $\left(-\frac{7}{2}, \frac{9}{2}\right)$

and radius r of circle (8) is

$$r = \sqrt{\left(\frac{7}{2}\right)^2 + \left(-\frac{9}{2}\right)^2 - 0}$$

$\Rightarrow \quad r = \sqrt{\frac{49}{4} + \frac{81}{4}}$

$\Rightarrow \quad r = \sqrt{\frac{49 + 81}{4}} = \sqrt{\frac{130}{4}}$

$\Rightarrow \quad r = \sqrt{\frac{65}{2}}$

Example 2. Find the equations of circles if their centres and radii are as follow:

(i) $(0,0)$, 2 (ii) $(2,0)$, 5 (iii) $(0,-3)$, 3

(iv) $(8,-4)$, 1 (v) $(3,6)$, 6 (vi) $(-2,-5)$, 10

Sol.

(i) Given that centre of circle is $(0,0)$ and radius is 2 i.e. $h=0$, $k=0$ and $r=2$.
We know that the equation of circle, when centre and radius is given, is

$$(x-h)^2+(y-k)^2=r^2$$
$$\Rightarrow \quad (x-0)^2+(y-0)^2=2^2$$
$$\Rightarrow \quad x^2+y^2=4$$
$$\Rightarrow \quad x^2+y^2-4=0$$

which is the required equation of circle.

(ii) Given that centre of circle is $(2,0)$ and radius is 5 i.e. $h=2$, $k=0$ and $r=5$.
We know that the equation of circle, when centre and radius is given, is

$$(x-h)^2+(y-k)^2=r^2$$
$$\Rightarrow \quad (x-2)^2+(y-0)^2=5^2$$
$$\Rightarrow \quad x^2+4-4x+y^2=25$$
$$\Rightarrow \quad x^2+y^2-4x-21=0$$

which is the required equation of circle.

(iii) Given that centre of circle is $(0,-3)$ and radius is 3 i.e. $h=0$, $k=-3$ and $r=3$.
We know that the equation of circle, when centre and radius is given, is

$$(x-h)^2+(y-k)^2=r^2$$
$$\Rightarrow \quad (x-0)^2+(y-(-3))^2=3^2$$
$$\Rightarrow \quad x^2+(y+3)^2=3^2$$
$$\Rightarrow \quad x^2+y^2+9+6y=9$$
$$\Rightarrow \quad x^2+y^2+6y=0$$

which is the required equation of circle.

(iv) Given that centre of circle is $(8,-4)$ and radius is 1 i.e. $h=8$, $k=-4$ and $r=1$.
We know that the equation of circle, when centre and radius is given, is

$$(x-h)^2+(y-k)^2=r^2$$
$$\Rightarrow \quad (x-8)^2+(y-(-4))^2=1^2$$
$$\Rightarrow \quad (x-8)^2+(y+4)^2=1^2$$
$$\Rightarrow \quad x^2+64-16x+y^2+16+8y=1$$
$$\Rightarrow \quad x^2+y^2-16x+8y+79=0$$

which is the required equation of circle.

(v) Given that centre of circle is $(3,6)$ and radius is 6 i.e. $h=3$, $k=6$ and $r=6$.
We know that the equation of circle, when centre and radius is given, is

$$(x-h)^2+(y-k)^2=r^2$$
$$\Rightarrow \quad (x-3)^2+(y-6)^2=6^2$$
$$\Rightarrow \quad x^2+9-6x+y^2+36-12y=36$$
$$\Rightarrow \quad x^2+y^2-6x-12y+9=0$$

which is the required equation of circle.

(vi) Given that centre of circle is $(-2,-5)$ and radius is 10 i.e. $h=-2$, $k=-5$ and $r=10$.

We know that the equation of circle, when centre and radius is given, is

$$(x-h)^2+(y-k)^2=r^2$$

$$\Rightarrow \quad (x-(-2))^2+(y-(-5))^2=10^2$$

$$\Rightarrow \quad (x+2)^2+(y+5)^2=100$$

$$\Rightarrow \quad x^2+4+4x+y^2+25+10y=100$$

$$\Rightarrow \quad x^2+y^2+4x+10y-71=0$$

which is the required equation of circle.

Process to find the equation of circle which passes through the three given points:

Suppose that the circle passes through the following three points $(x_1\,,y_1)$, $(x_2\,,y_2)$ and $(x_3\,,y_3)$.

Step 1: Write the general equation of the circle, i.e.,

$$x^2+y^2+2gx+2fy+c=0 \qquad (1)$$

Step 2: Since the circle passes through the points $(x_1\,,y_1)$, $(x_2\,,y_2)$ and $(x_3\,,y_3)$, so these points will satisfy the equation (1). Hence substitute x-coordinates and y-coordinates of these points one by one in equation (1) and we will get three linear equations in three unknowns. The three linear equations are as follow

$$x_1{}^2+y_1{}^2+2gx_1+2fy_1+c=0 \qquad (2)$$

$$x_2{}^2+y_2{}^2+2gx_2+2fy_2+c=0 \qquad (3)$$

$$x_3{}^2+y_3{}^2+2gx_3+2fy_3+c=0 \qquad (4)$$

Step 3: Solve equations (2), (3) and (4) for g, f and c.

Step 4: Use the values of g, f and c in equation (1), we will get the required equation of the circle.

Example 3. Find the equations of circles which passes through the following three points:

(i) $(0\,,0)$, $(3\,,0)$ and $(0\,,4)$

(ii) $(0\,,0)$, $(-5\,,2)$ and $(3\,,6)$

(iii) $(2\,,-3)$, $(1\,,4)$ and $(-1\,,2)$

Sol. (i) Let the equation of the circle is

$$x^2+y^2+2gx+2fy+c=0 \qquad (1)$$

Circle (1) passes through the point $(0\,,0)$, therefore

$$0^2+0^2+2g(0)+2f(0)+c=0$$

i.e. $c=0$ (2)

Circle (1) passes through the point $(3\,,0)$, therefore

$$3^2+0^2+2g(3)+2f(0)+0=0$$

$$\Rightarrow \quad 9+6g=0$$

$\Rightarrow \quad g = -\frac{9}{6}$

$\Rightarrow \quad g = -\frac{3}{2}$ (3)

Also, the circle (1) passes through the point $(0\,,4)$, therefore

$$0^2 + 4^2 + 2g(0) + 2f(4) + 0 = 0$$

$\Rightarrow \quad 16 + 8f = 0$

$\Rightarrow \quad f = -\frac{16}{8}$

$\Rightarrow \quad f = -2$ (4)

Using (2), (3) and (4) in equation (1), we have

$$x^2 + y^2 + 2\left(-\frac{3}{2}\right)x + 2(-2)y + 0 = 0$$

$\Rightarrow \quad x^2 + y^2 - 3x - 4y = 0$

which is the required equation of the circle.

(ii) Let the equation of the circle is

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad (1)$$

Circle (1) passes through the point $(0\,,0)$, therefore

$$0^2 + 0^2 + 2g(0) + 2f(0) + c = 0$$

i.e. $\quad c = 0$ (2)

Circle (1) passes through the point $(-5\,,2)$, therefore

$$(-5)^2 + 2^2 + 2g(-5) + 2f(2) + c = 0$$

$\Rightarrow \quad 25 + 4 - 10g + 4f + 0 = 0$

$\Rightarrow \quad -10g + 4f = -29$ (3)

Also, the circle (1) passes through the point $(3\,,6)$, therefore

$$3^2 + 6^2 + 2g(3) + 2f(6) + c = 0$$

$\Rightarrow \quad 9 + 36 + 6g + 12f + 0 = 0$

$\Rightarrow \quad 6g + 12f = -45$

$\Rightarrow \quad 2g + 4f = -15$ (4)

Solving equations (3) and (4):

Applying $equation$ (3) + 5 × $equation$ (4), we get

$$\begin{array}{r} -10g + 4f = -29 \\ \underline{10g + 20f = -75} \\ 24f = -104 \end{array}$$

$\Rightarrow \quad f = -\frac{104}{24}$

$\Rightarrow \quad f = -\frac{13}{3}$ (5)

Using equation (5) in equation (4), we get

$$2g + 4\left(-\frac{13}{3}\right) = -15$$

$\Rightarrow \quad 6g - 52 = -45$

$\Rightarrow \quad g = \frac{7}{6} \qquad (6)$

Now, using equations (2), (5) and (6) in equation (1), we have

$$x^2 + y^2 + 2\left(\frac{7}{6}\right)x + 2\left(-\frac{13}{3}\right)y + 0 = 0$$

$$\Rightarrow \quad x^2 + y^2 + \frac{7}{3}x - \frac{26}{3}y = 0$$

$$\Rightarrow \quad 3x^2 + 3y^2 + 7x - 26y = 0$$

which is the required equation of the circle.

(iii) Let the equation of the circle is

$$x^2 + y^2 + 2gx + 2fy + c = 0 \qquad (1)$$

Circle (1) passes through the point $(2, -3)$, therefore

$$2^2 + (-3)^2 + 2g(2) + 2f(-3) + c = 0$$

$$\Rightarrow \quad 4 + 9 + 4g - 6f + c = 0$$

$$\Rightarrow \quad 4g - 6f + c = -13 \qquad (2)$$

Circle (1) passes through the point $(1, 4)$, therefore

$$1^2 + 4^2 + 2g(1) + 2f(4) + c = 0$$

$$\Rightarrow \quad 1 + 16 + 2g + 8f + c = 0$$

$$\Rightarrow \quad 2g + 8f + c = -17 \qquad (3)$$

Also, the circle (1) passes through the point $(-1, 2)$, therefore

$$(-1)^2 + 2^2 + 2g(-1) + 2f(2) + c = 0$$

$$\Rightarrow \quad 1 + 4 - 2g + 4f + c = 0$$

$$\Rightarrow \quad -2g + 4f + c = -5 \qquad (4)$$

Applying $equation\ (2) - equation\ (3)$, we get

$$2g - 14f = 4 \qquad (5)$$

Applying $equation\ (3) - equation\ (4)$, we get

$$4g + 4f = -12 \qquad (6)$$

Applying $2 \times equation\ (5) - equation\ (6)$, we get

$$-28f - 4f = 8 + 12$$

$$\Rightarrow \quad -32f = 20$$

$$\Rightarrow \quad f = -\frac{5}{8} \qquad (7)$$

Using equation (7) in equation (5), we get

$$2g - 14\left(-\frac{5}{8}\right) = 4$$

$$\Rightarrow \quad 2g = -\frac{35}{4} + 4$$

$$\Rightarrow \quad g = -\frac{19}{8} \qquad (8)$$

Using equations (7) and (8) in equation (4), we get

$$-2\left(-\frac{19}{8}\right)+4\left(-\frac{5}{8}\right)+c=-5$$

$$\Rightarrow \qquad \frac{19}{4}-\frac{5}{2}+c=-5$$

$$\Rightarrow \qquad c=-5-\frac{19}{4}+\frac{5}{2}$$

$$\Rightarrow \qquad c=-\frac{29}{4} \qquad (9)$$

Now, using equations (7), (8) and (9) in equation (1), we have

$$x^2+y^2+2\left(-\frac{19}{8}\right)x+2\left(-\frac{5}{8}\right)y-\frac{29}{4}=0$$

$$\Rightarrow \qquad 4x^2+4y^2-19x-5y-29=0$$

which is the required equation of the circle.

<u>Equation of Circle in Diametric Form</u>: Let $A(x_1,y_1)$ and $B(x_2,y_2)$ be the end points of diameter of a circle then equation of circle is given by

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

Example 4. Find the equations of circles if end points of their diameters are as follow:

(i) $(1,5)$ and $(3,6)$

(ii) $(1,0)$ and $(-2,-5)$

(iii) $(0,0)$ and $(8,-6)$

(iv) $(-3,2)$ and $(-7,9)$

Sol.

(i) Given that end points of diameter of circle are $(1,5)$ and $(3,6)$.
Comparing these points with (x_1,y_1) and $(x_2\,,y_2)$ respectively, we get
$x_1=1\,,y_1=5\,,x_2=3$ and $y_2=6$.
We know that the equation of circle in diametric form is

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

$$\Rightarrow \qquad (x-1)(x-3)+(y-5)(y-6)=0$$

$$\Rightarrow \qquad x^2-3x-x+3+y^2-6y-5y+30=0$$

$$\Rightarrow \qquad x^2+y^2-4x-11y+33=0$$

which is the required equation of circle.

(ii) Given that end points of diameter of circle are $(1,0)$ and $(-2,-5)$.
Comparing these points with (x_1,y_1) and $(x_2\,,y_2)$ respectively, we get
$x_1=1\,,y_1=0\,,x_2=-2$ and $y_2=-5$.
We know that the equation of circle in diametric form is

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

$$\Rightarrow \qquad (x-1)(x-(-2))+(y-0)(y-(-5))=0$$

$$\Rightarrow \qquad (x-1)(x+2)+y(y+5)=0$$

$$\Rightarrow \qquad x^2+2x-x-2+y^2+5y=0$$

$$\Rightarrow \qquad x^2+y^2+x+5y-2=0$$

which is the required equation of circle.

(iii) Given that end points of diameter of circle are $(0,0)$ and $(8,-6)$.

Comparing these points with (x_1, y_1) and $(x_2\ , y_2)$ respectively, we get
$x_1 = 0\ , y_1 = 0\ , x_2 = 8$ and $y_2 = -6$.
We know that the equation of circle in diametric form is

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$
$$\Rightarrow \quad (x-0)(x-8)+(y-0)(y-(-6))=0$$
$$\Rightarrow \quad x(x-8)+y(y+6)=0$$
$$\Rightarrow \quad x^2-8x+y^2+6y=0$$
$$\Rightarrow \quad x^2+y^2-8x+6y=0$$

which is the required equation of circle.

(iv) Given that end points of diameter of circle are $(-3,2)$ and $(-7,9)$.
Comparing these points with (x_1, y_1) and $(x_2\ , y_2)$ respectively, we get
$x_1 = -3\ , y_1 = 2\ , x_2 = -7$ and $y_2 = 9$.
We know that the equation of circle in diametric form is

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$
$$\Rightarrow \quad (x-(-3))(x-(-7))+(y-2)(y-9)=0$$
$$\Rightarrow \quad (x+3)(x+7)+(y-2)(y-9)=0$$
$$\Rightarrow \quad x^2+7x+3x+21+y^2-9y-2y+18=0$$
$$\Rightarrow \quad x^2+y^2+10x-11y+39=0$$

which is the required equation of circle.

EXERCISE-I

1. Equation of circle with centre at $(2,0)$ and radius 7 is:
   (a) $x^2 + 4 - 4x + y^2 = 14$ (b) $x^2 + 4 - 4x + y^2 = 49$
   (c) $x^2 - 4 + 4x + y^2 = 49$ (d) None of these
2. Equation of circle whose centre is origin and radius v is:
   (a) $x^2 + y^2 + 2gx + 2fy + c = 0$ (b) $x^2 + y^2 = v^2$
   (c) $x^2 - 2vx + y^2 = v^2$ (d) $x^2 + y^2 = 0$
3. Equation of circle in diametric form is:
   (a) $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$ (b) $(x - x_1)(y - y_1) + (x - x_2)(y - y_2) = 0$
   (c) $(x - y_1)(x - x_2) + (y - x_1)(y - x_2) = 0$ (d) $(x - y)(x - y) + (y_1 - x_1)(y_2 - x_2) = 0$
4. Find the centres and radii of the following circles:
   (i) $9x^2 + 9y^2 - 12x - 30y + 24 = 0$
   (ii) $x^2 + y^2 - 6y - 24 = 0$
   (iii) $x^2 + y^2 + 20x - 5 = 0$
5. Find the equations of circles whose centres and radii are as follow:
   (i) $(8,8)\ , 2$ (ii) $(6,3)\ , 6$
6. Find the equations of circles if end points of their diameters are as follows:
   (i) $(2,6)$ and $(3,16)$ (ii) $(-1,-1)$ and $(4,5)$
7. Find the equations of circles which passes through the following three points:

(i) $(-3\,,0)$, $(0\,,5)$ and $(0\,,0)$
(ii) $(2\,,1)$, $(0\,,5)$ and $(-1\,,2)$
(iii) $(-6\,,5)$, $(-3\,,-4)$ and $(2\,,1)$

ANSWERS

1. (b)
2. (b)
3. (a)
4. (i) $\left(\frac{2}{3},\frac{5}{3}\right);\sqrt{\frac{5}{9}}$ (ii) $(0,3);\sqrt{33}$
   (iii) $(-10,0);\sqrt{105}$
5. (i) $x^2+y^2-16x-16y+124=0$
   (ii) $x^2+y^2-12x-6y+9=0$
6. (i) $x^2+y^2-5x-22y+102=0$
   (ii) $x^2+y^2-3x-4y-9=0$
7. (i) $x^2+y^2+3x-5y=0$
   (ii) $x^2+y^2-2x-6y+5=0$
   (iii) $x^2+y^2+6x-2y-15=0$

Software

5.2 MATLAB Or SciLab software

Introduction to MATLAB

In the introduction we will describe how MATLAB handles numerical expressions and mathematical formulae. The name MATLAB stands for MATrix LABoratory. Primarily, it is developed by Cleve Moler in the 1970's. MATLAB was written originally to provide easy access to matrix software and derived from FORTRAN subroutines LINPACK (linear system package) and EISPACK (Eigen system package). It is again rewritten in C in the 1980's with more functionality, which included plotting routines. Then the MathWorks Inc. was created (1984) to market to continue development of MATLAB. It has been designed to supersede LINPACK and EISPACK.

MATLAB is a high-performance language for technical computing. It provides an interactive environment to perform reports and data analysis. It also allows the implementation of computing algorithms, plotting graphs and other matrix functions. It contains built-in editing and debugging tools. These are the excellent features of MATLAB for teaching and research.

MATLAB has many advantages compared to conventional computer languages (e.g., C, FORTRAN) for solving technical problems. MATLAB is an interactive system whose basic data element is an array that does not require dimensioning. It has powerful built-in routines that enable a very wide variety of computations. It also has easy to use graphics commands that make the visualization of results immediately available. There are toolboxes for signal processing, symbolic computation, control theory, simulation, optimization, and several other fields of applied science and engineering.

Starting with MATLAB

After completion of installation process and logging into your account, we can enter in MATLAB by double-clicking on the MATLAB shortcut *icon* available on our desktop. When we start MATLAB, a special window called the MATLAB desktop appears which contains some *other* windows. The major tools within the MATLAB desktop are:

1. **The Command Window**: It is used to enter commands and data.
2. **The Graphic Window**: It is used to display plots and graphs.
3. **The Edit Window**: It which is used to create and modify M-files. (M-files are files that contain a program or script in MATLAB commands).
4. **The Command History**: It displays a log of statement that is ran in the current and previous MATLAB sessions.
5. **The Workspace**: It contains variables that are created or imported by users into MATLAB from data files or other programs.
6. **The Current Directory**: It is a reference location that MATLAB uses to find files.
7. **The Help Browser**: It is a Web browser integrated into the MATLAB desktop that displays HTML documents.
8. **The Start button**: It provides easy access to tools, demos and documentation for MathWorks products. Through this users can create and run MATLAB shortcuts, which are groups of MATLAB statements.

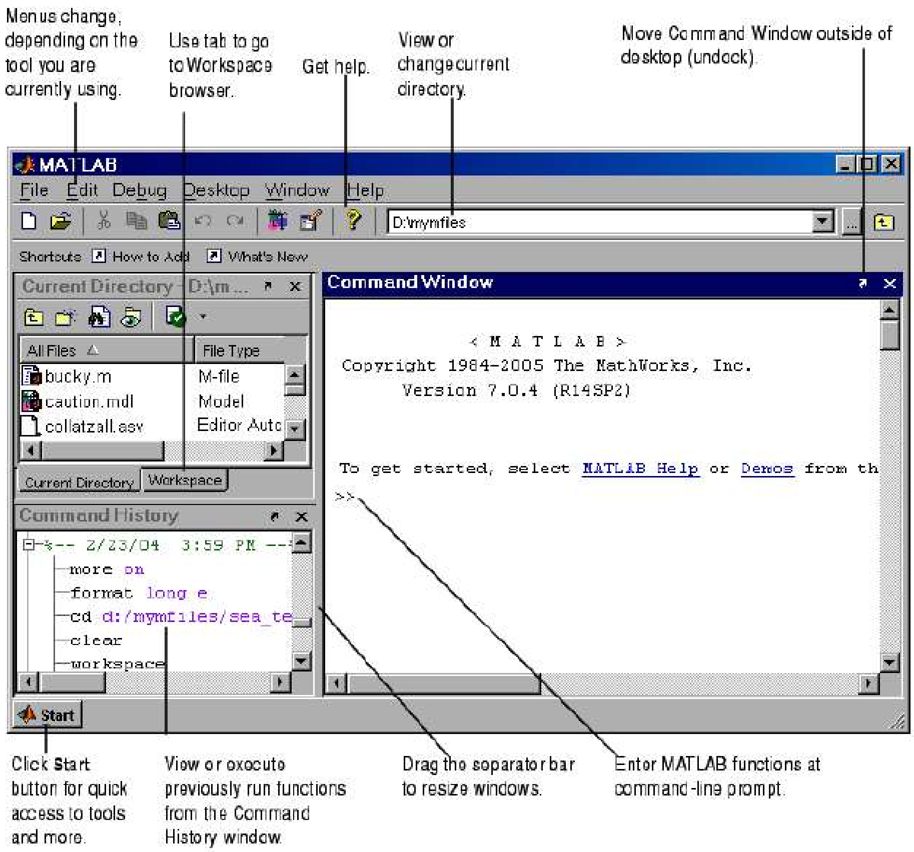

Fig. 5.9: The graphical interface to the MATLAB (Version 7.0.4) workspace

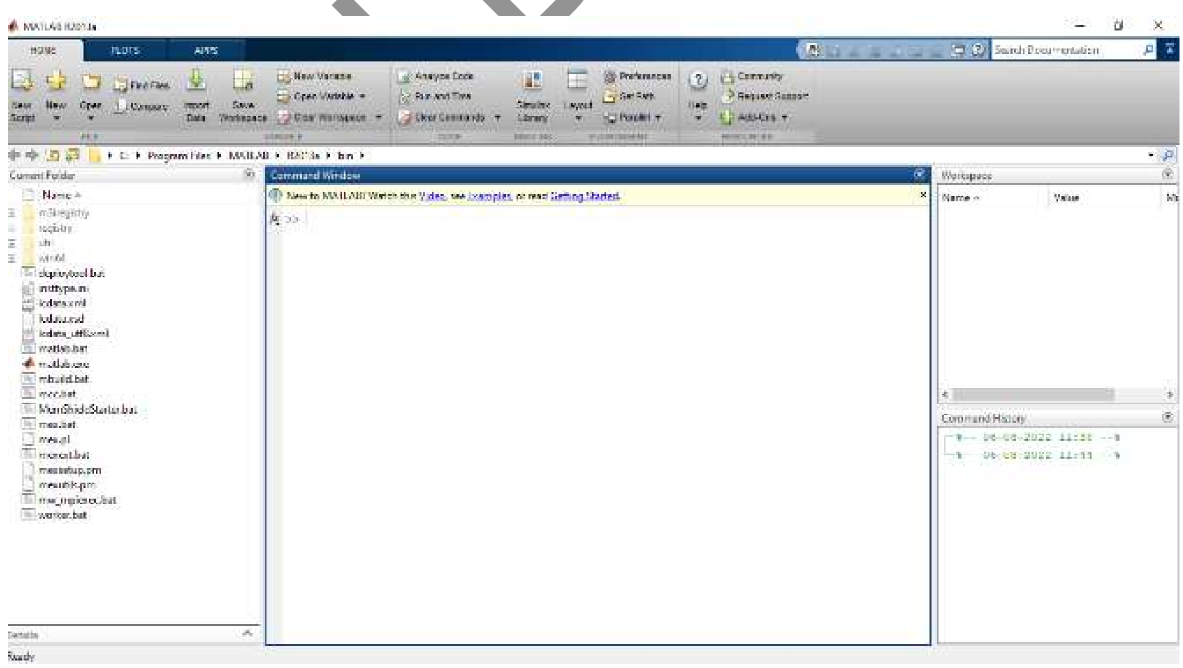

Fig. 5.10: The graphical interface to the MATLAB (Version R2013a) workspace

When MATLAB is started for the first time, the screen looks like the one that shown in the Fig. 5.9 or Fig. 5.10. This illustration also shows the default configuration of the MATLAB desktop. You can customize the arrangement of tools and documents to suit your needs. The screen will produce the MATLAB prompt **>>** (or **EDU >>**), which indicates that MATLAB is waiting for a command to be entered.

>> for full version

and

EDU>> for educational version.

Quitting MATLAB

In order to quit MATLAB, type **quit** or **exit** after the prompt, followed by pressing the enter or return key.

Entering Commands

To execute commands, every command has to be followed by enter key. MATLAB commands are case sensitive and **lower case** letters are used throughout. To execute an M-file (such as Demo_1.m), simply enter the name of the file without its extension (as in Demo_1).

The Semicolon Symbol (;)

If the semicolon symbol (;) is typed at the end of a command, the output of the command is not displayed.

The Percent Symbol (%)

If the percent symbol (%) is typed at the beginning of a line, then the line is designated as a comment. When the enter key is pressed, the line is not executed in this case.

The clc Command

Typing **clc** command and pressing enter key cleans the command window. Once the clc command is executed, a clear window is displayed.

The clear Command

The **clear** command remove all the variables from the memory.

Help

To obtain help on a particular topic in the MATLAB-list of built-in functions, e.g., Determinant, type help det after prompt.

Special Variable Names and Constants

1. **ans** It represents a value computed by an expression but not stored in a variable name.
2. **i, j** Imaginary unit/operator defined as $\sqrt{-1}$.
3. **inf** Infinity (∞)
4. **eps** Smallest floating point number.
5. **pi** π = 3.141592653589793
6. **NaN** Stands for not a number. E.g., 0/0.
7. **clock** It represents the current time in a row vector of six elements containing year, month, day, hour, minute, and seconds.
8. **date** It represents the current date in a character string format.

Note: (i) Overwriting/using these variables and constants should be avoided in programming.

(ii) MATLAB is a case sensitive language for function, script and variable names for all the platforms. For instance, Ab, ab, aB and AB are the names of four different variables.

Arithmetic Operations

Name of Arithmetic Operation	Symbol	Exapmle
Addition	+	10+5 = 15
Subtraction	-	10-5 = 5
Multiplication	*	10*5 = 50
Right Division	/	10/5 = 2
Left Division	\	10\5 = 5/10 = ½
Exponentiation	^	10^5 = 10^5 = 100000

Display Formats

Command	Description
format short	Fixed point with four decimal digits
format short e	Scientific notation with four decimal digits
format short g	Best of five digits fixed or floating point
format long	Fixed point with fourteen decimal digits
format long e	Scientific notation with fifteen decimal digits
format long g	Best of fifteen digits fixed or floating point
format bank	Two decimal digits
format compact	Suppresses the display of blank lines
format loose	Keeps the display of blank lines (default)

Trigonometric and Inverse Trigonometric Functions

Function Name	Description
sin(x)	Sine of argument x in radians
cos(x)	Cosine of argument x in radians
tan(x)	Tangent of argument x in radians
sec(x)	Secant of argument x in radians
csc(x)	Cosecant of argument x in radians
cot(x)	Cotangent of argument x in radians
asin(x)	Inverse sine, results in radians.
acos(x)	Inverse cosine, results in radians.
atan(x)	Inverse tangent, results in radians.
asec(x)	Inverse secant, results in radians.
acsc(x)	Inverse cosecant, results in radians.
acot(x)	Inverse cotangent, results in radians.

Some General Commands

Command Name	Description
Clc	It clears the command window.
Clear	It clears the Workspace, all variables are removed.
clear all	Same as the command clear.
clear a b c	It clears only the variables a, b and c from the Workspace.
Clf	It clears the figure window.
Who	Lists variables currently in the Workspace.
whos	Lists variables currently in the Workspace with their sizes.

<u>Using MATLAB as a calculator</u>

Let's start with a very simple interactive calculation at the very beginning. For example, suppose that we have to calculate an expression, $2 \times 9 - 5$, then we will type it at the prompt command (>>) as follows:

```
>> 2*9-5
ans =
      13
```

Here we have not assigned any specify variable name as an output variable, so MATLAB used a default variable **ans** (short for answer) to store the results of the current expression. The variable **ans** is created (or overwritten, if it is already existed). To avoid this, we may assign a value to a variable name. For example, if we type

```
>> a = 2*9-5
a =
    13
```

Here the value of the expression is assigned to the variable a. This variable name can be used to refer to the results of the previous computations. Therefore, computing 2a+10 will result in

```
>> 2*a+10
ans =
       36
```

Note: From above we can learn how to create a variable in MATLAB. So the syntax to create a variable in MATLAB is:

variable name = a value or an expression

Further, to evaluate the value of $\sin\left(\frac{\pi}{3}\right) + 2\cos\left(\frac{\pi}{2}\right)$ and to assign the value of this expression into the variable p, we will type the following:

```
>> p = sin(pi/3)+2*cos(pi/2)
p =
     0.8660
```

Merits

1. MATLAB is relatively easy to learn.
2. MATLAB may behave as a *calculator* or as a *programming language.*
3. A large set of toolboxes is available. A toolbox is a collection of MATLAB functions specific for a subject, e.g. the signal processing toolbox or the control toolbox. Over 8000 functions available for various disciplines.
4. At any time, variables (results of simulations) are stored in the workspace for debugging and inspection.
5. MATLAB combine nicely calculation and it has excellent visualization (plots) capabilities.
6. MATLAB can solve complex algebraic equations.
7. MATLAB can process and communicate the signals.
8. MATLAB is interpreted (not compiled), errors are easy to fix.
9. MATLAB is optimized to be relatively fast when performing matrix operations.
10. Quick code development.

Demerits

1. Very expensive for non students, although there are some free clones, such as Octave or Scilab that are MATLAB compatible (but not 100%).
2. MATLAB is not a *general* purpose programming language such as C, C++, or FORTRAN.
3. MATLAB is designed for scientific computing, and is not well suitable for other applications.
4. MATLAB is an interpreted language, slower than a compiled language such as C++.
5. Code execution can be slow if programmed carelessly without vectorization.
6. MATLAB commands are specific for MATLAB usage. Most of them do not have a direct equivalent with other programming language commands.

Introduction to Scilab:

Scilab is a scientific software package developed by INRIA and ENPC. It is an open-source software that is used for data analysis and computation. It is also an alternative for MATLAB as this is not open-source. Scilab is named as Scientific Laboratory which resolves the problem related to numeric data and scientific visualization. It is capable of interactive calculations as well as automation of computations through programming. It provides all basic operations on matrices through built-in functions so that the trouble of developing and testing code for basic operations are completely avoided. Further, the numerous toolboxes that are available for various specialized applications make it an important tool for research. Being compatible with Matlab, all available Matlab M-files can be directly used in Scilab with the help of the Matlab to Scilab translator. The greatest features of Scilab are that it is multiplatform and is free. It is available for many operating systems including Windows, Linux and MacOS X. Some basic features of Scilab are given below:

1. It is capable to solve different algebraic equations.
2. It supports the development of certain complicated algorithms.
3. Capable of the model the previous computations.
4. Performs visualization in Bar Graphs, lines, Histograms, MathML annotation.

When we start up Scilab, we see a window shown in Fig. 5.11.

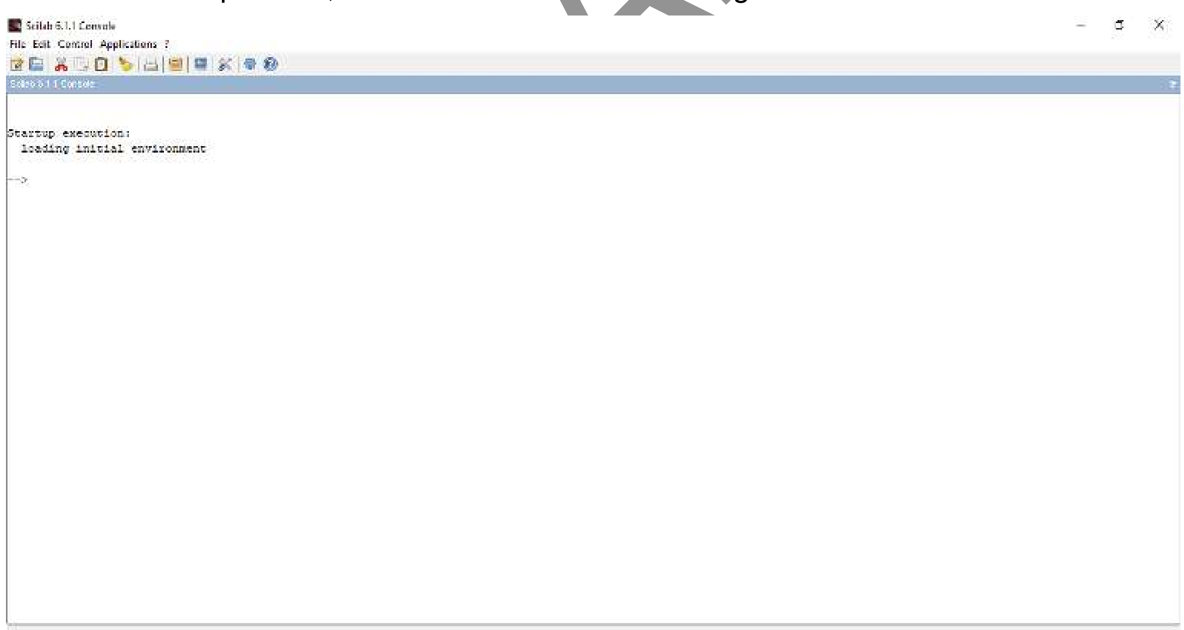

Fig. 5.11

The user enters Scilab commands after the prompt **-->**. But many of the commands are also available through the menu at the top. The most important menu for a beginner is the “Help” menu. Clicking on the “Help” menu opens up the *Help Browser*, showing a list of topics on which help is available (see the Fig. 5.12).

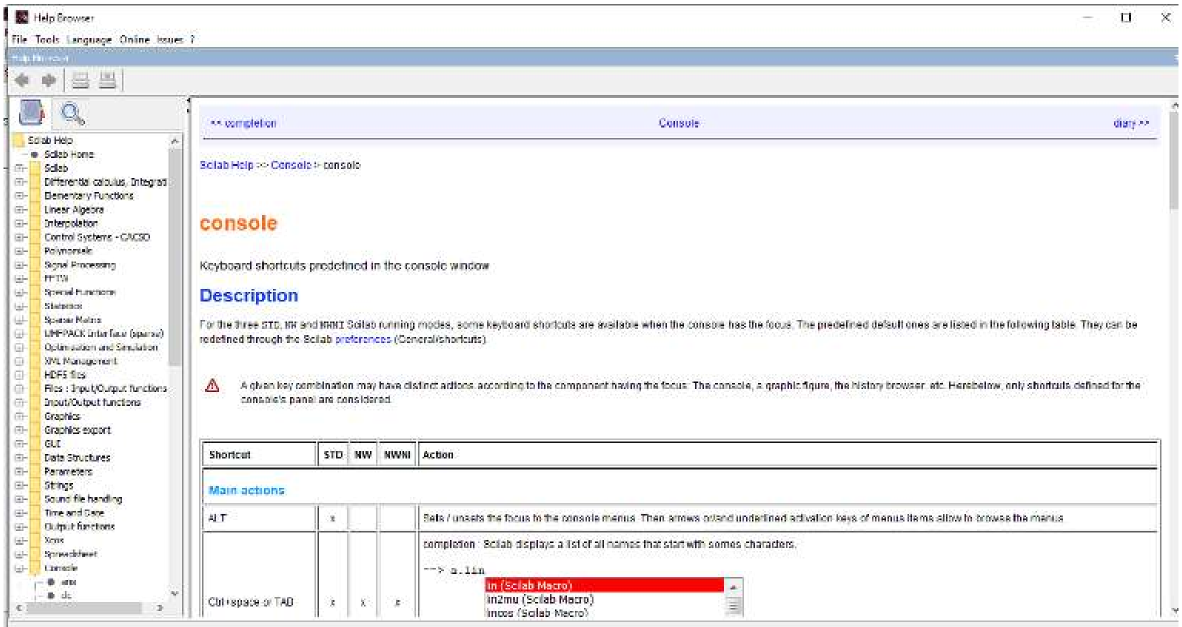

Fig. 5.12

Clicking on the relevant topic takes you to hyperlinked documents similar to web pages. The Help Browser has two tabs – *Table of Contents* and *Search*.

Table of Contents contains an alphabetically arranged list of topics and we may use the Search tab to search the help for particular topic by typing the topic in it.

Some calculations in Scilab and their results are given below:

```
--> 2+3
 ans  =

   5.

--> a=2+3
 a  =

   5.

--> p=sin(%pi/3)+2*cos(%pi/3)
 p  =

   1.8660254

--> b=2+3-5*6
 b  =

  -25.
```

Here we can see that **ans** is used as the default variable. Further, the predefined constants in Scilab are shown in the table given below:

Constant	Meaning
%pi	$\pi = 3.14159 \ldots$
%e	$e = 2.71828 \ldots$
%i	iota i.e. $\sqrt{-1}$
%eps	Epsilon
%inf	Infinity i.e. ∞
%nan	Not a number

Trigonometric and Inverse Trigonometric Functions

Function Name	**Description**
sin(x)	Sine of argument x in radians
cos(x)	Cosine of argument x in radians
tan(x)	Tangent of argument x in radians
sec(x)	Secant of argument x in radians
csc(x)	Cosecant of argument x in radians
cotg(x)	Cotangent of argument x in radians
asin(x)	sine inverse (radians)
acos(x)	element wise cosine inverse (radians)
atan(x)	2^{nd} quadrant and 4^{th} quadrant inverse tangent
asec(x)	computes the element-wise inverse secant of the argument
acsc(x)	computes the element-wise inverse cosecant of the argument
acot(x)	computes the element-wise inverse cotangent of the argument

EXERCISE-II

1. How to write infinity in MATLAB?
   (a) inf (b) infinity
   (c) undefined (d) None of these
2. How to write imaginary unit *iota* in Scilab?
   (a) i (b) j

(c) $\%i$ (d) None of these

3. What will be the output of 0/0 is MATLAB?
   (a) inf (b) eps
   (c) NaN (d) None of these
4. Is MATLAB case sensitive language?
5. How to assign the value of expression 2+9×3-5 to the variable q in MATLAB?
6. How to assign the value of expression 5×3-8×4 to the variable p in Scilab?
7. Write the syntaxes of tangent and inverse tangent function into MATLAB.
8. Write the syntaxes of cotangent and inverse cotangent function into Scilab.

ANSWERS

1. (a) 2. (c)
3. (c) 4. Yes
5. $q = 2 + 9 * 3 - 5$ 6. $p = 5 * 3 - 8 * 4$
7. tan $(value)$ and $a$tan $(value)$
8. $cotg(x)$ and $acot(x)$