Page No.
Date

Chapter = 1

Rules

Law of exponents for real numbers:

If a and b are real numbers & m and n are rational numbers then following rule hold:

i) $a^m \times a^n = a^{m+n}$

e.g. $2^3 \times 2^2 \Rightarrow 2^{3+2} \Rightarrow 2^5 \Rightarrow 32$

ii) $(a^m)^n = a^{mn}$

e.g. $(2^3)^4 \Rightarrow 2^{12} \Rightarrow 4096$

iii) $\frac{a^m}{a^n} = a^{m-n}$

e.g. $\frac{3^5}{3^2} \Rightarrow 3^{5-2} \Rightarrow 3^3 \Rightarrow 27$

iv) $(a \cdot b)^m = a^m \cdot b^m$

e.g. $(2 \times 3)^5 \Rightarrow 2^5 \times 3^5$

v) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

e.g. $\left(\frac{3}{2}\right)^5 \Rightarrow \frac{3^5}{2^5} \Rightarrow \frac{243}{32}$

Scanned with CamScanner

vi) $a^{-n} = \frac{1}{a^n} = \left(\frac{1}{a}\right)^n$ where $a \neq 0$

vii) $a^m = a^n \Rightarrow m = n$ where $m \neq 0$

viii) $a^m = b^m = a = b$ where $m \neq 0$

e.g $x^3 = 2^3$

Simplify the following

i) $(3a^4b^3)(18a^3b^5)$

Solve:- $= 3 \times 18 \times a^4 \times a^3 \times b^3 \times b^5$

$= 54 \times a^{4+3} \times b^{3+5}$

$54 \times a^7 \cdot b^8$

$54a^7b^8$

ii) $\frac{3a^7b^6}{18a^6b^8}$

$\Rightarrow \frac{3 \times a^7 \times b^6}{18 \times a^6 \times b^8}$

6

$\Rightarrow \frac{a^{7-6}\, b^{6-8}}{6} \Rightarrow \frac{a^1\, b^{-2}}{6}$

iii) $\left(\frac{-2a^2}{b^3}\right)^3$

$\Rightarrow \frac{(-2a^2)^3}{(b^3)^3} \Rightarrow \frac{(-2)^3 \times (a^2)^3}{b^9}$

Scanned with CamScanner

Page No.
Date

$\Rightarrow \frac{(-2)\times(-2)\times(-2)\times a^6}{b^9}$

$\Rightarrow \frac{-8\times a^6}{b^9}$

$\Rightarrow -8.a^6 b^{-9}$

iv) $(a^{-3}b)^2 \ (a^6 b^{-5})$

$= (a^{-3})^2 . b^2$

$\Rightarrow a^{-6} . b^2 \ a^6 b^{-5}$

$\Rightarrow a^{-6+6} \ b^{2+(-5)}$

$\Rightarrow a^0 \ b^{2-5}$

$\Rightarrow 1 \times b^{-3}$

$= b^{-3} = \frac{1}{b^3}$

v) $\frac{(2x^{-2}y^2)^3}{(2xy^{-1})^4}$

$\Rightarrow \frac{(2)^3(x^{-2})^3(y^2)^3}{(2)^4(x)^4(y^{-1})^4}$ $\Rightarrow \frac{8x^{-6}y^6}{16x^4y^{-4}}$

$\Rightarrow \frac{x^{-6-4} \ y^{6+4}}{2}$

$\Rightarrow \frac{x^{-10} \ y^{10}}{2}$

Scanned with CamScanner

v) $\frac{(a^{-3}b^2)(ab^3)^{-2}}{(a^{-1}b)^3}$

$\Rightarrow \frac{(a^{-3}b^2)(a)^{-2}(b^3)^3}{(a^2)^3(b)^3}$

$\Rightarrow \frac{a^{-3}b^2 \; a^{-2}b^6}{a^{-3}\; b^3}$

$\Rightarrow \frac{a^{-3-2}\; b^{2+6}}{a^{-3}\; b^3}$

$\Rightarrow \frac{a^{-5}\; b^{-4}}{a^{-3}\; b^3}$

vi) If $a^x = b$, $b^y = c$ and $c^z = a$ prove that $xyz = 1$

Solv:- $c^z = a$

$\Rightarrow (b^y)^z = a$ (use $c = b^y$

$\Rightarrow b^{yz} = a$

$\Rightarrow (a^x)^{yz} = a$ (use $b = a^x$

$\Rightarrow a^{xyz} = a^1$

$\Rightarrow xyz = 1$

vii) Prove that :-

$$\left(\frac{a^m}{a^{-n}}\right)^{m-n} \times \left(\frac{a^n}{a^{-l}}\right)^{n-l} \times \left(\frac{a^l}{a^{-m}}\right)^{l-m} = 1$$

Scanned with CamScanner

Page No.
Date

Solve $(a^{m+n})^{m-n} \times (a^{n+l})^{n-l} \times (a^{l+m})^{l-m} = 1$

$= a^{m^2-n^2} \times a^{n^2-l^2} \times a^{l^2-m^2} = 1$

$\Rightarrow a^{m^2-n^2+n^2-l^2+l^2-m^2} = 1$

$\Rightarrow a^0 = 1$

$\Rightarrow 1 = 1$

ix) If $2160 = 2^a 3^b 5^c$ find a, b and c Hence calculate the value of $3^a \times 2^{-b} \times 5^{-c}$

Solu:- $2160 = 2^a 3^b 5^c$

$(2)^4 \times (3)^3 \times 5 = 2^a \cdot 3^b \cdot 5^c$

$a = 4$, $b = 3$ and $c = 1$

2	2160
2	1080
2	540
2	270
3	135
3	45
3	15
5	5
	1

Now $3^a \times 2^{-b} \times 5^{-c} = 3^4 \times 2^{-3} \times 5^{-1}$

$= \Rightarrow \frac{81}{2^3 \times 5^1}$

$\Rightarrow \frac{81}{8 \times 5}$ $\Rightarrow \frac{81}{40}$

x) Solve for x

a) $2^{2x+1} = 8$

give that $2^{2x+1} = 8$

$\Rightarrow 2^{2x+1} = 2^3$

Scanned with CamScanner

$\Rightarrow 2x+1 = 3$

$= 2x = 3-1$

$\therefore 2x = 2$

$x = \frac{2}{2}$

$\Rightarrow x = 1$

b) $3^{4x+1} = (27)^{x+1}$

$\Rightarrow 3^{4x+1} = (3^3)^{x+1}$

$\Rightarrow 3^{4x+1} = 3^{3x+3}$

$\Rightarrow 4x+1 = 3x+3$

$\Rightarrow 4x-3x = 3-1$

$\Rightarrow x = 2$

xi) If $x=3$ and $y=-2$ find the value of $x^x + y^y$

$\Rightarrow 3^3 + (-2)^{(-2)}$

$\Rightarrow 3\times3\times3 + \frac{1}{(-2)^2}$

$\Rightarrow \frac{27}{1} + \frac{1}{4}$

$\Rightarrow \frac{27\times4+1}{4}$

$\Rightarrow \frac{108+1}{4} \Rightarrow \frac{109}{4} = 27\frac{1}{4}$

Scanned with CamScanner

b) $x^y + y^x$

$= 3^{(-2)} + (-2)^3$

$\Rightarrow \frac{1}{3^2} + (-2) \times (-2) \times (-2)$

$\Rightarrow \frac{1}{3 \times 3} - 8$

$\Rightarrow \frac{1}{9} - \frac{8}{1} \Rightarrow \frac{1 - 8 \times 9}{9}$

$\Rightarrow \frac{1 - 72}{9}$

$\Rightarrow \frac{-71}{9} \Rightarrow -7\frac{8}{9}$

Scanned with CamScanner

Page No.
Date 14/8/19

Chapter = 2

Factorize

Formula

1. $(a+b)^2 = a^2+b^2+2ab$
2. $(a-b)^2 = a^2+b^2-2ab$
3. $a^2-b^2 = (a+b)(a-b)$
4. $a^3+b^3 = (a+b)(a^2-ab+b^2)$
5. $a^3-b^3 = (a-b)(a^2+ab+b^2)$
6. $(a+b)^3 = a^3+b^3+3ab(a+b)$
7. $(a-b)^3 = a^3-b^3-3ab(a-b)$

Factorize the fallowing

1. $3a^2-9ab$
   $3a(a-3b)$

2. $2(x+y)^3-6(x+y)$
   $2(x+y)[(x+y)^2-3]$
   $= 2(x+y)(x^2+y^2+2xy-3)$

Scanned with CamScanner

3. $2p(a-b) + 3q(5a-5b) + 4r(2b-2a)$

$= 2p(a-b) + 3q \times 5(a-b) + 4r \times 2(b-a)$

$= 2p(a-b) + 15q(a-b) + 8r(b-a)$

$= 2p(a-b) + 15q(a-b) - 8r(a-b)$

$= (a-b)\,2p + 15q - 8r$

4. $4x^3 - 6x^2$

$2x^2(x-3)$

5. $25abc^2 - 15a^2b^2c$

$5abc(5c - 3ab)$

6. $2\pi r^2 - 4\pi r$

$2\pi r(r-2)$

7. $18m + 16m$

$2(9m + 8m)$

8. $ab(a^2+b^2-c^2) + bc(a^2+b^2-c^2) - ca(a^2+b^2-c^2)$

$(a^2+b^2-c^2)\,(ab+bc-ca)$

9. $28p^2q^2r - 42pq^2r^2$

$7pq^2r(4p - 6r)$

10. $14x^2y^2 - 10x^2y + 8xy^2$

$2xy(7xy - 5x + 4y)$

Scanned with CamScanner

Page No.
Date

11. $10a(2x+y)^3 - 15b(2x+y)^2 + 35(2x+y)$

$5(2x+y)[(2a(2x+y)^2 - 3b(2x+y) + 7]$

12. $x^3 - 2x^2y + 3xy^2 - 6y^3$

$x^2(x-2y) + 3y^2(x-2y)$

$(x-2y)(x^2+3y^2)$

13. $ab + bc + ax + cx$

$b(a+c) + x(a+c)$

$= (a+c)(b+x)$

14. $a^3 + a - 3a^2 - 3$

$a(a^2+1) - 3(a^2+1)$

$= (a-3)(a^2+1)$

15. $a^4 - 2a^3 - 4a + 8$

$a^3(a-2) - 4(a-2)$

$= (a-2)(a^3-4)$

Scanned with CamScanner

16. $16(a+b)^2 - 4a - 4b$

$16(a+b)^2 - 4(a+b)$

$= 4(a+b)[4(a+b) - 1]$

$= 4(a+b)(4a+4b-1)$

17. $x^3 - x^2 + ax + x - a - 1$

$x^3 - x^2 + ax - a + x - 1$

$= x^2(x-1) + a(x-1) + 1(x-1)$

$= (x-1)(x^2+a+1)$

18. $x^3 - 3x^2 + x - 3$

$x^2(x-3) + 1(x-3)$

$= (x-3)(x^2+1)$

19. $6ab - b^2 + 12ac - 2bc$

$= b(6a-b) + 2c(6a-b)$

$= (6a-b)(b+2c)$

20. $x^3(2x-3y) - x^2(2x-3y)^2$

$x^2(2x-3y)(x-3y)$

Scanned with CamScanner

21. $x^3 - 3x^2 + x - 3$

$= x^3 + x - 3x^2 - 3$

$= x(x^2+1) - 3(x^2+1)$

$= (x-3)(x^2+1)$ Ans

22. $a^2b - ab + 3a - 3b$

$ab(a-b) + 3(a-b)$

$= (ab+3)(a-b)$ Ans

23. $m(x-3y)^2 + n(3y-x) + 5x - 15y$

$m(x-3y)^2 + n(3y-x) + 5(x-3y)$

$(x-3y)\, m(x-3y) + n + 5$

$(x-3y)(mx - 3my - x + 5)$ Ans

24. $a^3 + ab(1-2a) - 2b^2$

$a^3 + ab - 2a^2b - 2b^2$

$= a(a^2+b) - 2b(a^2+b)$ Ans

25 $3x^2y - 3xy + 12x - 12$

$3xy(x-1) + 12(x-1)$

$= (3xy+12)(x-1)$

$= 3(xy+4)$ $= (xy+4)(x-1)$

Scanned with CamScanner

Page No.
Date

Q6. $6ab - b^2 + 12ac - 2bc$

$b(6a-b) + 2c(6a-b)$

$(6a-b)(b+2c)$ Ans

Q7. $a^2b + ab^2 - abc - b^2c + axy + bxy$

$ab(a+b) - bc(a+b) + xy(a+b)$

$(a+b)(ab-bc+xy)$ Ans

Q8. $p(x+y)^2 + px+py + 7x + 7y$

$p(x+y)^2 + p(x+y) + 7(x+y)$

$= (x+y)\,p(x+y) + p + 7$

$(x+y)(px+py+p+7$ Ans

Scanned with CamScanner

Chapter = 3

Expansions

i) $(a+b)^2 = a^2+b^2+2ab$

ii) $(a-b)^2 = a^2+b^2-2ab$

iii) $(a+b)^3 = a^3+b^3+3ab(a+b)$

iv) $(a-b)^3 = a^3-b^3-3ab(a-b)$

v) $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

vi) $(a+b-c)^2 = a^2+b^2+(-c)^2+2ab+2b(-c)+2(-c)a$
$= a^2+b^2+c^2+2ab-2bc-2ca$

- Find the square of following :-

i) $2a+b$

$= (2a+b)^2$

$= (2a)^2+(b)^2+2a\times 2\times b$

$= 4a^2+b^2+4ab$

ii) $\frac{3a}{2b} - \frac{2b}{3a}$

$= \left(\frac{3a}{2b} - \frac{2b}{3a}\right)^2$

$= \left(\frac{3a}{2b}\right)^2 + \left(\frac{2b}{3a}\right)^2 - 2\times\frac{3a}{2b}\times\frac{2b}{3a}$

$= \frac{9a^2}{4b^2} + \frac{4b^2}{9a^2} - \frac{12ab}{6ab}$

$= \frac{9a^2}{4b^2} + \frac{4b^2}{9a^2} - 2$ Ans

Scanned with CamScanner

Page No.
Date

3) $(a+2b+c)^2$

$= (a)^2 + (2b)^2 + (c)^2 + 2a \times 2b + 2 \times 2b \times c + 2 \times c \times a$

$= a^2 + 4b^2 + c^2 + 4ab + 4bc + 2ca$ Ans

* Evaluate the following

1. $\left(\frac{2x}{7} - \frac{7y}{4}\right)^2$

$\left(\frac{2x}{7}\right)^2 + \left(\frac{7y}{4}\right)^2 - 2 \times \frac{2x}{7} \times \frac{7y}{4}$

$\frac{4x^2}{49} + \frac{49y^2}{16} - xy$ Ans

2. $(2x+3y)(2x-3y)$

$= (2x)^2 - (3y)^2$

$= 4x^2 - 9y^2$ Ans

3. $(x-1)(x+1)(x^2+1)(x^4+1)$

$= (x^2-1^2)(x^2+1)(x^4+1)$

$= (x^2-1)(x^2+1)(x^4+1)$

$= [(x^2)^2 - 1^2](x^4+1)$

$= (x^4-1)(x^4+1)$ Ans

$(x^4)^2 - 1^2$

(x^8-1) Ans

Scanned with CamScanner

Page No.
Date

4. $(2a+3b)^3$

$(2a)^3 + (3b)^3 + 3 \times 2a \times 3b\,(2a+3b)$

$= 8a^3 + 27b^3 + 18ab\,(2a+3b)$

$= 8a^3 + 27b^3 + 36a^2b + 54ab^2$

5. $(3x-4y)^3$

$(3x)^3 - (4y)^3 - 3 \times 3x \times 4y\,(3x-4y)$

$27x^3 - 64y^3 - 36xy\,(3x-4y)$

$27x^3 - 64y^3 - 108x^2y + 144xy^2$

6. 103×97

$(100+3)(100-3)$

$(100)^2 - (3)^2$

$10000 - 9$

9991 Ans

7. 103×103

$(100+3)(100+3)$

$= (100+3)^2$

$= (100)^2 + (3)^2 + 2 \times 100 \times 3$

$= 10000 + 9 + 600$

10609 Ans

8. $(97)^2$

$(100-3)^2$

$(100)^2 + (3)^2 - 2 \times 100 \times 3$

$10000 + 9 - 600$

$10009 - 600$

$= 9409$ Ans

9. $(0.99)^2$

$(1-0.01)^2$

$= (1)^2 + (0.01)^2 - 2 \times 1 \times 0$

$= 1 + 0.0001 - 0.02$

$= 1.0001 - 0.02$

$= 0.9801$ Ans

Scanned with CamScanner

* Expand / evaluate the following

1) $(3x+4y)^2$

$= (3x)^2 + (4y)^2 + 2 \times 3x \times 4y$

$= 9x^2 + 16y^2 + 24xy$

2) $\left(\frac{x}{2} + \frac{2y}{3}\right)^2$

$\left(\frac{x}{2}\right)^2 + \left(\frac{2y}{3}\right)^2 + 2 \times \frac{x}{2} \times \frac{2y}{3}$

$\frac{x^2}{4} + \frac{4y^2}{9} + \frac{2xy}{3}$

3) $(x-3y)^2$

$= (x)^2 + (3y)^2 - 2 \times x \times 3y$

$x^2 + 9y^2 - 6xy$

4. $\left(\frac{x}{2} - \frac{y}{3}\right)^2$

$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 - 2 \times \frac{x}{2} \times \frac{y}{3}$

$\frac{x^2}{4} + \frac{y^2}{9} - \frac{2xy}{3}$

5. $\left(\frac{2x}{y} - \frac{y}{2x}\right)^2$

$= \left(\frac{2x}{y}\right)^2 + \left(\frac{y}{2x}\right)^2 - 2 \times \frac{2x}{y} \times \frac{y}{2x}$

$= \frac{4x^2}{y} + \frac{y^2}{4x^2} - 2$

Page No.
Date

6. $(2x+y)^3$

$(2x)^3 + (y)^3 + 3 \times 2x \times y\,(2x+y)$

$8x^3 + y^3 + 6xy\,(2x+y)$

$8x^3 + y^3 + 12x^2y + 6xy^2$

~~7.~~ $(2x+\frac{1}{2}y)^3$

$(2x)^3 + (y)^3 + 2 \times 2x \times y\,(2x+y)$

$= 8x^3 + y^3 + 6xy\,(2x+y)$

$\therefore 8x^3 + y^3 + 12x^2y + 6xy^2$

~~9.~~ $(x+4y)^3$

$(x)^3 + (4y)^3 + 3 \times x \times 4y\,(x+4y)$

$x^3 + 64y^3 + 12xy\,(x+4y)$

$x^3 + 64y^3 + 12x^2y + 48xy$

8. $\left(\frac{x}{2} - \frac{2}{y}\right)^3$

$\left(\frac{x}{2}\right)^3 - \left(\frac{2}{y}\right)^3 - 3 \times \frac{x}{2} \times \frac{2}{y}\left(\frac{x}{2} - \frac{2}{y}\right)$

$= \frac{x^3}{8} - \frac{8}{y^3} - \frac{3x}{y}\left(\frac{x}{2} - \frac{2}{y}\right)$

Scanned with CamScanner

$= \frac{x^3}{8} - \frac{8}{y^3} - \frac{3x^2}{2y} + \frac{6x}{y^2}$ Ans

9. $(3x - 2y)^3$

$(3x)^3 - (2y)^3 - 3 \times 3x \times 2y (3x - 2y)$

$(27x)^3 - 8y^3 - 18xy (3x - 2y)$

$27x^3 - 8y^3 - 54x^2y + 36xy^2$ Ans

10. $(a + 2b)^3$

$= a^3 + (2b)^3 + 3 \times a \times 2b (a + 2b)$

$= a^3 + 8b^3 + 6ab (a + 2b)$

$= a^3 + 8b^3 + 6a^2b + 12ab^2$ Ans

11. 104×96

$(100+4)(100-4)$

$(100)^2 - (4)^2$

$10000 - 16$

$= 9984$

12. $(101)^2$

$= (100+1)^2$

$= (100)^2 + 1^2 + 2 \times 1 \times 100$

$= 10000 + 1 + 200$

$= 10201$ Ans

13. 102×98

$(100+2)(100-2)$

$(100)^2 - (2)^2$

$10000 - 4$

14. $(102)^2$

$(100+2)$

$= (100)^2 + (2)^2 + 2 \times 100 \times 2$

$= 10000 + 4 + 400$

10404 Ans

Scanned with CamScanner

15. $(98)^2$

$(100-2)^2$

$(100)^2 + (2)^2 - 2 \times 100 \times 2$

$10000 + 4 - 400$

9604 Ans

16. $(99)^2$

$(100-1)^2$

$(100)^2 + (1)^2 - 100 \times 1 \times 2$

$10000 + 1 - 200$

9801 Ans

17. $(0.98)^2$

$(1-0.02)^2$

$= 1^2 + (0.02)^2 - 2 \times 1 \times$

$= 1 + 0.0004 - 0.04$

0.9604 Ans

18. $(1.01)^2$

$(1+0.01)^2$

$1^2 + (0.01)^2 + 2 \times 1 \times 0.01$

$1 + 0.0001 + 0.02$

1.0201 Ans

19. $195 \times 195 - 105 \times 105$

$= (195)^2 - (105)^2$

$= (195-105)(195+105)$

$= (90)(300)$

$= 27000$ Ans

Scanned with CamScanner

Chapter = 4

Partial fraction

★ **Polynomial fraction:-** A fraction of the form $\frac{P(x)}{Q(x)}$ is called polynomial fraction, where $Q(x)$ $P(x)$ and $Q(x)$ are some polynomial in x and $Q(x) \neq 0$.

02

For example:- $\frac{x^2}{x^2-x+1}$, $\frac{1}{(x-1)(x+1)}$, $\frac{x^3}{(x+1)(x+2)}$ etc.

★ **Proper fraction:** A polynomial fraction is said to be proper fraction is degree of numerator is less than the degree of denominator.

For example:- $\frac{x}{(x+1)(x+2)}$, $\frac{1}{(x-1)(x-2)}$, $\frac{x^2}{(x-1)(x^2+2x+7)}$ etc.

★ **Improper fraction:-** A poyomial fraction is said to be improper fraction if degree of numerator is greater than or equal to degree of denominator.

For example: $\frac{x^3}{(x-1)(2x+1)}$, $\frac{(x+1)(x+3}{(x+2)(x+4)}$, $\frac{x^3-1}{(2x+3)(x+5)}$

★ **Partial fraction :-** The different fraction into a the give fraction is decomposed are called partial fraction.

For example: $\frac{1}{(x+1)(x+2)} = \frac{1}{(x+1)} - \frac{1}{(x+2)}$

Scanned with CamScanner

Page No.
Date

Ques 1. Resolve the following into partial fraction:

1. $\frac{1}{(x+1)(x+2)}$

given fraction is $\frac{1}{(x+1)(x+2)}$

Here degree of numerator = 0
which is less than degree of denominator = 2
∴ fraction is proper.

let $\frac{1}{(x+1)(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)}$ — (1)

$1 = \frac{A}{(x+1)} \times (x+1)(x+2) + \frac{B}{(x+2)} \times (x+1)(x+2)$

$= 1 = A(x+2) + B(x+1)$ — (2)

Put $(x+1) = 0 \Rightarrow x = -1$ in eq (2) we get

$1 = A(x+2) + B(x+1)$
$1 = A(-1+2) + B(-1+1)$
$1 = A(1) + B(0)$
$= 1 = A$

again put $(x+2) = 0 \Rightarrow x = -2$ in eq (2) we get

$1 = A(x+2) + B(x+1)$
$1 = A(-2+2) + B(-2+1)$
$1 = A(0) + B(-1)$
$1 = -B$
$-1 = B$

using the value an eq (1) we get

$\frac{1}{(x+1)(x+2)} = \frac{1}{(x+1)} + \frac{-1}{(x+2)}$

$= \frac{1}{(x+1)} - \frac{1}{(x+2)}$

Scanned with CamScanner

2) $\frac{u}{(u-1)(u-2)}$

given fraction is $\frac{u}{(u-1)(u-2)}$

Here degree of numerator = 1
which is less than degree of denominator = 2
∴ fraction is proper.

let $\frac{u}{(u-1)(u-2)} = \frac{A}{(u-1)} + \frac{B}{(u-2)}$ — (1)

$1 = \frac{A}{(u-1)}(u-1)(u-2) + \frac{B}{(u-2)}(u-1)(u-2)$

$1 = A(u-2) + B(u-1)$ — (2)

Put $u-1=0 \Rightarrow u=1$ in eq (2) we get

$1 = A(1-2) + B(1-1)$

$1 = A(-1) + B(0)$

$1 = -A$

$A = -1$

also put $u-2=0 \Rightarrow u=2$ in eq (2) we get

$1 = A(u-2) + B(u-1)$

$1 = A(2-2) + B(2-1)$

$1 = A(0) + B(1)$

$1 = B$

using this value in eq (1) we get

$\frac{u}{(u-1)(u-2)} = \frac{-1}{(u-1)} + \frac{1}{(u-2)}$

Scanned with CamScanner

Page No.
Date

3) $\frac{2x+1}{(1-x)(2-x)}$

given fraction is $\frac{2x+1}{(1-x)(2-x)}$

Here degree of numerator = 1
which is less than degree of denominator = 2
∴ fraction is proper.

let $\frac{2x+1}{(1-x)(2-x)} = \frac{A}{(1-x)} + \frac{B}{(2-x)}$ — (1)

$2x+1 = \frac{A}{(1-x)}(1-x)(2-x) + \frac{B}{(2-x)}(1-x)(2-x)$

$2x+1 = A(2-x) + B(1-x)$ — (2)

Put $1-x=0 \Rightarrow x=1$ in eq (2) we get

$2(1)+1 = A(2-1) + B(1-1)$

$3 = A(1) + B(0)$

$3 = A$

$A = 3$

also put $2-x=0 \Rightarrow x=2$ in eq (2) we get

$2x+1 = A(2-x) + B(1-x)$

$2(2)+1 = A(2-2) + B(1-2)$

$5 = A(0) + B(-1)$

$5 = -B$

$B = -5$

using the value on eq (1) we get

$\frac{2x+1}{(1-x)(2-x)} = \frac{3}{(1-x)} - \frac{5}{(2-x)}$

Scanned with CamScanner

Lesson - 5
Complex Numbers

* **Complex number** - Any number of the form $a+ib$ is called a complex number, where 'a' and 'b' are real number and $i = \sqrt{-1}$

e.g. $3+2i$, $5-6i$, $-3i-2$ etc.

NOTE: (i) $a+ib$ is the standard form of complex numbers

(ii) ib is called purely imaginary

(iii) If $z = a+ib$ is an complex number, then real part of z is a and imaginary part of z is b and these are represented by $Re(z) = a$ and $Im(z) = b$ resp.

Note: $i = \sqrt{-1}$

$i^2 = i \times i = \sqrt{-1} \times \sqrt{-1} = \sqrt{-1}^2 = -1$

$i^3 = i^2 \times i = -1 \times i = -i$

$i^4 = i^2 \times i^2 = -1 \times (-1) = 1$

Q1 Evaluate the following:-

i) $i^5 = i^2 \times i^2 \times i = (-1) \times (-1) \times i = 1 \times i = i$

ii) $i^6 = i^2 \times i^2 \times i^2 = (-1) \times (-1) \times (-1) = -1$

iii) $i^7 = i^2 \times i^2 \times i^2 \times i = (-1) \times (-1) \times (-1) \times i = -1 \times i = -i$

iv) $i^8 = i^4 \times i^4 = 1 \times 1 = 1$

v) $i^{-16} = i^4 \times i^{-4} = 1 \times \frac{1}{i^4} = 1 \times \frac{1}{1} = 1$

vi) $i^{200} = i^4 \times 50 = (i^4)^{50} = (1)^{50} = 1$

vii) $i^{493} = i^{4 \times 123 + 1} = (i^4)^{123}$

Scanned with CamScanner

(viii) i^{4003}

(ix)

x) $i^5 \cdot i^4 \times 1 = 1 \times i = i$

Q2. Find the real and imaginary parts of following complex number:

i) $10 + 11i$

Let, $z = 10 + 11i$

$Re(z) = 10$ and $Im(z) = 11$

ii) $13 - i$

Let, $z = 13 - i$

$Re(z) = 13$ and $Im(z)$

iii) $12i$

Let, $z = 12i$

$Re(z) = 0$ and $Im(z) = 12$

iv) -10

Let, $z = -10$

$Re(z) = -10$ and $Im(z) = 0$

Q3. Evaluate the following:

i) $i^{16} = i^4 \times i^4 \times i^4 \times i^4 = 1 \times 1 \times 1 \times 1 = 1$

ii) $i^{20} = i^{4 \times 5} = (i^4)^5 = (1)^5 = 1$

iii) $i^{-8} = i^{4 \times -2} = (i^4)^{-2} = (1)^{-2} =$

iv) $i^{-12} = i^{4 \times -3} = (i^4)^{-3} = (1)^{-3} =$

v) $i^{256} = i^{4 \times 64} = (i^4)^{64} = (1)^{64} = 1$

vii) $i^{1001} = i$

viii) $i^{2003} =$

ix) $i^{497} =$

x) $1 + i^2 + i^4 = 1 + (-1) + 1 = 1 - 1 + 1 = 1$

Scanned with CamScanner

xi) i^{23}

xii) $1 - i + i^2 - i^3 = 1 - (\sqrt{-1}) + (-1) + 1 = 1 - \sqrt{1} - 1 + 1$

Q4. Write down the real and imaginary parts of following complex numbers.

i) $3 + 2i$

Let, $z = 3 + 2i$

$Re(z) = 3$ and $Im(z) = 2$

ii) $5 - 6i$

Let, $z = 5 - 6i$

$Re(z) = 5$ and $Im(z) = -6$

iii) $-3 - i$

Let, $z = -3 - i$

$Re(z) = -3$ and $Im(z) = -1$

iv) $-6 + 7i$

Let, $z = -6 + 7i$

$Re(z) = -6$ and $Im(z) = 7$

v) $1 + i$

Let, $z = 1 + i$

$Re(z) = 1$ and $Im(z) = 1$

vi) 5

Let, $z = 5$

$Re(z) = 5$ and $Im(z) = 0$

vii) $-3i$

Let, $z = -3i$

$Re(z) = 0$ and $Im(z) = -3$

Scanned with CamScanner

viii) $5i$

Let, $z = 5i$

$Re(z) = 0$ and $Im(z) = 6$

ix) 0

Let, $z = 0$

$Re(z) = 0$ and $Im(z) = 0$

x) $-1-i$

Let, $z = -1-i$

$Re(z) = -1$ and $Im(z) = -1$

xi) $\frac{1}{3} - \frac{2}{8}i$

Let, $z = \frac{1}{3} - \frac{2}{3}i$

$Re(z) = \frac{1}{3}$ and $Im(z) = -\frac{2}{3}$

xii) $2i-5$

Let, $z = 2i-5$

$Re(z) = -5$ and $Im(z) = 2$

xiii) $-3i-7$

Let, $z = -3i-7$

$Re(z) = -7$ and $Im(z) = -3$

* Conjugate of a complex number.

Let $z = a+ib$ is any complex numbers then conjugate z is $a-ib$ and it is represent by $\bar{z} = a-ib$

e.g. $\overline{2+13i} = 2-13i$

$\overline{-10-20i} = -10+20i$

$\overline{20i} = -20i$

Scanned with CamScanner

* Properties of conjugate : Let z_1 and z_2 be any two complex numbers then.

i) $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$

ii) $\overline{z_1 - z_2} = \overline{z_1} - \overline{z_2}$

iii) $\overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}$

iv) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$, where $z_2 \neq 0$

v) $\overline{(\overline{z_1})} = z_1$

vi) $z_1 + \overline{z_1} = 2\,Re\,(z_1)$

Q5. Find the conjugate of all the parts of question no-④

i) $3 + 2i$

Let, $z = 3+2i \Rightarrow \overline{z} = \overline{3+2i} \Rightarrow \overline{z} = 3-2i$

ii) $5 - 6i$

Let $z = 5-6i$

$\overline{z} = \overline{5-6i}$

$\overline{z} = 5+6i$

iii) $-3 - i$

Let $z = -3-i$

$\overline{z} = \overline{-3-i}$

$\overline{z} = -3+i$

iv) $-6 + 7i$

Let, $z = -6+7i$

$\overline{z} = \overline{-6+7i}$

$\overline{z} = -6-7i$

v) $1 + i$

Let, $z = 1+i$

$\overline{z} = \overline{1+i}$

$\overline{z} = 1-i$

Scanned with CamScanner

vi) $-3i$

let, $z = -3i$

$\bar{z} = \overline{-3i}$

$\bar{z} = 3i$

vii) $5i$

let, $z = 5i$

$\bar{z} = \overline{5i}$

$\bar{z} = -5i$

viii) $-1-i$

let, $z = -1$

$\bar{z} = -1$

$\bar{z} = -1$

ix) $\frac{1}{3} - \frac{2}{3}i$

let, $z = \frac{1}{3} - \frac{2}{3}i$

$\bar{z} = \overline{\frac{1}{3} - \frac{2}{3}i}$

$\bar{z} = \frac{1}{3} + \frac{2}{3}i$

x) $2i-5$

let, $z = 2i-5$

$\bar{z} = \overline{2i-5}$

$\bar{z} = -2i-5$

xi) $-3i-7$

let, $z = -3i-7$

$\bar{z} = \overline{-3i-7}$

$\bar{z} = 3i-7$

* Modulus of a complex numbers:

let $z = a+ib$ is only complex number then modulus of z is $\sqrt{a^2+b^2}$ and it is represented by:-

$$|z| = \sqrt{a^2+b^2}$$

e.g. $|2+7i| = \sqrt{(2)^2+(7)^2} = \sqrt{4+49} = \sqrt{53}$

$|-7-i| = \sqrt{(-7)^2+(-1)^2} = \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$

Q Find the modulus of all the parts of question no - (4).

Scanned with CamScanner

i) $3+2i$

let, $|z| = \sqrt{a^2+b^2}$

$|3+2i| = \sqrt{(3)^2+(2)^2} = \sqrt{9+4} = \sqrt{13}$

ii) $5-6i$

let, $|z| = \sqrt{a^2+b^2}$

$|5-6i| = \sqrt{(5)^2+(-6i)^2} = \sqrt{25+36} = \sqrt{61}$

iii) $-3-i$

let, $|z| = \sqrt{a^2+b^2}$

$|-3-i| = \sqrt{(-3)^2+(-1)^2} = \sqrt{9+1} = \sqrt{10}$

iv) $-6+7i$

let, $|z| = \sqrt{a^2+b^2}$

$|-6+7i| = \sqrt{(-6)^2+(7)^2} = \sqrt{36+49} = \sqrt{85}$

v) 5

let, $|z| = \sqrt{a^2+b^2}$

$|5| = \sqrt{(5)^2} = \sqrt{25}$

vi) $1+i$

let, $|z| = \sqrt{a^2+b^2}$

$|1+i| = \sqrt{(1)^2+(1)^2} = \sqrt{1+1} = \sqrt{2}$

vii) $-3i$

let, $|z| = \sqrt{a^2+b^2}$

$|-3i| = \sqrt{(-3)^2} = \sqrt{9} = 3$

viii) $5i$

let, $|z| = \sqrt{a^2+b^2}$

$|5i| = \sqrt{(5)^2} = \sqrt{25} = 5$

Scanned with CamScanner

ix) 0

let, $|z| = \sqrt{a^2+b^2}$

$|0| = \sqrt{(0)^2} = \sqrt{0}$

x) $-1-i$

let, $|z| = \sqrt{a^2+b^2}$

$|-1-i| = \sqrt{(-1)^2+(-1)^2} = \sqrt{1+1} = \sqrt{2}$

xi) $\frac{1}{3} - \frac{2}{3}i$

let, $|z| = \sqrt{a^2+b^2}$

$\left|\frac{1}{3} - \frac{2}{3}\right| = \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{-2}{3}\right)^2} = \sqrt{\frac{1}{9}+\frac{4}{9}} = \sqrt{\frac{5}{9}}$

xii) $2i-5$

let, $|z| = \sqrt{a^2+b^2}$

$|2i-5| = \sqrt{(-5)^2+(2)^2} = \sqrt{25+4} = \sqrt{29}$

xiii) $-3i-7$

let, $|z| = \sqrt{a^2+b^2}$

$|-3i-7| = \sqrt{(-3)^2+(-7)^2} = \sqrt{9+49} = \sqrt{58}$

★ If $z_1 = a+ib$ and $z_2 = c+id$ then:

(i) $z_1+z_2 = (a+ib)+(c+id)$

$= a+ib+c+id$

$= (a+c)+i(b+d)$

ii) $z_1-z_2 = (a+ib)-(c+id)$

$= a+ib-c+id$

$= (a-c)+i(b+d)$

Scanned with CamScanner

iii) $z_1 \times z_2 = (a+ib)(c+id)$

$= ac + iad + ibc + i^2bd$

$= ac + iad + ibc + (-1)bd$

$= ac + iad + ibc - bd$

$= (ac - bd) + i(ad + bc)$

iv) $\frac{z_1}{z_2} = \frac{a+ib}{c+id} \Rightarrow \frac{a+ib}{c+id} \times \frac{c-id}{c-id}$

$\Rightarrow \frac{(a+ib)(c-id)}{(c+id)(c-id)}$

$\Rightarrow \frac{ac - iad + ibc - i^2bd}{c^2 - (id)^2}$

$\Rightarrow \frac{ac - iad + ibc - (-1)bd}{c^2 - i^2d^2}$

$\Rightarrow \frac{ac - iad + ibc + bd}{c^2 - (-1)d^2}$

$\Rightarrow \frac{(ac+bd) + i(bc - ad)}{c^2 + d^2}$

$\Rightarrow \frac{(ac+bd)}{(c^2+d^2)} + i\frac{(bc-ad)}{(c^2+d^2)}$

Scanned with CamScanner

Page No.
Date

Qus 7. Find z_1+z_2, z_1-z_2, $z_1 \cdot z_2$ and $\frac{z_1}{z_2}$ for following pairs

i) $z_1 = 2+5i$, $z_2 = 1+3i$

Given that

Now $z_1+z_2 = (2+5i)+(1+3i)$

$= 2+5i+1+3i$

$= 3+8i$

ii) $z_1-z_2 = (2+5i)-(1+3i)$

$= 2+5i-1-3i$

$= 1+8i$

iii) $z_1 \cdot z_2 = (2+5i)(1+3i)$

$= 2+6i+5i+15i^2$

$= 2+6i+5i+15(-1)$

$= 2+6i+5i-15$

$= 2+11i-15$

$= -3+11i$

iv) $\frac{z_1}{z_2} = \frac{(2+5i)}{(1+3i)} \times \frac{(1-3i)}{(1-3i)}$

$= \frac{2-6i+5i-15i^2}{1^2-(3i)^2}$

$= \frac{2-11i-15(-1)}{1-9i^2}$

$= \frac{2-11i+15}{1-9(-1)}$

$= \frac{2-11i+15}{1+9} \Rightarrow \frac{17-11i}{10}$

Scanned with CamScanner

Page No.
Date

Ques 2 given that $z_1 = 2-3i$, $z_2 = -5-6i$

i) $z_1 + z_2 = (2-3i) + (-5-6i)$

$= 2-3i-5-6i$

$= 2-5i(-6-3i)$

$z_1 + z_2 = -3-9i$

ii) $z_1 - z_2 = (2-3i) - (-5-6i)$

$= 2-3i+5+6i$

$= 7+3i$

iii) $z_1 z_2 = (2-3i)(-5-6i)$

$= -10-12i+15i+81i^2$

$= -10-12i+15i+81(-1)$

$= -10+3i-18$

$= -28+3i$

iv) $\frac{z_1}{z_2} = \frac{2-3i}{-5-6i} \times \frac{-5+6i}{-5+6i}$

$\Rightarrow \frac{(2-3i)(-5+6i)}{(-5-6i)(-5+6i)}$

$\Rightarrow \frac{-10+12i+15i-18i^2}{(-5)^2-(6i)^2}$

$\Rightarrow \frac{-10+27i-18(-1)}{25-36i^2}$

$\Rightarrow \frac{-10+27i+18}{25-36(-1)}$

$\Rightarrow \frac{-10+27i+18}{25+36} \Rightarrow \frac{8+27i}{61} = \frac{8}{61} + \frac{27i}{61}$

Scanned with CamScanner

3. given that $z_1 = -1-i$, $z_2 = 2-i$

i) $z_1 + z_2 = (-1-i) + (2-i)$
$= -1-i+2-i$
$= 1-2i$

ii) $z_1 - z_2 = (-1-i) - (2-i)$
$= -1-i-2+i$
$= -3$

iii) $z_1 z_2 = (-1-i)(2-i)$
$= -2+i-2i+i^2$
$= -2-i+(-1)$
$= -2-i-1$
$= -3-i$

iv) $\frac{z_1}{z_2} = \frac{(-1-i)}{(2-i)} \times \frac{(2+i)}{(2+i)}$

$= \frac{-2-i-2i-i^2}{(2)^2-(i)^2}$

$= \frac{-2-3i-(-1)}{4-(-1)}$

$= \frac{-2-3i+1}{4+1}$

$= \frac{-1-3i}{5}$

Scanned with CamScanner

4. given that $z_1 = 3-5i$, $z_2 = i-3$

i) $z_1 + z_2 = (3-5i) + (i-3)$
$= 3-5i+i-3$
$= 0-4i$
$= -4i$

ii) $z_1 - z_2 = (3-5i) - (i-3)$
$= 3-5i-i+3$
$= 6-6i$

iii) $z_1 z_2 = (3-5i)(i-3)$
$= 3i-9-5i^2+15i$
$= 3i-9-5(-1)+15i$
$= 18i-9+5$
$= 18i-4$

iv) $\frac{z_1}{z_2} = \frac{(3-5i)}{(i-3)} \times \frac{(i+3)}{(i+3)}$

$= \frac{3i+9-5i^2-15i}{(i)^2-(3)^2}$

$\Rightarrow \frac{3i+9-5(-1)-15i}{i^2-9}$

$\Rightarrow \frac{3i+9+5-15i}{1-9}$

$\Rightarrow \frac{-12i+4}{-10}$

Scanned with CamScanner

Page No.
Date

5. given that $z_1 = -6-7i$, $z_2 = 6+7i$

i) $z_1 + z_2 = (-6-7i) + (6+7i)$
$= -6 - 7i + 6 + 7i$
$= 0$

ii) $z_1 - z_2 = (-6-7i) - (6+7i)$
$= -6 - 7i - 6 - 7i$
$= -12 - 14i^2$
$= -12 - 14(-1)$
$= -12 - 14i$

iii) $z_1 \cdot z_2 = (-6-7i)(6+7i)$
$= -36 - 42i - 42i - 49i^2$
$= -36 - 84i - 49(-1)$
$= -36 - 84i + 49$
$= 13 - 84i$

iv) $\frac{z_1}{z_2} = \frac{(-6-7i)}{(6-7i)} \times \frac{(6+7i)}{(6+7i)}$

$\Rightarrow \frac{36 + 42i - 42i + 49i^2}{(6)^2 - (7i)^2}$

$\Rightarrow \frac{36 + 49(-1)}{36 - 49i^2}$

$\Rightarrow \frac{36 - 49}{36 - 49(-1)}$

$\Rightarrow \frac{36-49}{36+49} \Rightarrow \frac{-13}{85}$

6)
i)
ii)
iii)
iv)

Scanned with CamScanner

6) given that $z_1 = 2i - 5$, $z_2 = 1 - i$

i) $z_1 + z_2 = (2i - 5) + (1 - i)$
$= 2i - 5 + 1 - i$
$= (2i - i) - 4$
$= i - 4$

ii) $z_1 - z_2 = (2i - 5) - (1 - i)$
$= 2i - 5 - 1 + i$
$= -6 + 3i$

iii) $z_1 \cdot z_2 = (2i - 5)(1 - i)$
$= 2i - 2i^2 - 5 + 5i$
$= 2i - 2(-1) - 5 + 5i$
$= 2i + 2 - 5 + 5i$
$= 7i - 3$

iv) $\frac{z_1}{z_2} = \frac{(2i - 5)}{(1 - i)} \times \frac{(1 + i)}{(1 + i)}$

$\Rightarrow \frac{2i + 2i^2 - 5 - 5i}{(1)^2 - (i)^2}$

$\Rightarrow \frac{2i + 2(-1) - 5 - 5i}{1 - (-1)}$

$\Rightarrow \frac{2i - 2 - 5 - 5i}{1 + 1}$

$\Rightarrow \frac{-3i - 7}{2}$

Scanned with CamScanner

7 given that $z_1 = 3-3i$, $z_2 = -3+7i$

i) $z_1+z_2 = (3-3i)+(-3+7i)$
$= 3-3i-3+7i$
$= 0+4i$

ii) $z_1-z_2 = (3-3i)-(-3+7i)$
$= 3-3i+3-7i$
$= 6-10i^2$

iii) $z_1 \cdot z_2 = (3-3i)(-3+7i)$
$= -9+21i+9i-21i^2$
$= -9+21i+9i-21(-1)$
$= -9+21i+9i+21$
$= -9+30i+21$
$= 12+30i$

iv) $\frac{z_1}{z_2} = \frac{(3-3i)}{(-3+7i)} \times \frac{(-3-7i)}{(-3-7i)}$

$\Rightarrow \frac{9-21i+9i+21i^2}{(-3)^2-(7i)^2}$

$\Rightarrow \frac{9-21i+9i+21(-1)}{9-49i^2}$

$\Rightarrow \frac{9-21i+9i-21}{9-49(-1)}$

$\Rightarrow \frac{9-21i+9i-21}{9+49}$ $\Rightarrow \frac{-12-12i}{58}$

Scanned with CamScanner

8 Given that $z_1 = 10+11i$, $z_2 = 11+10i$

i) $z_1 + z_2 = (10+11i) + (11+10i)$
$= 10+11i+11+10i$
$= 21 + 21i$

ii) $z_1 - z_2 = (10+11i) - (11+10i)$
$= 10+11i-11-10i$
$= -1+i$

iii) $z_1 \cdot z_2 = (10+11i)(11+10i)$
$= 110 + 100i + 121i + 110i^2$
$= 110 + 100i + 121i + 110(-1)$
$= 110 + 221i - 110$
$= 221i$

iv) $\frac{z_1}{z_2} = \frac{(10+11i) \times (11-10i)}{(11+10i)\ (11-10i)}$

$\Rightarrow \frac{110 - 100i + 121i - 110i^2}{(11)^2 - (10i)^2}$

$\Rightarrow \frac{110 + 21i - 110(-1)}{121 - 100(-1)}$

$\Rightarrow \frac{110 + 21i + 110}{121 + 100}$

$\Rightarrow \frac{220 + 21i}{221}$

Scanned with CamScanner

Page No.
Date

★ Find the value of x and y for following equation:-

i) $x+iy = 3+5i$
$\Rightarrow x = 3$
and $y = 5$

ii) $x+iy = -8+3i$
$\Rightarrow x = -8$
and $y = 3$

iii) $(x-3)+2yi = -9-11i$
given that
$(x-3)+2yi = -9-11i$
$\Rightarrow x-3 = -9$ and $2y = -11$
$x = -9+3$ $\quad y = \frac{-11}{2}$
$x = -6$

iv) $(x+8)+(5-y)i = 10-20i$
give that and $5-y = -20$
$\Rightarrow x+8 = 10$ $\quad -y = -20-5$
$x = 10-8$ $\quad -y = -25$
$x = 2$ $\quad y = 25$

v) $\frac{1}{x}+\frac{1}{y}i = -20-10i$

given that and $\frac{1}{y} = -10$
$\Rightarrow \frac{1}{x} = -20$ $\quad y = \frac{1}{-10}$

$x = \frac{1}{-20}$

vi) $(x+y)+(x-y)i = 8+4i$
given that
$\Rightarrow x+y = 8$ —(1) and $x-y = 4$ —(2)

Scanned with CamScanner

$x+y=8$
$x-y=4$
$2x=12$

$x=\frac{12}{2}6$

$x=6$

Put $x=6$ in eq ① we get

$x+y=8$
$6+y=8$
$y=8-6$
$y=2$

vii) $(x-2)+iy=5-3i$

$\Rightarrow x-2=5$ and $y=-3$

$x=5+2$
$x=7$

viii) $(x+3)+(2-y)i=-6-7i$

$\Rightarrow x+3=-6$ and $2-y=-7$

$x=-6-3$ $\quad -y=-7-2$

$x=-9$ $\quad -y=-9$

$y=9$

ix) $(x+y)-2yi=7+4i$

$\Rightarrow x+y=7$ — ① and $-2y=4$

$y=4+2$

$y=6$

Put $y=6$ in eq ① we get

$x+y=7$
$x+6=7$
$x=7-6$
$x=1$

Scanned with CamScanner

x) $\frac{1}{x} + \frac{1}{y}i = 3 + 10i$

$\frac{1}{x} = 3$ and $\frac{1}{y} = 10$

$x = \frac{1}{3}$ $\quad y = \frac{1}{10}$

xi) $(x+y) + (x-y)i = 4 - 8i$

$x+y = 4$ - (1) and $x-y = -8$ - (2)

$$\begin{array}{r} x+y=4 \\ x-y=-8 \\ \hline 2x=-4 \end{array}$$

$x = \frac{-4}{2} = -2$

Put $x = -2$ in eq (1) we get

$x + y = 4$

$-2 + y = 4$

$y = 4 + 2$

$y = 6$

xiij $(2x-y) + (x+y)i = 3 + 12i$

$\Rightarrow 2x - y = 3$ - (1) and $x + y = 12$ - (2)

$$\begin{array}{r} 2x-y=3 \\ x+y=12 \\ \hline 3x=15 \end{array}$$

$x = \frac{15}{3} = 5$

$x = 5$

Scanned with CamScanner

Put $x = 5$ in eq (1) we get

$$2x - y = 3$$
$$2 \times 5 - y = 3$$
$$10 - y = 3$$
$$-y = 3 - 10$$
$$-y = -7$$
$$y = 7$$

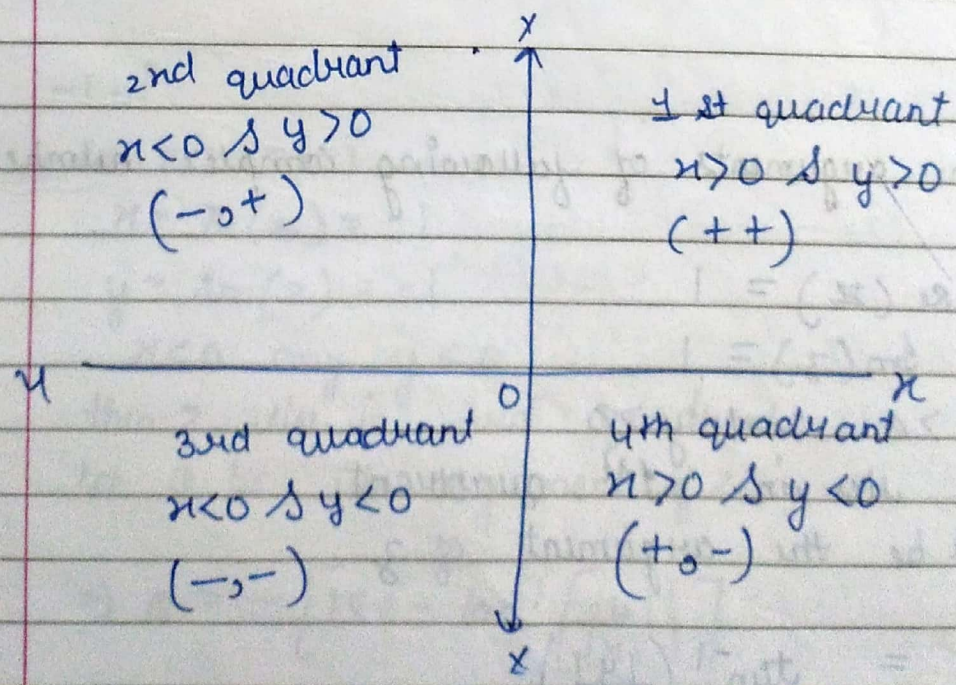
$$\tan^{-1} x \neq \frac{1}{\tan x}$$

$$\tan^{-1} x \neq \frac{x}{\tan}$$

$$\tan^{-1}(\tan x) = x$$

For example $\therefore \tan^{-1}(\tan 60^\circ) = 60^\circ$

θ	0	30°	45°	60°	90°
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞



Scanned with CamScanner

Page No.
Date

Amplitude / Argument / Angle of a complex number

Let $z = x + iy$ is any complex number

$x = Re(z)$

and $y = Im(z)$

If $x > 0$ and $y > 0$ then z lies in 1st quadrant

If $x < 0$ and $y > 0$ then z lies in 2nd quadrant

If $x < 0$ and $y < 0$ then z lies in 3rd quadrant

If $x > 0$ and $y < 0$ then z lies in 4th quadrant

Let θ be the amplitude of z then

(1) $\tan^{-1}\left(\frac{|y|}{|x|}\right)$ if z lies in 1st quadrant.

(2) $180 - \tan^{-1}\left(\frac{|y|}{|x|}\right)$ if z lies in 2nd quadrant

(3) $-\left[180^\circ - \tan^{-1}\left(\frac{|y|}{|x|}\right)\right]$ if z lies in 3rd quadrant

(4) $-\tan^{-1}\left(\frac{|y|}{|x|}\right)$ if z lies in 4th quadrant.

Ques 1. Find the arguments of following complex number

i) $1 + i$

$x = Re(z) = 1$

$y = Im(z) = 1$

$x > 0$ and $y > 0$

then z lies in 1st quadrant

Let θ be the argument of z.

$\Rightarrow \theta = \tan^{-1}\left(\frac{|y|}{|x|}\right)$

$$= \tan^{-1}\left(\frac{|1|}{|1|}\right)$$

$$= \tan^{-1}(1)$$

$$= \tan^{-1}(\tan 45^\circ)$$

$$= 45^\circ$$

ii) $1-i$

$x = Re(z) = 1$

$y = Im(z) = -1$

then z lies in 4th quadrant

let θ be the argument of z

$$\Rightarrow \theta = -\tan^{-1}\left(\frac{|y|}{|x|}\right)$$

$$= -\tan^{-1}\left(\frac{|-1|}{|1|}\right)$$

$$= -\tan^{-1}\left(\frac{|1|}{|1|}\right)$$

$$= -\tan^{-1}(1)$$

$$= -\tan^{-1}(\tan 45^\circ)$$

$$= -45^\circ$$

iii) $-1-i$

Let $z = -1-i$

$x = Re(z) = -1$

$y = Im(z) = -1$

$x < 0$ any $y < 0$

then z lies in 3rd quadrant.

Let θ be the argument of z.

$$\Rightarrow \theta = -\left[180^\circ - \tan^{-1}\left(\frac{|y|}{|x|}\right)\right]$$

Scanned with CamScanner

$$= -\left[180^\circ - \tan^{-1}\left(\frac{|-1|}{|-1|}\right)\right]$$

$$= -\left[180^\circ - \tan^{-1}\left(\frac{|1|}{|1|}\right)\right]$$

$$= -[180^\circ - \tan^{-1}(1)]$$
$$= -[180^\circ - \tan^{-1}(\tan 45^\circ)]$$
$$= -[180 - 45^\circ]$$
$$= -135^\circ$$

(iv) $-1+i$

Let $z = -1+i$

$x = Re(z) = -1$

$y = Im(z) = 1$

$x < 0$ and $y > 0$

then z lies in 2nd quadrant

let θ be the argument of z

$$\Rightarrow \theta = 180^\circ - \tan^{-1}\left(\frac{|y|}{|x|}\right)$$

$$= 180^\circ - \tan^{-1}\left(\frac{|1|}{|-1|}\right)$$

$$= 180^\circ - \tan^{-1}\left(\frac{|1|}{|1|}\right)$$

$$= 180^\circ - \tan^{-1}(\tan 45^\circ)$$
$$= 180^\circ - 45^\circ$$
$$= 135^\circ$$

Scanned with CamScanner

v) $1+\sqrt{3}$

Let $z = 1+\sqrt{3}$

$x = Re(z) = 1$

$y = Im(z) = \sqrt{3}$

$x > 0$ and $y \geq 0$

then z lies in 1st quadrant

Let θ be the argument of z.

$$\Rightarrow \theta = \tan^{-1}\left(\frac{|y|}{|x|}\right)$$

$$= \tan^{-1}\left(\frac{|\sqrt{3}|}{|1|}\right)$$

$$= \tan^{-1}(|\sqrt{3}|)$$

$$= \tan^{-1}(\tan 60^\circ)$$

$$= 60^\circ$$

vi) $1-i\sqrt{3}$

Let $z = 1-i\sqrt{3}$

$x = Re(z) = 1$

$y = Im(z) = -\sqrt{3}$

$x > 0$ and $y < 0$

then z lies in 4th quadrant

Let θ be the argument of z

$$\Rightarrow \theta = -\tan^{-1}\left(\frac{|y|}{|x|}\right)$$

$$= -\tan^{-1}\left(\frac{|-\sqrt{3}|}{|1|}\right)$$

$$= -\tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$$

$$= -\tan^{-1}(\tan 60^\circ)$$

Scanned with CamScanner

vii) $\sqrt{3}+i$

Let $z = \sqrt{3}+i$

$x = Re(z) = \sqrt{3}$

$y = Im(z) = 1$

$x > 0$ and $y > 0$

then z lies in 1st quadrant

Let θ be the argument of z.

$$\Rightarrow \theta = \tan^{-1}\left(\frac{|y|}{|x|}\right)$$

$$= \tan^{-1}\left(\frac{|1|}{|\sqrt{3}|}\right)$$

$$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$= \tan^{-1}(\tan 30^\circ)$$

$$= 30^\circ$$

viii) $-\sqrt{3}-i$

Let $z = -\sqrt{3}-i$

$x = Re(z) = -\sqrt{3}$

$y = Im(z) = -1$

$x < 0$ and $y < 0$

then z lies in 3rd quadrant

Let θ be the argument of z

$$\Rightarrow \theta = -\left[180^\circ - \tan^{-1}\left(\frac{|y|}{|x|}\right)\right]$$

$$= -\left[180^\circ - \tan^{-1}\left(\frac{|-1|}{|-\sqrt{3}|}\right)\right]$$

$$= -\left[180^\circ - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]$$

Scanned with CamScanner

$= -[180^\circ - \tan^{-1}(\tan 30^\circ)]$

$= -[180^\circ - 30^\circ]$

$= -[150^\circ]$

$= -150^\circ$

ix) $2 - 2i$

Let $z = 2 - 2i$

$x = Re(z) = 2$

$y = Im(z) = -2$

$x > 0$ and $y < 0$

then z lies in 4th quadrant

Let θ be the argument of z

$\Rightarrow \theta = -\tan^{-1}\left[\frac{|y|}{|x|}\right]$

$\Rightarrow -\tan^{-1}\left[\frac{|-2|}{|2|}\right]$

$= -\tan^{-1}\left(\frac{2}{2}\right)$

$= -\tan^{-1}(1)$

$= -\tan^{-1}(\tan 45^\circ)$

$= -45^\circ$

x) $-2 + 2i$

Let $z = -2 + 2i$

$x = Re(z) = -2$

$y = Im(z) = 2$

$x < 0$ and $y > 0$

then z lies in 2nd quadrant

Let θ be the argument of z

Scanned with CamScanner

$\Rightarrow \theta = 180^\circ - \tan^{-1}\left(\frac{|y|}{|x|}\right)$

$= 180^\circ - \tan^{-1}\left(\frac{|2|}{|-2|}\right)$

$= 180^\circ - \tan^{-1}\left(\frac{2}{2}\right)$

$= 180^\circ - \tan^{-1}(1)$

$= 180^\circ - \tan^{-1}(\tan 45^\circ)$

$= 180^\circ - 45^\circ$

$= 135^\circ$

xi) $2+5i$

Let $z = 2+5i$

$x = Re(z) = 2$

$y = Im(z) = 5$

$x > 0$ and $y > 0$

then z lies in 1st quadrant

Let θ be the argument of z

$\Rightarrow \theta = \tan^{-1}\left(\frac{|y|}{|x|}\right)$

$= \tan^{-1}\left(\frac{|5|}{|2|}\right)$

$= \tan^{-1}\left(\frac{5}{2}\right)$

xii) $2-3i$

Let $z = 2-3i$

$x = Re(z) = 2$

$y = Im(z) = -3$

$x > 0$ and $y < 0$

then z lies in 4th quadrant

Let θ be the argument of z

Scanned with CamScanner

$$= \theta = -\tan^{-1}\left(\frac{|y|}{|x|}\right)$$

$$= -\tan^{-1}\left(\frac{|-3|}{|2|}\right)$$

$$= -\tan^{-1}\left(\frac{3}{2}\right)$$

Cartesian	Polar
(x, y)	(r, θ)
$z = 1+i$	$\sqrt{2}(\cos 45^\circ + i \sin 45^\circ)$

$x = r\cos\theta$

$y = r\sin\theta$

$z = x+iy = r\cos\theta + ir\sin\theta$

$r(\cos\theta + i\sin\theta)$

$r\,\mathrm{cis}\,\theta$

1. Convert $1+i$ into polar form.

Let $z = 1+i$

$x = Re(z) = 1$

$y = Im(z) = 1$

Let $x = r\cos\theta$

$y = r\sin\theta$

Here $r = |z| = \sqrt{x^2+y^2} = \sqrt{(1)^2+(1)^2} = \sqrt{1+1} = \sqrt{2}$

Now $x > 0$ and $y > 0$

$\Rightarrow z$ lies in 1st quadrant

$$\Rightarrow \theta = \tan^{-1}\left(\frac{|y|}{|x|}\right)$$

$$= \tan^{-1}\left(\frac{|1|}{|1|}\right)$$

$$= \tan^{-1}(1)$$

Scanned with CamScanner

$= \tan^{-1}(1)$

$= \tan^{-1}(\tan 45^\circ)$

$= 45^\circ$

New $z = x + iy$

$r\cos\theta + ir\sin\theta$

$= \sqrt{2}\cos 45^\circ + i\sqrt{2}\sin 45^\circ$

2. Convert $\sqrt{2}(\cos 45^\circ + i\sin 45^\circ)$ into cartesian form

Let $z = \sqrt{2}(\cos 45^\circ + i\sin 45^\circ)$

$= \sqrt{2}\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)$

$= \frac{\sqrt{2}}{\sqrt{2}} + i\frac{\sqrt{2}}{\sqrt{2}}$

$= 1 + i$

Scanned with CamScanner

Chapter = 6

Logarithms

1. **Logarithms:-** If a is any real number, $a \neq 1$, x or is any rational numbers such that $a^x = N$ then $\log_a N = x$ which is read as logarithm of N to the base "a" is equal to "x"

exponential form: $a^x = N$

logarithms form: $\log_a N = x$

For example :-

$$2^3 = 8 \iff \log_2 8 = 3$$

Ques 1. Convert the following into logarithms forms:

i) $2^3 = 8$

given that $2^3 = 8$

$\Rightarrow \log_2 8 = 3$

ii) $3^2 = 9$

given that $3^2 = 9$

$\Rightarrow \log_3 9 = 2$

iii) $4^3 = 64$

given that $4^3 = 64$

$\log_4 64 = 3$

iv) $5^3 = 125$

given that $5^3 = 125$

$\Rightarrow \log_5 125 = 3$

v) $10^{-3} = 0.001$

given that $10^{-3} = 0.001$

$\Rightarrow \log_{10} 0.001 = -3$

(vi) $10^3 = 1000$

given that $10^3 = 1000$

$\Rightarrow \log_{10} 1000 = 3$

Scanned with CamScanner

Page No.
Date

vii) $5^{-4} = \frac{1}{625}$

given that
$5^{-4} = \frac{1}{625}$

$\Rightarrow \log_5\left(\frac{1}{625}\right) = -4$

viii) $2^{-10} = \frac{1}{1024}$

given that
$2^{-10} = \frac{1}{1024}$

$= \log_2\left(\frac{1}{1024}\right) = -10$

ix) $3^{-4} = \frac{1}{81}$

given that
$3^{-4} = \frac{1}{81}$

$\Rightarrow \log_3\left(\frac{1}{81}\right) = -4$

x) $a^b = c$

given that
$a^b = c$
$\Rightarrow \log_a c = b$

xi) $10^{-4} = 0.001$

given that
$10^{-4} = 0.001$

$\Rightarrow \log_{10} 0.001 = -4$

xii) $2^8 = 256$

given that
$2^8 = 256$

$\Rightarrow \log_2 256 = 8$

Ques 2. Convert the following into exponential form.

i) $\log_5 125 = 3$

given that
$\log_5 125 = 3$

$\Rightarrow 5^3 = 125$

ii) $\log_2 1024 = 10$

given that
$\log_2 1024 = 10$

$\Rightarrow 2^{10} = 1024$

Scanned with CamScanner

iii) $\log_3 81 = 4$

given that

$\log_3 81 = 4$

$\Rightarrow 3^4 = 81$

iv) $\log_3 \left(\frac{1}{81}\right) = -4$

given that

$\log_3 \left(\frac{1}{81}\right) = -4$

$\Rightarrow 3^{-4} = \frac{1}{81}$

v) $\log_4 \left(\frac{1}{64}\right) = -3$

given that

$\log_4 \left(\frac{1}{64}\right) = -3$

$\Rightarrow 4^{-3} = \frac{1}{64}$

(vi) $\log_{10} (0.001) = -3$

given that

$\log_{10} (0.001) = -3$

$\Rightarrow 10^{-3} = 0.001$

vii) $\log_{10} 1000 = 3$

given that

$\log_{10} 1000 = 3$

$\Rightarrow 10^3 = 1000$

(viii) $\log_a b = c$

given that

$\log_a b = c$

$\Rightarrow a^c = b$

ix) $\log_5 625 = 4$

given that

$\log_5 625 = 4$

$\Rightarrow 5^4 = 625$

x) $\log_2 512 = 9$

given that

$\log_2 512 = 9$

$\Rightarrow 2^9 = 512$

Scanned with CamScanner

Properties of logarithms :- If a, b and c are real number and $a \neq 1$, then

(i) Product property :- $\log_a (b \times c) = \log_a b + \log_a c$

For example :- $\log_2 (8 \times 16) = \log_2 8 + \log_2 16$

(ii) Quotient property :- $\log_a \left(\frac{b}{c}\right) = \log_a b - \log_a c$

for example :- $\log_4 \left(\frac{16}{256}\right) = \log_4 16 - \log_4 256$

(iii) Power property :- $\log_a b^c = c \log_a b$

for example = $\log_2 2^7 = 7 \log_2 2$

(iv) $\log_a a = 1$ for example : $\log_2 2 = 1$

(v) $\log_a 1 = 0$ for example = $\log_{10} 1 = 0$. $\log_e 1 = 0$

(vi) Change of base property :-

$$\log_a b = \frac{\log_e b}{\log_e a} = \frac{\log_{10} b}{\log_{10} a}$$

for example : $\log_2 8 = \frac{\log_5 8}{\log_5 2}$

Scanned with CamScanner

vii) One to one property

$\log_a b = \log_a c$

$\Leftrightarrow b = c$

For example: If $\log_5 x = \log_5 2$

$\Rightarrow x = 2$

Qus 3 Evaluate the following

i) $\log_2 (8 \times 16)$

$\log_2 8 + \log_2 16$ (product property)

$= \log_2 2^3 + \log_2 2^4$

$= 3 \cdot \log_2 2 + 4 \cdot \log_2 2$ (power property)

$= 3 \times 1 + 4 \times 1$

$= 3 + 4$

$= 7$

ii) $\log_4 \left(\frac{16}{0.256}\right)$

$= \log_4 16 - \log_4 0.256$

$= \log_4 4^2 - \log_4 4^4$

$= 2 \log_4 4 - 4 \log_4 4$ (Power property)

$= 2 \times 1 - 4 \times 1$

$= 2 - 4$

$= -2$

Scanned with CamScanner

iii) $\log_3 9^3$

$= 3 \cdot \log_3 9$ (power property)

$= 3 \times 1$ $\quad \because \log_3 3^2 = 2 \log_3 3 = 2 \times 1 = 2$

$= 3$

iv) $\log_{10} \left(\frac{1}{10}\right)^8$

$= 8 \times \log_{10} \left(\frac{1}{10}\right)$

$= 8 \times \log_{10} 10^{-1}$

$= 8 \times (-1) \times \log_{10} 10$

$= -8 \times 1$

$= -8$

or $\log_{10} = \frac{1}{10^8}$

$= \log_{10} 10^{-8}$

$= -8 \times \log_{10} 10$

$= -8 \times 1$

$= -8$

Q.4 Prove that;

i) $\log_b a \times \log_c b \times \log_a c = 1$ where a, b, c are real number and $a, b, c \neq 1$.

Ans i) L.H.S

$\log_b a \cdot \log_c b \cdot \log_a c = \frac{\log_e a}{\log_e b} \times \frac{\log_e b}{\log_e c} \times \frac{\log_e c}{\log_e a}$

$= 1$

R.H.S

Scanned with CamScanner

ii) $\log_3 25 + \log_3 15 - \log_3 5 = \log_3 75$

Ans ii) L.H.S

$\log_3 25 + \log_3 15 - \log_3 5$

$= \log_3 (25 \times 15) - \log_3 5$

$= \log_3 (375) - \log_3 5$

$= \log_3 \left(\frac{375}{5}\right)$ (75)

$= \log_3 75 = R.H.S$

iii) $\log_5 16 - \log_5 8 + \log_5 10 = \log_5 20$

Ans (iii) L.H.S

$\log_5 16 - \log_5 (8 \times 10) + \log_5 10$ (8)

$= \log_5 \left(\frac{16}{8}\right) + \log_5 10$ (2)

$= \log_5 2 + \log_5 10$

$= \log_5 (2 \times 10)$

$= \log_5 20$ R.H.S

iv) $2 \log_2 4 + \log_2 9 - \log_2 6 = \log_2 24$

Ans iv) L.H.S

$\log_2 4^2 + \log_2 9 - \log_2 6$

Scanned with CamScanner

$= \log_2 16 + \log_2 9 - \log_2 6$

$= \log_2 (16 \times 9) - \log_2 6$

$= \log_2 144 - \log_2 6$

$= \log_2 \left(\frac{144}{6}\right)^{24}$

$= \log_2 24$ R.H.S

vi) $3\log_2 4 + 4\log_2 3 = \log_2 5184$

Ans L.H.S

$\log_2 4^3 + \log_2 3^4$

$= \log_2 64 + \log_2 81$

$= \log_2 (64 \times 81)$

$= \log_2 5184$ R.H.S

Scanned with CamScanner

Page No.
Date

Chapter = 8

Matrics and Determinants

Matrix :- A system of mn numbers arranged in the form of an ordered set of m rows and n columns is called a matrix.

Ex:- $\begin{bmatrix} 2 & 0 & 1 \\ 1 & 5 & -1 \end{bmatrix}_{2\times 3}$, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$, $\begin{bmatrix} 1 \cdot 3 & 5 \end{bmatrix}_{1\times 1}$

Note:- i) Rows are horizontal lines and these are represented by R_i (i^{th} row)

ii) Columns are vertical lines and these are represented by c_i (i^{th} column)

Taking an example,

$$A = \begin{bmatrix} 2 & 0 & 7i & -i \\ i & -1 & 1+i & 2+i \\ 3 & 5 & -3 & i-3 \end{bmatrix} \begin{matrix} \rightarrow R_1 \\ \rightarrow R_2 \\ \rightarrow R_3 \end{matrix}$$

$\downarrow c_1 \quad \downarrow c_2 \quad \downarrow c_3 \quad \downarrow c_4$

order of matrix A = 3×4

Number of element in matrix A = 12

$A = [a_{ij}]$ $\quad a_{33} = -3$ $\quad a_{24} = 2+i$

$a_{12} = 0$ $\quad a_{22} = -1$ $\quad a_{32} = 5$

$a_{13} = 7i$ $\quad a_{31} = 3$

$a_{21} = i$ $\quad a_{14} = -1$

Scanned with CamScanner

Ques 1- Find the order and no of element in the matrix

$$A = [a_{ij}] = \begin{bmatrix} 1 & 2 & -3 & 1 \\ 6 & 5 & 1-i & 6 \\ 8 & -9 & -3 & 4 \end{bmatrix}_{3\times 4}$$

also find $a_{13}, a_{21}, a_{24}, a_{14}, a_{33}, a_{31}, a_{32}$

Order of matrix A = 3×4

Number of element in matrix = 12

$a_{13} = -3$ $\quad a_{33} = -3$

$a_{21} = 0$ $\quad a_{31} = 8$

$a_{24} = 6$ $\quad a_{32} = -9$

$a_{14} = 1$

Ques 2. Find the order and no of element in the matrix

$$B = [b_{ij}] = \begin{bmatrix} 3 & -5 & 6i \\ -i & 7i & 1+i \\ 6 & 0 & -3 \end{bmatrix}_{3\times 3}$$

also find $b_{23}, b_{11}, b_{13}, b_{21}, b_{33}, b_{32}$

Order of matrix B = 3×3

Number of element in matrix = 9

$b_{23} = 1+i$ $\quad b_{33} = -3$

$b_{11} = 3$ $\quad b_{32} = 0$

$b_{13} = 6i$

$b_{21} = -i$

Scanned with CamScanner

Page No.
Date

Ques 3. Construct a matrix $A = (a_{ij})$ of order 2×3 where $a_{ij} = \frac{i-j}{2}$

Any matrix $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2\times 3}$

Now $a_{ij} = \frac{i-j}{2}$

$a_{11} = \frac{1-1}{2} = \frac{0}{2} = 0$

$a_{12} = \frac{1-2}{2} = \frac{-1}{2}$

$a_{13} = \frac{1-3}{2} = \frac{-2}{2} = -1$

$a_{24} = \frac{2-1}{2} = \frac{1}{2}$

$a_{22} = \frac{2-2}{2} = \frac{0}{2} = 0$

$a_{23} = \frac{2-3}{2} = \frac{-1}{2}$

$A = \begin{bmatrix} 0 & -\frac{1}{2} & -1 \\ \frac{1}{2} & 0 & \frac{-1}{2} \end{bmatrix}_{2\times 3}$

Ques 4. Construct $A = (a_{ij})$ of order 3×2 where $a_{ij} = \frac{i+j}{2}$

Scanned with CamScanner

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}_{3\times 2}$$

Now: $a_{ij} = \frac{i+j}{2}$

$a_{11} = \frac{1+1}{2} = \frac{2}{2} = 1$ $\quad a_{22} = \frac{2+2}{2} = \frac{4}{2} = 2$

$a_{12} = \frac{1+2}{2} = \frac{3}{2}$ $\quad a_{31} = \frac{3+1}{2} = \frac{4}{2} = 2$

$a_{21} = \frac{2+1}{2} = \frac{3}{2}$ $\quad a_{32} = \frac{3+2}{2} = \frac{5}{2}$

$\therefore A = \begin{bmatrix} 1 & \frac{3}{2} \\ \frac{3}{2} & 2 \\ 2 & \frac{5}{2} \end{bmatrix}_{3\times 2}$

Ques 5. Construct a matrix $B = (b_{ij})$ of order 3×3 where $b_{ij} = i - j + 2$

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}_{3\times 3}$$

Now $B = b_{ij} = i - j + 2$

$b_{11} = 1 - 1 + 2 = 2$ $\quad b_{21} = 2 - 1 + 2 = 3$

$b_{12} = 1 - 2 + 2 = 1$ $\quad b_{22} = 2 - 2 + 2 = 2$

$b_{13} = 1 - 3 + 2 = 0$ $\quad b_{23} = 2 - 3 + 2 = 1$

Scanned with CamScanner

$b_{31} = 3-1+2 = 4$

$b_{32} = 3-2+2 = 3$

$b_{33} = 3-3+2 = 2$

$$\therefore \begin{bmatrix} 2 & 1 & 0 \\ 3 & 2 & 1 \\ 4 & 3 & 2 \end{bmatrix}_{3\times 2}$$

Ans 6. Construct matrix $C = (c_{ij})$ of order 2×2 where

$$c_{ij} = \frac{(i-j)^2}{2}$$

$$C = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$

Now $C = \frac{(i-j)^2}{2}$

$$c_{11} = \frac{(1-1)^2}{2} = \frac{1-1}{2} = \frac{0}{2} = 0$$

$$c_{12} = \frac{(1-2)^2}{2} = \frac{1-4}{2} = \frac{-3}{2}$$

$$c_{21} = \frac{(2-1)^2}{2} = \frac{4-1}{2} = \frac{3}{2}$$

$$c_{22} = \frac{(2-2)^2}{2} = \frac{4-4}{2} = \frac{0}{2} = 0$$

$$\therefore C \begin{bmatrix} 0 & \frac{-3}{2} \\ \frac{3}{2} & 0 \end{bmatrix}_{2\times 2}$$

Scanned with CamScanner

Page No.
Date

Definations

Types of matrixes

1. Rectangular matrix :- A matrix of order $m \times n$ (where $m \neq n$) is called a rectangular matrix.

e.g. $\begin{bmatrix} 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}_{2\times 3}, \begin{bmatrix} 1, 1 \end{bmatrix}_{1\times 3}, \begin{bmatrix} 2 & 3 \\ 0 & 0 \\ 5 & 6 \end{bmatrix}_{3\times 2}$ etc

2. Square matrix :- A matrix of order $m \times n$ (where $m = n$) is called square matrix.

$\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}_{2\times 2}, \begin{bmatrix} -1 \end{bmatrix}_{1\times 1}, \begin{bmatrix} i & 0 & 1 \\ 1 & 0 & i \\ -1 & 0 & -i \end{bmatrix}_{3\times 3}$ etc.

3. Row matrix :- A matrix having only one row is called a row matrix.

e.g. $\begin{bmatrix} i & 0 & 5 \end{bmatrix}_{1\times 3}, \begin{bmatrix} 1 & -1 \end{bmatrix}_{1\times 2}, \begin{bmatrix} 1 \end{bmatrix}_{1\times 1}$ etc.

4. Column matrix :- A matrix having only one column is called a column matrix etc.

e.g $\begin{bmatrix} 1 \end{bmatrix}_{1\times 1}, \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}_{3\times 1}, \begin{bmatrix} i \\ -i \end{bmatrix}_{2\times 1}$

5. Diagonal Matrix :- A square matrix is said to be diagonal matrix if all of its non-diagonal element are zero.

Scanned with CamScanner

e.g. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}_{3\times3}$ $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}_{2\times2}$, $\begin{bmatrix} 1 \end{bmatrix}_{1\times1}$ etc

6. Scalar Matrix :- A diagonal matrix is said to be scalar matrix if all of its diagonal element are same

e.g. $\begin{bmatrix} -5 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -5 \end{bmatrix}_{3\times3}$, $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2\times2}$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $[1]$ etc

7. Identity / Unit Matrix :- A diagonal matrix is said to be Identity matrix if all of its diagonal are equal to 1.

e.g. $\underset{I_3}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times3}}$, $\underset{I_2}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times2}}$, $\underset{I_1}{[1]}$ etc

8. Null / Zero Matrix :- A matrix is said to be Null matrix if all of its element are equal to zero.

e.g. $\underset{O_{2\times3}}{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{2\times3}}$, $\underset{O_{3\times1}}{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}$, $\underset{O_{1\times2}}{[0\ 0]}$, $\underset{O_{2\times2}}{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}$ etc

9. Upper triangle matrix :- A square matrix is said to be upper triangular matrix if all the elements below the main diagonal are zero

Scanned with CamScanner

e.g. $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & 9 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2\times 2}$, $\begin{bmatrix} 5 & 0 \\ 0 & 6 \end{bmatrix}$, $[1]$ ect

10. lower triangle matrix :- A square matrix is said to lower triangular matrix if all the elements above the main diagonal are zero.

e.g $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 5 & 0 \\ 3 & 6 & 9 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 \\ 0 & 6 \end{bmatrix}$, $\begin{bmatrix} 5 & 0 \\ 0 & 6 \end{bmatrix}$, $[1]$ ect

Properties of Matrics :-

Let $A = [aij]_{m\times n}$ and $B = [bij]_{m\times n}$ are two matrices & K is any scalar then

1) $A+B = [aij]_{m\times n} + [bij]_{m\times n} = [aij + bij]_{m\times n}$

2) $A-B = [aij]_{m\times n} - [bij]_{m\times n} = [aij - bij]_{m\times n}$

⇒ Adding and subtraction are possible only if the ordered of matrices are same.

3) $kA = k(aij]_{m\times n} = [kaij]_{m\times n}$

Ques 1. Find $A+B$, $A-B$, $2A$, $-3B$, $2A-3b$, $A+2b$, $-A$ if

i) $A = \begin{bmatrix} 2 & -3 \\ -4 & 5 \end{bmatrix}$ and $B \begin{bmatrix} 0 & 8 \\ -6 & 3 \end{bmatrix}$

Scanned with CamScanner

Ans i) Given that $A = \begin{bmatrix} 2 & -3 \\ -4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 8 \\ -6 & 3 \end{bmatrix}$

a) $A+B = \begin{bmatrix} 2 & -3 \\ -4 & 5 \end{bmatrix} + \begin{bmatrix} 0 & 8 \\ -6 & 3 \end{bmatrix}$

$= \begin{bmatrix} 2+0 & -3+8 \\ -4+(-6) & 5+3 \end{bmatrix}$

$= \begin{bmatrix} 2 & 5 \\ -4-6 & 8 \end{bmatrix}$

$= \begin{bmatrix} 2 & 5 \\ -10 & 8 \end{bmatrix}$ Ans

Now $A-B = \begin{bmatrix} 2 & -3 \\ -4 & 5 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -6 & 3 \end{bmatrix}$

$= \begin{bmatrix} 2-0 & -3-8 \\ -4-(-6) & 5-3 \end{bmatrix}$

$= \begin{bmatrix} 2 & -11 \\ -4+6 & 2 \end{bmatrix}$

$= \begin{bmatrix} 2 & -11 \\ 2 & 2 \end{bmatrix}$ Ans

again $2A = 2\begin{bmatrix} 2 & -3 \\ -4 & 5 \end{bmatrix}$

$= \begin{bmatrix} 4 & -6 \\ -8 & 10 \end{bmatrix}$ Ans

Scanned with CamScanner

Page No.
Date

Now, $-3B = -3\begin{bmatrix} 0 & 8 \\ -6 & 3 \end{bmatrix}$

$-3B = \begin{bmatrix} 0 & -24 \\ 18 & -9 \end{bmatrix}$ Ans

Now $2A - 3B = 2\begin{bmatrix} 2 & -3 \\ -4 & 5 \end{bmatrix} - 3\begin{bmatrix} 0 & 8 \\ -6 & 3 \end{bmatrix}$

$= \begin{bmatrix} 4 & -6 \\ -8 & 10 \end{bmatrix} - \begin{bmatrix} 0 & 24 \\ -18 & 9 \end{bmatrix}$

$= \begin{bmatrix} 4-0 & -6-24 \\ -8-(-18) & 10-9 \end{bmatrix}$

$= \begin{bmatrix} 4 & -30 \\ -8+18 & 1 \end{bmatrix}$

$= \begin{bmatrix} 4 & -30 \\ 10 & 1 \end{bmatrix}$ Ans

Now $A + 2B = \begin{bmatrix} 2 & -3 \\ -4 & 5 \end{bmatrix} + 2\begin{bmatrix} 0 & 8 \\ -6 & 3 \end{bmatrix}$

$= \begin{bmatrix} 2 & -3 \\ -4 & 5 \end{bmatrix} + \begin{bmatrix} 0 & 16 \\ -12 & 6 \end{bmatrix}$

$= \begin{bmatrix} 2+0 & -3+16 \\ -4+(-12) & 5+6 \end{bmatrix}$

$= \begin{bmatrix} 2 & 13 \\ -4-12 & 11 \end{bmatrix} = \begin{bmatrix} 2 & 13 \\ -16 & 11 \end{bmatrix}$ Ans

Scanned with CamScanner

Now $-A \Rightarrow -\begin{bmatrix} 2 & -3 \\ -4 & 5 \end{bmatrix}$

$= \begin{bmatrix} -2 & 3 \\ 4 & -5 \end{bmatrix}$ Ans

Q.) Find $A+B$, $A-B$, $2A$, $-3B$, $2A-3b$, $A+2b$, $-A$ if

$A \begin{bmatrix} -7 & 10 \\ 8 & 5 \end{bmatrix}$ and $B. \begin{bmatrix} -1 & -4 \\ 7 & 3 \end{bmatrix}$

a) $A+B := \begin{bmatrix} -7 & 10 \\ 8 & 5 \end{bmatrix} + \begin{bmatrix} -1 & -4 \\ 7 & 3 \end{bmatrix}$

$= \begin{bmatrix} -7+(-1) & 10+(-4) \\ 8+7 & 5+3 \end{bmatrix}$

$= \begin{bmatrix} -7-1 & 10-4 \\ 15 & 8 \end{bmatrix}$

$= \begin{bmatrix} -8 & 6 \\ 15 & 8 \end{bmatrix}$ Ans

b) $A-B = \begin{bmatrix} -7 & 10 \\ 8 & 5 \end{bmatrix} - \begin{bmatrix} -1 & -4 \\ 7 & 3 \end{bmatrix}$

$= \begin{bmatrix} -7-(-1) & 10-(-4) \\ 8-7 & 5-3 \end{bmatrix}$

$= \begin{bmatrix} -7+1 & 10+4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -6 & 14 \\ 1 & 2 \end{bmatrix}$ Ans

Scanned with CamScanner

c) $2A$:-

$$2\begin{bmatrix} -7 & 10 \\ 8 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -14 & 20 \\ 16 & 10 \end{bmatrix}$$ Ans

d) $-3B$ =

$$-3\begin{bmatrix} -1 & -4 \\ 7 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 12 \\ -21 & -9 \end{bmatrix}$$ Ans

e) $2A - 3b$:-

$$2\begin{bmatrix} -7 & 10 \\ 8 & 5 \end{bmatrix} - 3\begin{bmatrix} -1 & -4 \\ 7 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -14 & 20 \\ 16 & 10 \end{bmatrix} - \begin{bmatrix} -3 & -12 \\ 21 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} -14-(-3) & 20-(-12) \\ 16-21 & 10-9 \end{bmatrix}$$

$$= \begin{bmatrix} -14+3 & 20+12 \\ -5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -11 & 32 \\ -5 & 1 \end{bmatrix}$$ Ans

a) $A + 2b$:-

$$\begin{bmatrix} -7 & 10 \\ 8 & 5 \end{bmatrix} + 2\begin{bmatrix} -1 & -4 \\ 7 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & 16 \\ 8 & 5 \end{bmatrix} + \begin{bmatrix} -2 & -8 \\ 14 & 6 \end{bmatrix}$$

Scanned with CamScanner

$$= \begin{bmatrix} -7+(-2) & 10+(-8) \\ 8+14 & 5+6 \end{bmatrix}$$

$$= \begin{bmatrix} -7-2 & 10-8 \\ 22 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & 2 \\ 22 & 11 \end{bmatrix}$$ Ans

c) $-A = -\begin{bmatrix} -7 & 10 \\ 8 & 5 \end{bmatrix}$

$$= \begin{bmatrix} 7 & -10 \\ -8 & -5 \end{bmatrix}$$ Ans

Ques 2. Find $2A - B + 3C$, $A+B+C$, $A-B-C$, $A - 2B + 3C$, $2A - B + 2C$ if

$$A = \begin{bmatrix} 1 & 5 & 0 \\ 3 & -2 & 4 \end{bmatrix}, B \begin{bmatrix} -4 & 3 & 6 \\ 0 & 5 & 7 \end{bmatrix} \text{ and } C \begin{bmatrix} -3 & -1 & 0 \\ -2 & 0 & 4 \end{bmatrix}$$

a) $2A - B + 3C$:-

$$= 2\begin{bmatrix} 1 & 5 & 0 \\ 3 & -2 & 4 \end{bmatrix} - \begin{bmatrix} -4 & 3 & 6 \\ 0 & 5 & 7 \end{bmatrix} + 3\begin{bmatrix} -3 & -1 & 0 \\ -2 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 10 & 0 \\ 6 & -4 & 8 \end{bmatrix} - \begin{bmatrix} -4 & 3 & 6 \\ 0 & 5 & 7 \end{bmatrix} + \begin{bmatrix} -9 & -3 & 0 \\ -6 & 0 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 2-(-4) & 10-3 & 0-6 \\ 6-0 & -4-5 & 8-7 \end{bmatrix} + \begin{bmatrix} -9 & -3 & 0 \\ -6 & 0 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 2+4 & 7 & -6 \\ 6 & -9 & 1 \end{bmatrix} + \begin{bmatrix} -9 & -3 & 0 \\ -6 & 0 & 12 \end{bmatrix}$$

Scanned with CamScanner

$$= \begin{bmatrix} 6+(-9) & 7+(-3) & -6+0 \\ 6+(-6) & -9+0 & 1+12 \end{bmatrix}$$

$$= \begin{bmatrix} 6-9 & 7-3 & -6 \\ 6-6 & -9 & 13 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 4 & -6 \\ 0 & -9 & 13 \end{bmatrix}$$ Ans

b) $A+B+C$

$$= \begin{bmatrix} 1 & 5 & 0 \\ 3 & -2 & 4 \end{bmatrix} + \begin{bmatrix} -4 & 3 & 6 \\ 0 & 5 & 7 \end{bmatrix} + \begin{bmatrix} -3 & -1 & 0 \\ -2 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1+(-4) & 5+3 & 0+6 \\ 3+0 & -2+5 & 4+7 \end{bmatrix} + \begin{bmatrix} -3 & -1 & 0 \\ -2 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1-4 & 8 & 6 \\ 3 & 3 & 11 \end{bmatrix} + \begin{bmatrix} -3 & -1 & 0 \\ -2 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -3+(-3) & 8+(-1) & 6+0 \\ 3+(-2) & 3+0 & 11+4 \end{bmatrix}$$

$$= \begin{bmatrix} -3-3 & 8-1 & 6 \\ 3-2 & 3 & 15 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 7 & 6 \\ 1 & 3 & 15 \end{bmatrix}$$ Ans

Scanned with CamScanner

c) A - B - C :-

$$= \begin{bmatrix} 1 & 5 & 0 \\ 3 & -2 & 4 \end{bmatrix} - \begin{bmatrix} -4 & 3 & 6 \\ 0 & 5 & 7 \end{bmatrix} - \begin{bmatrix} -3 & -1 & 0 \\ -2 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1-(-4) & 5-3 & 0-6 \\ 3-0 & -2-5 & 4-7 \end{bmatrix} - \begin{bmatrix} -3 & -1 & 0 \\ -2 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4 & 2 & -6 \\ 3 & -7 & -3 \end{bmatrix} - \begin{bmatrix} -3 & -1 & 0 \\ -2 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 5-(-3) & 2-(-1) & -6-0 \\ 3-(-2) & -7-0 & -3-4 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 3 & -6 \\ 5 & -7 & -7 \end{bmatrix}$$ Ans

d) A - 2B + 3C :-

$$= \begin{bmatrix} 1 & 5 & 0 \\ 3 & -2 & 4 \end{bmatrix} - 2\begin{bmatrix} -4 & 3 & 6 \\ 0 & 5 & 7 \end{bmatrix} + 3\begin{bmatrix} -3 & -1 & 0 \\ -2 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 5 & 0 \\ 3 & -2 & 4 \end{bmatrix} - \begin{bmatrix} -8 & 6 & 12 \\ 0 & 10 & 14 \end{bmatrix} + \begin{bmatrix} -9 & -3 & 0 \\ -6 & 0 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 1-(-8) & 5-6 & 0-12 \\ 3-0 & -2-10 & 4-14 \end{bmatrix} + \begin{bmatrix} -9 & -3 & 0 \\ -6 & 0 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 1+8 & -1 & -12 \\ 3 & -12 & -10 \end{bmatrix} + \begin{bmatrix} -9 & -3 & 0 \\ -6 & 0 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -1 & -12 \\ 3 & -12 & -10 \end{bmatrix} + \begin{bmatrix} -9 & -3 & 0 \\ -6 & 0 & 12 \end{bmatrix}$$

Scanned with CamScanner

$$= \begin{bmatrix} 9+(-9) & -1+(-3) & -12+0 \\ 3+(-6) & -12+0 & -10+12 \end{bmatrix}$$

$$= \begin{bmatrix} 9-9 & -1-3 & -12 \\ 3-6 & -12 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -4 & -12 \\ -3 & -12 & 2 \end{bmatrix} \text{Ans}$$

e) $2A - B + 2C$

$$= 2\begin{bmatrix} 1 & 5 & 0 \\ 3 & -2 & 4 \end{bmatrix} - \begin{bmatrix} -4 & 3 & 6 \\ 0 & 5 & 7 \end{bmatrix} + 2\begin{bmatrix} -3 & -1 & 0 \\ -2 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 10 & 0 \\ 6 & -4 & 8 \end{bmatrix} - \begin{bmatrix} -4 & 3 & 6 \\ 0 & 5 & 7 \end{bmatrix} + \begin{bmatrix} -6 & -2 & 0 \\ -4 & 0 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 2-(-4) & 10-3 & 0-6 \\ 6-0 & -4-5 & 8-7 \end{bmatrix} + \begin{bmatrix} -6 & -2 & 0 \\ -4 & 0 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 2+4 & 10-3 & 0-6 \\ 6-0 & -4-5 & 8-7 \end{bmatrix} + \begin{bmatrix} -6 & -2 & 0 \\ -4 & 0 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 7 & -6 \\ 6 & -9 & 1 \end{bmatrix} + \begin{bmatrix} -6 & -2 & 0 \\ -4 & 0 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 6+(-6) & 7+(-2) & -6+0 \\ 6+(-4) & -9+0 & 1+8 \end{bmatrix}$$

$$= \begin{bmatrix} 6-6 & 7-2 & -6+0 \\ 6-4 & -9+0 & 1+8 \end{bmatrix} = \begin{bmatrix} 0 & 5 & -6 \\ 2 & -9 & 9 \end{bmatrix}$$

Scanned with CamScanner

Matrics Multiplication:

Matrics Multiplication Let $A\,[a_{ij}]_{m\times n}$ and $B\,[b_{ij}]_{n\times p}$

then $C = A \cdot B = [a_{ij}]_{m\times n}\,[b_{ij}]_{n\times p} = [c_{ij}]_{m\times p}$

Qus 1. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$, find AB and BA

Ans 1. $A \cdot B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2} \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}_{2\times 2}$

$$= \begin{bmatrix} (1)(4)+(2)(2) & (1)(3)+(2)(1) \\ (3)(4)+(4)(2) & (3)(3)+(4)(1) \end{bmatrix}$$

$$= \begin{bmatrix} 4+4 & 3+2 \\ 12+8 & 9+4 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ 20 & 13 \end{bmatrix}$$ Ans

$B \cdot A$ $\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}_{2\times 2} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2}$

$$= \begin{bmatrix} (4)(1)+(3)(3) & (4)(2)+(3)(4) \\ (2)(1)+(1)(3) & (2)(2)+(1)(4) \end{bmatrix}$$

$$= \begin{bmatrix} 4+9 & 8+12 \\ 2+3 & 4+4 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 20 \\ 5 & 8 \end{bmatrix}$$ Ans

Scanned with CamScanner

Page No.
Date

Ques 2. If $A = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$ and $B = \begin{bmatrix} -7 & 3 \end{bmatrix}$ find AB and BA

Ans 2. $A \cdot B = \begin{bmatrix} -3 \\ 4 \end{bmatrix}_{2\times1}$ $B \begin{bmatrix} -7 & 3 \end{bmatrix}_{1\times2}$

$$= \begin{bmatrix} (-3)(-7) & (-3)(3) \\ (4)(-7) & (4)(3) \end{bmatrix}$$

$$= \begin{bmatrix} 21 & -9 \\ -28 & 12 \end{bmatrix}$$

$B.A$ $\begin{bmatrix} -7 & 3 \end{bmatrix}_{1\times2}$ $\begin{bmatrix} -3 \\ 4 \end{bmatrix}_{2\times1}$

$$= \begin{bmatrix} (-7)(-3) + (3)(4) \end{bmatrix}$$

$$= \begin{bmatrix} 21 + 12 \end{bmatrix}$$

$$= \begin{bmatrix} 33 \end{bmatrix}$$ Any

Ques 3. If $A = \begin{bmatrix} 5 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 \\ -4 & -5 \end{bmatrix}$ find AB and BA

Ans 3. $A \cdot B$ $\begin{bmatrix} 5 & 8 \end{bmatrix}_{1\times2}$ $\begin{bmatrix} -2 & 3 \\ -4 & -5 \end{bmatrix}_{2\times2}$

$$= \begin{bmatrix} (5)(-2) + (8)(-4) & (5)(3) + (8)(-5) \end{bmatrix}$$

$$= \begin{bmatrix} -10 + -32 & 15 - 40 \end{bmatrix}$$

$$= \begin{bmatrix} -42 & -25 \end{bmatrix}$$

$B \cdot A = \begin{bmatrix} -2 & 3 \\ -4 & 5 \end{bmatrix}_{2\times2}$ $\begin{bmatrix} 5 & 8 \end{bmatrix}_{1\times2}$

$2 \neq 1$ B·A does not exit

Scanned with CamScanner

Page No.
Date

Ques 5. If $A = \begin{bmatrix} 9 & 3 \\ -7 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ find AB and BA

$$AB = \begin{bmatrix} 9 & 3 \\ -7 & 5 \end{bmatrix}_{2\times2} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}_{2\times2}$$

$$= \begin{bmatrix} (9)(1)+(3)(2) & (9)(2)+(3)(1) \\ (-7)(1)+(5)(2) & (-7)(2)+(5)(1) \end{bmatrix}$$

$$= \begin{bmatrix} 9+6 & 18+3 \\ -7+10 & -14+5 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 21 \\ 3 & 19 \end{bmatrix}$$

$$BA \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}_{2\times2} \begin{bmatrix} 9 & 3 \\ -7 & 5 \end{bmatrix}_{2\times2}$$

$$= \begin{bmatrix} (1)(9)+(2)(-7) & (1)(3)+(2)(5) \\ (2)(9)+(1)(-7) & (2)(3)+(1)(5) \end{bmatrix}$$

$$= \begin{bmatrix} 9-14 & 3+10 \\ 18-7 & 6+5 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & 13 \\ 11 & 11 \end{bmatrix}$$

Scanned with CamScanner

Ques 6. If $A = \begin{bmatrix} 5 & -7 \\ 3 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}$ find AB and BA

$$AB = \begin{bmatrix} 5 & -7 \\ 3 & -2 \end{bmatrix}_{2\times2} \begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}_{2\times2}$$

$$= \begin{bmatrix} (5)(4)+(-7)(-2) & (5)(-2)+(-7)(1) \\ (3)(4)+(-2)(-2) & (3)(-2)+(-2)(1) \end{bmatrix}$$

$$= \begin{bmatrix} 20+14 & -10-7 \\ 12+4 & -6-2 \end{bmatrix}$$

$$= \begin{bmatrix} 34 & -17 \\ 16 & -8 \end{bmatrix}$$

$$BA = \begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}_{2\times2} \begin{bmatrix} 5 & -7 \\ 3 & -2 \end{bmatrix}_{2\times2}$$

$$= \begin{bmatrix} (4)(5)+(-2)(3) & (4)(-7)+(2)(-2) \\ (-2)(5)+(1)(3) & (-2)(-7)+(1)(-2) \end{bmatrix}$$

$$= \begin{bmatrix} 20-6 & -28+4 \\ -10+3 & 14-2 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & -24 \\ -7 & 12 \end{bmatrix}$$

Scanned with CamScanner

Page No.
Date

Ques 7. If $A = \begin{bmatrix} 1 & 2 \end{bmatrix}$ and $B \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ find AB and BA

$A.B \quad \begin{bmatrix} 1 & 2 \end{bmatrix}_{1\times 2} \quad \begin{bmatrix} 3 \\ 4 \end{bmatrix}_{2\times 1}$

$= \begin{bmatrix} (1)(3) & (1)(4) \\ (2)(3) & (2)(4) \end{bmatrix}$

$= \begin{bmatrix} 3 & 4 \\ 6 & 8 \end{bmatrix}$

$B.A \quad \begin{bmatrix} 3 \\ 4 \end{bmatrix}_{2\times 1} \quad \begin{bmatrix} 1 & 2 \end{bmatrix}_{1\times 2}$

$= \begin{bmatrix} (3)(1) & (3)(2) \\ (4)(1) & (4)(2) \end{bmatrix}$

$= \begin{bmatrix} 3 & 6 \\ 4 & 8 \end{bmatrix}$

Ques 8. If $A = \begin{bmatrix} -5 \\ 7 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -8 \\ 4 & 9 \end{bmatrix}$ find AB and BA

$AB = \begin{bmatrix} -5 \\ 7 \end{bmatrix}_{2\times 1} \quad \begin{bmatrix} 3 & -8 \\ 4 & 9 \end{bmatrix}_{2\times 2}$

$= \begin{bmatrix} (-5)(3) & (-5)(4) \\ (7)(-8) & (7)(9) \end{bmatrix}$

$= \begin{bmatrix} -15 & -20 \\ -56 & 63 \end{bmatrix}$

Scanned with CamScanner

B.A $\begin{bmatrix} 3 & -8 \\ 4 & 9 \end{bmatrix}_{2\times2} \quad \begin{bmatrix} -5 \\ 7 \end{bmatrix}_{2\times1}$

$= \begin{bmatrix} (3)(-5) & (-8)(7) \\ (4)(-5) & (9)(7) \end{bmatrix}$

$= \begin{bmatrix} -15 & -56 \\ -20 & 63 \end{bmatrix}$

Qus 9. find the Determine of following:

(i) $A = \begin{bmatrix} 5 & 2 \\ 2 & 4 \end{bmatrix}$

$\Rightarrow |A| = \begin{vmatrix} 5 & 2 \\ 2 & 4 \end{vmatrix}$

$= (5)(4) - (2)(2)$

$= 20 - 4$

$= 16$

(ii) $B = \begin{bmatrix} -2 & -3 \\ 5 & 6 \end{bmatrix}$

$|B| = \begin{vmatrix} -2 & -3 \\ 5 & 6 \end{vmatrix}$

$= (-2)(6) - (5)(-3)$

$= -12 + 15$

$= 3$

Scanned with CamScanner

iii) $C = [5]$

$|C| = |5| = 5$

iv) $D = [-5]$

$|D| = |-5| = -5$

Q#1: Find the determinants of following:-

(i) $A = \begin{bmatrix} 2 & 7 \\ 8 & 4 \end{bmatrix}$

$|A| = \begin{vmatrix} 2 & 7 \\ 8 & 4 \end{vmatrix}$

$= (2)(4) - (8)(7)$

$= 8 - 56$

$= -48$

ii) $B = \begin{bmatrix} -3 & 5 \\ 2 & -3 \end{bmatrix}$

$|B| = \begin{vmatrix} -3 & 5 \\ 2 & -3 \end{vmatrix}$

$= (-3)(-3) - (2)(5)$

$= 9 - 10$

$= -1$

iii) $C = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$|C| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

$= (a)(d) - (c)(b)$

$= ad - cb$

(iv) $D = \begin{bmatrix} -1 & 0 \\ 5 & 8 \end{bmatrix}$

$|D| = \begin{vmatrix} -1 & 0 \\ 5 & 8 \end{vmatrix}$

$= (-1)(8) - (5)(0)$

$= -8 - 0$

$= -8$

Scanned with CamScanner

v) $E = \begin{bmatrix} 9 & 3 \\ -5 & 7 \end{bmatrix}$

$|E| = \begin{vmatrix} 9 & 3 \\ -5 & 7 \end{vmatrix}$

$= (9)(7) - (-5)(3)$

$= 63 + 15$

$= 78$

(vi) $F = \begin{bmatrix} 9 & 3 \\ -9 & -3 \end{bmatrix}$

$|F| = \begin{vmatrix} 9 & 3 \\ -9 & -3 \end{vmatrix}$

$= (9)(-3) - (-9)(3)$

$= -27 + 27$

$= 0$

(vii) $G = [7]$

$|G| = |7| = 7$

(viii) $H = [-7]$

$|H| = |-7| = -7$

ix) $I = [10]$

$|I| = |10| = 10$

(x) $J = [-10]$

$|J| = |-10| = -10$

Expanding 3×3 Determinants by Laplace method

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$= +a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$= a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{31}a_{23}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

Scanned with CamScanner

Page No.
Date

Q.no 1 Evaluate the following determinants.

i) $\begin{vmatrix} 3 & 5 & 6 \\ 1 & 2 & 3 \\ 3 & 0 & 2 \end{vmatrix}$

$= 3\begin{vmatrix} 2 & 3 \\ 0 & 2 \end{vmatrix} - 5\begin{vmatrix} 1 & 3 \\ 3 & 2 \end{vmatrix} + 6\begin{vmatrix} 1 & 2 \\ 3 & 0 \end{vmatrix}$

$= 3(4-0) - 5(2-9) + 6(0-6)$

$= 3(4) - 5(-7) + 6(-6)$

$= 12 + 35 - 36$

$= 11$

ii) $\begin{vmatrix} -2 & -3 & -1 \\ 5 & -6 & 7 \\ -2 & 1 & 2 \end{vmatrix}$

$= +(-2)\begin{vmatrix} -6 & 7 \\ 1 & 2 \end{vmatrix} - (-3)\begin{vmatrix} 5 & 7 \\ -2 & 2 \end{vmatrix} + (-1)\begin{vmatrix} 5 & -6 \\ -2 & 1 \end{vmatrix}$

$= -2(-12-7) + 3(10-(-14)) - 1(5-12)$

$= -2(-19) + 3(10+14) - 1(-7)$

$= 38 + 3(24) + 7$

$= 38 + 72 + 7$

$= 117$

Scanned with CamScanner

iii) $\begin{vmatrix} + & - & + \\ 5 & 0 & -1 \\ 2 & +3 & 4 \\ -2 & -3 & -1 \end{vmatrix}$

$= 5\begin{vmatrix} 3 & 4 \\ 3 & -1 \end{vmatrix} - 0$

$= -0\begin{vmatrix} 2 & 4 \\ -2 & -1 \end{vmatrix} + 3\begin{vmatrix} 5 & -1 \\ -2 & -1 \end{vmatrix} - (3)\begin{vmatrix} 5 & -1 \\ 2 & 4 \end{vmatrix}$

$= 0 + 3(-5-2) + 3(20+2)$

$= 3(-7) + 3(22)$

$= -21 + 66$

$= 45$ Ans

iv) $\begin{vmatrix} + & - & + \\ 5 & 3 & 4 \\ 3 & -2 & 1 \\ -2 & -1 & -3 \end{vmatrix}$

$= 5\begin{vmatrix} -2 & 1 \\ -1 & -3 \end{vmatrix} - 3\begin{vmatrix} 3 & 1 \\ -2 & -3 \end{vmatrix} + 4\begin{vmatrix} 3 & -2 \\ -2 & -1 \end{vmatrix}$

$= 5(6+1) - 3(-9+2) + 4(-3-4)$

$= 5(7) - 3(-7) + 4(-7)$

$= 35 + 21 - 28$

$= 56 - 28$

$= 28$ Ans

Scanned with CamScanner

Page No.
Date

v) $\begin{vmatrix} 4 & -1 & -2 \\ 5 & 0 & 6 \\ 3 & 7 & -6 \end{vmatrix}$ (+ − +)

$= 4\begin{vmatrix} 0 & 6 \\ 7 & -6 \end{vmatrix} - (-1)\begin{vmatrix} 5 & 6 \\ 3 & -6 \end{vmatrix} + (-2)\begin{vmatrix} 5 & 0 \\ 3 & 7 \end{vmatrix}$

$= 4(-6-42) + 1(-30-18) - 2(35-0)$

$= 4(-48) + 1(-48) - 2(35)$

$= -192 - 48 - 70$

$= -310$

vi) $\begin{vmatrix} 2 & 1 & -3 \\ 4 & 2 & -6 \\ 5 & 1 & 7 \end{vmatrix}$ (+ − +)

$= 2\begin{vmatrix} 2 & -6 \\ 1 & 7 \end{vmatrix} - 1\begin{vmatrix} 4 & -6 \\ 5 & 7 \end{vmatrix} + (-3)\begin{vmatrix} 4 & 2 \\ 5 & 1 \end{vmatrix}$

$= 2(14+6) - 1(28+30) - 3(4-10)$

$= 2(20) - 1(58) - 3(-6)$

$= 40 - 58 + 6$

$= 46 - 58$

$= -12$

Scanned with CamScanner

Page No.
Date

Cramer's rule for 2 unknown :-

$$a_{11}x + a_{12}y = b_1$$
$$a_{21}x + a_{22}y = b_2$$

$$D = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$|a_{11} \times a_{22} - a_{12} \times a_{21}|$$

$$D_1 \begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix} \qquad D_2 \begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}$$

i) If $D \neq 0$, then unique solution exists and solution is given by $x = \frac{D_1}{D}$ & $\frac{D_2}{D}$

ii) $D = 0 = D_1 = D_2$, then infinite many solution exists.

iii) If $D = 0$ and either $D_1 \neq 0$ or $D_2 \neq 0$ then no solution

ans 1. Solve the following system of equation by Cramer rule :

$$2x - y = -7$$
$$-x + 3y = 8$$

given that

$$2x - y = -7$$
$$-x + 3y = 8$$

Scanned with CamScanner

$$D = \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix} = (2)(3) - (-1)(-1)$$
$$= 6 - 1 = 5$$

$$D_1 = \begin{vmatrix} -7 & -1 \\ 8 & 3 \end{vmatrix} = (-7)(3) - (8)(-1)$$
$$= -21 + 8$$
$$= -13$$

$$D_2 = \begin{vmatrix} 2 & -7 \\ -1 & 8 \end{vmatrix} = (2)(8) - (-1)(-7)$$
$$= 16 - 7$$
$$= 9$$

$$x = \frac{D1}{D} = \frac{-13}{5}$$

$$y = \frac{D_2}{D} = \frac{9}{5}$$

Q. $x - 5y = 3$
$-2 + y = -7$

$$D = \begin{vmatrix} 1 & -5 \\ -2 & 1 \end{vmatrix} = 1 \times 1 - (-5)(-2)$$
$$1 - 10 = -9$$

$$D_1 = \begin{vmatrix} 3 & -5 \\ -7 & 1 \end{vmatrix} = (3 \times 1) - (-7)(-5)$$
$$3 - 35$$
$$= -32$$

$$D_2 = \begin{vmatrix} 1 & 3 \\ -2 & -7 \end{vmatrix} = (1)(-7) - (3)(-2)$$
$$-7 + 6 = -1$$

$$x = \frac{D1}{D} = \frac{+32}{-9} = \frac{32}{9} \qquad y = \frac{D_2}{D} = \frac{-1}{-9} = \frac{1}{9}$$

Scanned with CamScanner

3. $x + 3y = -7$
$-x + 5y = 4$

$$D \begin{vmatrix} 1 & 3 \\ -1 & 5 \end{vmatrix} \quad (1)(5) - (3)(-1)$$
$$5 + 3 = 8$$

$$D_1 \begin{vmatrix} -7 & 3 \\ 4 & 5 \end{vmatrix} \quad -35 - 12$$
$$= -47$$

$$D_2 \begin{vmatrix} 1 & -7 \\ -1 & 4 \end{vmatrix} = (1)(4) - (-7)(-1)$$
$$4 - 7 = -3$$

$$x = \frac{D_1}{D} = \frac{-47}{8} \qquad y = \frac{D_2}{D} = \frac{-3}{8}$$

4. $-x + y = 10$
$2x - y = -8$

$$D \begin{vmatrix} -1 & 1 \\ 2 & -1 \end{vmatrix} = (-1)(-1) - (1)(2)$$
$$1 - 2$$
$$= -1$$

$$D_1 \begin{vmatrix} 10 & 1 \\ -8 & -1 \end{vmatrix} = (10)(-1) - (1)(-8)$$
$$-10 + 8$$
$$= -2$$

$$D_2 \begin{vmatrix} -1 & 10 \\ 2 & -8 \end{vmatrix} \quad (-1)(-8) - (10)(2)$$
$$8 - 20$$
$$= -12$$

$$x = \frac{D_1}{D} = \frac{-2}{-1} = \frac{2}{1}$$

$$y = \frac{D_2}{D} = \frac{-12}{-1} = \frac{12}{1}$$

Scanned with CamScanner

5. $3x+4y=-2$
$x-y=8$

$$D \begin{vmatrix} 3 & 4 \\ 1 & -1 \end{vmatrix} \quad (3)(-1)-(4)(1)$$
$$-3-4=-7$$

$$D_1 \begin{vmatrix} -2 & 4 \\ 8 & -1 \end{vmatrix} \quad (2)(-1)-(4)(8)$$
$$2-32$$
$$=-30$$

$$D_2 \begin{vmatrix} 3 & -2 \\ 1 & 8 \end{vmatrix} = (3)(8)-(-2)(1)$$
$$24+2$$
$$=26$$

$$x=\frac{D_1}{D}=\frac{-30}{-7}=\frac{30}{7}$$

$$y=\frac{D_2}{D}=\frac{26}{-7}$$

Cramers rule for 3X3 Unknowns.

$$a_{11}x+a_{12}y+a_{13}z=b_1$$
$$a_{21}x+a_{22}y+a_{23}z=b_2$$
$$a_{31}x+a_{32}y+a_{33}z=b_3$$

$$D \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$D_1 \begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}$$

Scanned with CamScanner

$$D_2 = \begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}$$

$$D_3 = \begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}$$

i) If $D \neq 0$ then unique solution exists and solution is given by.

$$x = \frac{D_1}{D}, \quad y = \frac{D_2}{D}, \quad z = \frac{D_3}{D}$$

ii) If $D = 0 = D_1 = D_2 = D_3$ then infinite many solution

iii) If $D = 0$ but either $D_1 \neq 0$ or $D_2 \neq 0$ or $D_3 \neq 0$ then no solution.

(Ex. Write down the formula of ${}^nP_r$ and ${}^nC_r$ and hence evaluate ${}^7P_2$ and ${}^7C_2$

i) ${}^7P_2 = \frac{n!}{(n-r)!} = \frac{7!}{(7-2)!} = \frac{7!}{5!} = \frac{7 \times 6 \times 5!}{5!} = 42$

ii) ${}^7C_2 = \frac{n!}{(n-r)! \times r!} = \frac{7!}{(7-2)! \times 2!} = \frac{7!}{5! \times 2!} = \frac{7 \times 6 \times 5!}{5! \times 2 \times 1} = 21$

Permutation

5. $3x + 4y = -2$
$x - y = 8$

$$D \begin{vmatrix} 3 & 4 \\ 1 & -1 \end{vmatrix} \quad (3)(-1) - (4)(1)$$
$$-3 - 4 = -7$$

$$D_1 \begin{vmatrix} -2 & 4 \\ 8 & -1 \end{vmatrix} \quad (2)(-1) - (4)(8)$$
$$2 - 32$$
$$= -30$$

$$D_2 \begin{vmatrix} 3 & -2 \\ 1 & 8 \end{vmatrix} = (3)(8) - (-2)(1)$$
$$24 + 2$$
$$= 26$$

$$x = \frac{D_1}{D} = \frac{-30}{-7} = \frac{30}{7}$$

$$y = \frac{D_2}{D} = \frac{26}{-7}$$

Cramers rule for 3x3 Unknowns.

$$a_{11}x + a_{12}y + a_{13}z = b_1$$
$$a_{21}x + a_{22}y + a_{23}z = b_2$$
$$a_{31}x + a_{32}y + a_{33}z = b_3$$

$$D \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$D_1 \begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}$$

Scanned with CamScanner

$$D_2 = \begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}$$

$$D_3 = \begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}$$

i) If $D \neq 0$ then unique solution exists and solution is given by.

$$x = \frac{D_1}{D}, \quad y = \frac{D_2}{D}, \quad z = \frac{D_3}{D}$$

ii) If $D = 0 = D_1 = D_2 = D_3$ then infinite many solution

iii) If $D = 0$ but either $D_1 \neq 0$ or $D_2 \neq 0$ or $D_3 \neq 0$ then no solution.

ex1. write down the formula of ${}^nP_r$ and ${}^nC_r$ and hence evaluate ${}^7P_2$ and ${}^7C_2$

i) ${}^7P_2 = \frac{n!}{(n-r)!} = \frac{7!}{(7-2)!} = \frac{7!}{5!} = \frac{7 \times 6 \times 5!}{5!} = 42$

ii) ${}^7C_2 = \frac{n!}{(n-r)! \times r!} = \frac{7!}{(7-2)! \times 2!} = \frac{7!}{5! \times 2!} = \frac{7 \times 6 \times 5!}{5! \times 2 \times 1} = 21$

Permutation

Chapter = 8

Permutation and Combination

Factorial: If n is a natural number then factorial of n is given by $n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$ and $0! = 1$

For example: $5! = 5 \times 4 \times 3 \times 2 \times 1$
$= 120$

$2! = 2 \times 1$
$= 2$

Note:- (i) $n!$ can also be written as $\lfloor n$

ii) Factorial of negative number and fractional numbers do not exists.

iii) $7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
$= 7 \times 6 \times 5!$

Evaluate the following:-

i) $3! + 2!$
$3 \times 2 \times 1 + 2 \times 1$
$= 6 + 2$
$= 8$

ii) $(3+2)!$
$= 5!$
$= 5 \times 4 \times 3 \times 2 \times 1$
$= 120$

iii) $3 + 2!$
$= 3 + 2 \times 1$
$= 3 + 2$
$= 5$

iv) $3! + 2$
$= 3 \times 2 \times 1 + 2$
$= 6 + 2$
$= 8$

(v) $5! - 3!$
$5 \times 4 \times 3 \times 2 \times 1 - 3 \times 2 \times 1$
$= 120 - 6$
$= 114$

Scanned with CamScanner

vi) $(5-3)!$
$= 2!$
$= 2 \times 1$
$= 2$

(vii) $\frac{8!}{3!}$
$= \frac{8 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1}$
$= 20$

(viii) $\left(\frac{5}{2}\right)!$
$(1.7)!$
doesn't exists

ix) $3! \times 2!$
$= 3 \times 2 \times 1 \times 2 \times 1$
$= 12$

(x) $(3 \times 2)!$
$= 6!$
$= 6 \times 5 \times 4 \times 3 \times 2 \times 1$
$= 720$

(xi) $(-5)!$
Does n't exist

(xii) $-5!$
$-5 \times 4 \times 3 \times 2 \times 1$
$= -120$

(xiii) $(5.6)!$
Does n't exist

(xiv) $\left(\frac{10}{3}\right)!$
$= (3.33)!$
Does n't exist

(xv) $\left(\frac{26}{10}\right)!$
$= 2 \times 1$
$= 2$

Evaluate the following

i) $7!$
$= 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
$= 5040$

ii) $8!$
$= 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
$= 40320$

iii) $6! - 4!$
$6 \times 5 \times 4 \times 3 \times 2 \times 1 - 4 \times 3 \times 2 \times 1$
$= 720 - 24$
$= 696$

(iv) $3! + 4!$
$3 \times 2 \times 1 + 4 \times 3 \times 2 \times 1$
$= 6 + 24$
$= 30$

Scanned with CamScanner

v) $5! - 2!$

$5 \times 4 \times 3 \times 2 \times 1 - 2 \times 1$

$= 120 - 2$

$= 118$

(vi) $(5+2)!$

$= 7!$

$= 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 5040$

vii) $(6-4)!$

$= 2!$

$= 2 \times 1$

$= 2$

(viii) $4! \times 3!$

$= 4 \times 3 \times 2 \times 1 \times 3 \times 2 \times 1$

$= 144$

(ix) $2! \times 0!$

$= 2 \times 1 \times 1$

$= 2$

x) $(4 \times 2)!$

$= 8!$

$= 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 40320$

(xi) $\left(\frac{10!}{8!}\right)$

$= \frac{10 \times 9 \times 8!}{8!}$

$= 90$

(xii) $\left(\frac{10}{8}\right)!$

$\left(\frac{16}{8}\right)! \cdot \left(\frac{5}{4}\right)!$

$(0.8)!$

does not exist

xiii) $\frac{50!}{48!}$

$= \frac{50 \times 49 \times 48!}{48!}$

$= 2450$

(xiv) $(-3)!$

Doesn't exist

(xv) $(-7)!$

Doesn't exist

(xvi) $(3.2)!$

Doesn't exist

(xvii) $(4.3)!$

Doesn't exit

(xviii) $-6!$

$= -6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= -720$

(xix) $\frac{6!}{3! \times 2!}$

$= \frac{6 \times 5 \times 4 \times 3 \times 2!}{3 \times 2 \times 1 \times 2!}$

$= 60$

Scanned with CamScanner

1) ${}^5P_0 = \frac{5!}{(5-0)!} = \frac{5!}{5!} = 1$

2) ${}^5P_1 = \frac{5!}{(5-1)!} = \frac{5!}{4!} = \frac{5 \times 4!}{4!} = 5$

3 ${}^5P_2 = \frac{5!}{(5-2)!} = \frac{5!}{3!} = \frac{5 \times 4 \times 3!}{3!} = 20$

4. ${}^5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{5 \times 4 \times 3 \times 2!}{2!} = 60$

5. ${}^5P_4 = \frac{5!}{(5-4)!} = \frac{5!}{1!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{1} = 120$

6 ${}^5P_5 = \frac{5!}{(5-5)!} = \frac{5!}{0!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{1} = 120$

7) ${}^7P_4 = \frac{7!}{(7-4)!} = \frac{7!}{3!} = \frac{7 \times 6 \times 5 \times 4 \times 3!}{3!} = 840$

8) ${}^6P_3 = \frac{6!}{(6-3)!} = \frac{6!}{3!} = \frac{6 \times 5 \times 4 \times 3!}{3!} = 120$

9) ${}^4P_0 = \frac{4!}{(4-0)!} = \frac{4!}{4!} / = 1$

10) ${}^8P_8 = \frac{8!}{(8-8)!} = \frac{8!}{0!} = \frac{8!}{1} = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 =$

11) ${}^6P_1 = \frac{6!}{(6-1)!} = \frac{6!}{5!} = \frac{6 \times 5!}{5!} = 6$

Scanned with CamScanner

12) $^7P_3 = \frac{7!}{(7-3)!} = \frac{7!}{4!} = \frac{7\times6\times5\times4!}{4!} = 216$

13) $^3P_2 = \frac{3!}{(3-2)!} = \frac{3!}{1!} = \frac{3\times2\times1}{1} = 6$

14) $^3P_0 = \frac{3!}{(3-0)!} = \frac{3!}{3!} = 1$

15) $^4P_4 = \frac{4!}{(4-4)!} = \frac{4!}{0!} = \frac{4\times3\times2\times1}{1} = 24$

Ans 2(i) $^nC_n = \frac{n!}{(n-n)!\times n!}$

(i) $^5C_0 = \frac{5!}{(5-0)!\times0!} = \frac{5!}{5!\times0!} = \frac{1}{1} = 1$

ii) $^5C_1 = \frac{5!}{(5-1)!\times1!} = \frac{5!}{4!\times1!} = \frac{5\times4\times3\times2\times1}{4\times3\times2\times1} = 5$

iii) $^5C_2 = \frac{5!}{(5-2)!\times2!} = \frac{5!}{3!\times2!} = \frac{5\times4\times3!}{3!\times2\times1} = 10$

iv) $^5C_3 = \frac{5!}{(5-3)!\times3!} = \frac{5!}{2!\times3!} = \frac{5\times4\times3!}{2\times1\times3!} = 10$

v) $^5C_4 = \frac{5!}{(5-4)!\times4!} = \frac{5!}{1!\times4!} = \frac{5\times4\times3\times2\times1}{1\times4\times3\times2\times1} = 5$

Scanned with CamScanner

vi) ${}^5C_5 = \frac{5!}{(5-5)! \times 5!} = \frac{5!}{0! \times 5!} = \frac{1}{1} = 1$

vii) ${}^7C_4 = \frac{7!}{(7-4)! \times 4!} = \frac{7!}{3! \times 4!} = \frac{7 \times 6 \times 5 \times 4!}{3 \times 2 \times 1 \times 4!} = 35$

8) ${}^6C_3 = \frac{6!}{(6-3)! \times 3!} = \frac{6!}{3! \times 3!} = \frac{6 \times 5 \times 4 \times 3!}{3! \times 3 \times 2 \times 1} = 26$

9) ${}^4C_0 = \frac{4!}{(4-0)! \times 0!} = \frac{4!}{4! \times 0!} = \frac{1}{1} = 1$

10) ${}^8C_8 = \frac{8!}{(8-8)! \times 8!} = \frac{8!}{0! \times 8!} \quad \frac{1}{1} = 1$

11) ${}^6C_1 = \frac{6!}{(6-1)! \times 1!} = \frac{6!}{5! \times 1!} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1 \times 1} = 6$

12) ${}^7C_3 = \frac{7!}{(7-3)! \times 3!} = \frac{7!}{4! \times 3!} = \frac{7 \times 6 \times 5 \times 4!}{4! \times 3 \times 2 \times 1} = 35$

13) ${}^3C_2 = \frac{3!}{(3-2)! \times 2!} = \frac{3!}{1! \times 2!} = \frac{3 \times 2 \times 1}{1 \times 2 \times 1} = 3$

14) ${}^3C_0 = \frac{3!}{(3-0)! \times 0!} = \frac{3!}{3! \times 0!} = \frac{1}{1} = 1$

15) ${}^4C_4 = \frac{4!}{(4-4)! \times 4!} = \frac{4!}{0! \times 4!} = \frac{1}{1} = 1$

Scanned with CamScanner

Page No.
Date 28/09/19

Chapter = 9

Binomial theorem

$$(a+b)^n = \underbrace{{}^nC_0\, a^n b^0}_{T_1} + \underbrace{{}^nC_1\, a^{n-1} b^1}_{T_2} + \underbrace{{}^nC_2\, a^{n-2} b^2}_{T_3} + \cdots + \underbrace{{}^nC_n\, a^0 b^n}_{T_{n+1}}$$

Note:- i) Number of term in the binomial expension of $(a+b)^n$ is $n+1$.

ii) $T_1 = {}^nC_0\, a^n b^0$
$T_2 = {}^nC_1\, a^{n-1} b^1$
$T_3 = {}^nC_2\, a^{n-2} b^2$

General Term $= T_{r+1} = {}^nC_r\, a^{n-r} b^r$

ex $= T_{100} = {}^nC_{99}\, a^{n-99} b^{99}$

iii) r^{th} term from end $= (n-r-2)^{th}$ term from start
$= T_{n-r+2}$

iv) Middle term :-

$T_{\frac{n+1}{2}}$. $T_{\frac{n+3}{2}}$ if n is odd

$T_{\frac{n+2}{2}}$ if n is even

Ques. write down the general term in the binomial expansion of $(a+b)^n$

$$T_{r+1} = {}^nC_r\, a^{n-r} b^r$$

$$(2x+7y)^5 = T_{r+1} = {}^nC_r\, a^{n-r} b^r$$
$$= {}^5C_r\, (2x)^{5-r} (7y)^r$$

Scanned with CamScanner

Page No.
Date

Ques 2. How many terms are in the binomial expansion of $(7x - 5y)^{10}$

Ans There are 11 term in the binomial expansion of $(7x - 5y)^{10}$

Ques 3. which term is the middle term in binomial expansion of $(x + y)^5$

Ans Here $n = 5$, which is odd

Middle term = $T_{\frac{5+1}{2}}$ and $T_{\frac{5+3}{2}}$

$= T_{\frac{6}{2}}$ and $T_{\frac{8}{2}}$

$= T_3$ and T_4

Ques 4. which term is the middle term in the binomial expansion of $\left(x + \frac{1}{x}\right)^8$

Ans Here $n = 8$, which is even

middle term = $T_{\frac{n+2}{2}}$

$T_{\frac{8+2}{2}}$

$T_{\frac{10}{2}} = T_5 = 5^{th}$ term

Ques 5. write down the general term in the binomial expansion of $(a+b)^n$.

$T_{r+1} = {}^nC_r \, a^{n-r} b^r$

Scanned with CamScanner

Page No.
Date

Ques 6. How many terms in the binomial expansion of

(i) $(x+y)^{10}$

Ans There are 11 term in the binomal exp of $(x+y)^{10}$

(ii) $(x-y)^7$

Ans There are 8 term in the binomial exp of $(x-y)^7$

(iii) $(8x-3y)^8$

Ans There are 9 term in the binomial exp of $(5x-3y)^8$

(iv) $(2x+3y)^5$

Ans There are 6 term in the binomial exp of $(2x+3y)^5$

(v) $(x-\frac{1}{x})^6$

Ans There are 7 term in the binomial exp of $(x-\frac{1}{x})^6$

(vi) $(a+x)^3$

Ans There are 4 term in the binomial exp of $(a+x)^3$

Ques 7. Write the term is the middle term in the binomial expansion of $(x+y)^{10}$

Ans Here $n = 10$, which is even

Middle term $T_{\frac{10+2}{2}}$

$= T_{\frac{12}{2}}$

T_6

(ii) $(x-y)^7$

Here $n = 7$, which is odd

Middle term $= T_{\frac{7+1}{2}}$ and $T_{\frac{7+3}{2}}$

Scanned with CamScanner

$T_{\frac{8}{2}}$ and $T_{\frac{10}{2}}$

T_4 and T_5

iii) $(5x - 3y)^8$

Here $n = 8$, which is even

Middle term $= T_{\frac{8+2}{2}}$

$= T_{\frac{10}{2}}$

T_5

iv) $(2x + 3y)^5$

Here $n = 5$, which is odd

Middle term $= T_{\frac{5+1}{2}}$ and $T_{\frac{5+3}{2}}$

$= T_{\frac{6}{2}}$ and $T_{\frac{8}{2}}$

$= T_3$ and T_4

v) $(x - \frac{1}{x})^6$

Here $n = 6$, which is even

Middle term $= T_{\frac{6+2}{2}}$

$= T_{\frac{8}{2}}$

$= T_4$

Scanned with CamScanner

vi) $(2+2)^3$

Here $n=3$, which is odd

Middle term = $T_{\frac{3+1}{2}}$ and $T_{\frac{3+3}{2}}$

$= T_{\frac{4}{2}}$ and $T_{\frac{6}{2}}$

$= T_2$ and T_3

Ques: expand $(x+2y)^4$ using binomial Theorem

Ans $(x+2y)^4$

$= {}^4C_0 (x)^4 (2y)^0 + {}^4C_1 (x)^3 (2y)^1 + {}^4C_2 (x)^2 (2y)^2 + {}^4C_3 (x)^1 (2y)^3 +$

${}^4C_4 (x)^0 (2y)^4$

Now ${}^4C_0 = \frac{4!}{(4-0)!\times 0!} = \frac{4!}{4!\times 1} = \frac{1}{1} = 1$

${}^4C_1 = \frac{4!}{(4-1)!\times 1!} = \frac{4!}{3!\times 1} = \frac{4\times 3!}{3!\times 1} = 4$

${}^4C_2 = \frac{4!}{(4-2)!\times 2!} = \frac{4!}{2!\times 2!} = \frac{4\times 3\times 2!}{2!\times 2\times 1} = 6$

${}^4C_3 = \frac{4!}{(4-3)!\times 3!} = \frac{4!}{1!\times 3!} = \frac{4\times 3!}{1\times 3!} = 4$

${}^4C_4 = \frac{4!}{(4-4)!\times 4!} = \frac{4!}{0!\times 4!} = \frac{4!}{1\times 4!} = 1$

Put these value in eq - (1) we get

Scanned with CamScanner

$(x+2y)^4 = 1\times x^4\times 1 + 4\times x^3\times 2y + 6\times x^2\times 4y^2 + 4\times x\times 8y^3 + 1\times 1 \times 16y^4$

$= x^4 + 8x^3y + 24x^2y^2 + 32xy^3 + 16y^4$

Second method :-

Now $= {}^4C_0 = 1 = {}^4C_4$

${}^4C_1 = 4 = {}^4C_3$

${}^4C_2 = \frac{4!}{(4-2)!\times 2!} = \frac{4!}{2!\times 2!} = \frac{4\times 3\times 2!}{2!\times 2\times 1} = 6$

Note = i) ${}^nC_0 = 1 = {}^nC_n$

ii) ${}^nC_1 = n = {}^nC_{n-1}$

iii) ${}^nC_r = {}^nC_{n-r}$

Ques 2. Expand the following by binomial theorem

i) $(x+y)^5$

Now ${}^5C_0 = 1$ ${}^5C_5 = 1$

${}^5C_1 = 5$

${}^5C_2 = 10$

${}^5C_3 = 10$

${}^5C_4 = 5$

Scanned with CamScanner

Page No.
Date

$= {}^5C_0(x)^5(y)^0 + {}^5C_1(x)^4(y)^1 + {}^5C_2(x)^3(y)^2 + {}^5C_3(x)^2(y)^3 + {}^5C_4($

$(y)^4 + {}^5C_5(x)^0(y)^5$

$= 1 \times x^5 \times 1 + 5 \times x^4 \times y + 10 \times x^3 \times y^2 + 10 \times x^2 \times y^2 + 5 \times x \times y^4$
$+ 1 \times 1 \times y^5$

$= x^5 + 5x^4y + 10x^3y^2 + 10x^2y^2 + 5xy^4 + y^5$ Ans

(ii) $(2x+y)^4$

Now $= {}^4C_0 = 1$

${}^4C_1 = 4$

${}^4C_2 = \frac{n!}{(n-r)! \times r!} = \frac{4!}{(4-2)! \times 2!} = \frac{4!}{2! \times 2!} = \frac{4 \times 3 \times 2!}{2! \times 2 \times 1} = 6$

${}^4C_3 = 4$

${}^4C_4 = 1$

$= {}^4C_0(2x)^4(y)^0 + {}^4C_1(2x)^3(y)^1 + {}^4C_2(2x)^2(y)^2 + {}^4C_3(2x)^1(y)^3 + {}^4C_4$

$(2x)^0(y)^4$

$= 1 \times 16x^4 \times 1 + 4 \times 8x^3 \times y + 6 \times 4x^2 \times y^2 + 4 \times 2x \times y^3 + 1 \times 1 \times y^4$

$= 16x^4 + 32x^3y + 24x^2y^2 + 8y^3 + 1y^4$

Scanned with CamScanner

Wednesday

Page No.
Date 9/10/19

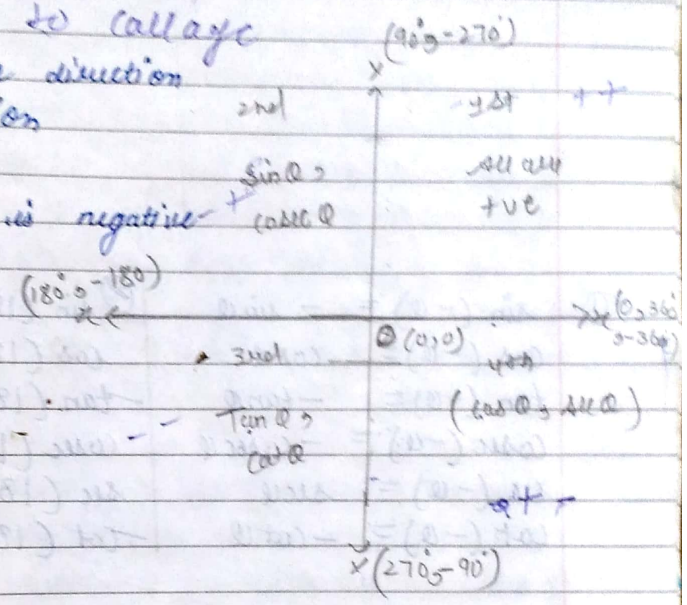
Chapter = 10

Trigonometry

School to college

Note:- Anti clock wise direction is positive direction

clock wise direction is negative direction.



$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

$$\sin\theta = \frac{1}{\operatorname{cosec}\theta}$$

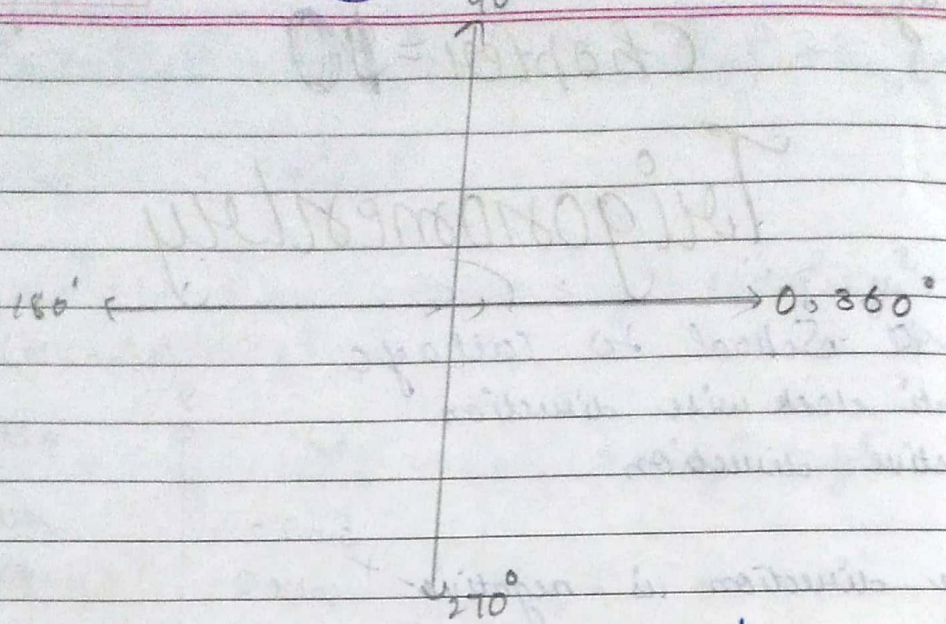
$$\cot\theta = \frac{1}{\tan\theta}$$

★ $\sin\theta$ and $\operatorname{cosec}\theta$ are +ve in 1st and 2nd quadrant but these are negative in 3rd and 4th quadrant

★ $\tan\theta$ and $\cot\theta$ are +ve in 1st and 3rd quadrant but these are -ve in 2nd and 4th quadrant.

★ $\cos\theta$ and $\sec\theta$ are +ve in 1st and 4th quadrant but these are -ve in 2nd and 3rd quadrant.

Scanned with CamScanner



① $\sin(-\theta) = -\sin\theta$
$\cos(-\theta) = \cos\theta$
$\tan(-\theta) = -\tan\theta$
$\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$
$\sec(-\theta) = \sec\theta$
$\cot(-\theta) = -\cot\theta$

3rd

② $\sin(180^\circ+\theta) = -\sin\theta$
$\cos(180^\circ+\theta) = -\cos\theta$
$\tan(180^\circ+\theta) = \tan\theta$
$\operatorname{cosec}(180^\circ+\theta) = -\operatorname{cosec}\theta$
$\sec(180^\circ+\theta) = -\sec\theta$
$\cot(180^\circ+\theta) = \cot\theta$

2nd

③ $\sin(180^\circ-\theta) = \sin\theta$
$\cos(180^\circ-\theta) = -\cos\theta$
$\tan(180^\circ-\theta) = -\tan\theta$
$\operatorname{cosec}(180^\circ-\theta) = \operatorname{cosec}\theta$
$\sec(180^\circ-\theta) = -\sec\theta$
$\cot(180^\circ-\theta) = -\cot\theta$

4th

④ $\sin(360^\circ-\theta) = -\sin\theta$
$\cos(360^\circ-\theta) = \cos\theta$
$\tan(360^\circ-\theta) = -\tan\theta$
$\operatorname{cosec}(360^\circ-\theta) = -\operatorname{cosec}\theta$
$\sec(360^\circ-\theta) = \sec\theta$
$\cot(360^\circ-\theta) = -\cot\theta$

⑧ $\sin(360^\circ+\theta) = \sin\theta$
$\cos(360^\circ+\theta) = \cos\theta$
$\tan(360^\circ+\theta) = \tan\theta$
$\operatorname{cosec}(360^\circ+\theta) = \operatorname{cosec}\theta$
$\sec(360^\circ+\theta) = \sec\theta$
$\cot(360^\circ+\theta) = \cot\theta$

Scanned with CamScanner

Page No.
Date

★ $90^\circ \pm \theta$, $270^\circ \pm \theta$

First check that the sign +ve or -ve, then change the function

sin ⟷ cos
Tan ⟷ cot
sec ⟷ Cosec

1st

$\sin(90^\circ - \theta) = \cos\theta$	$\sin(90^\circ + \theta) = \cos\theta$
$\cos(90^\circ - \theta) = \sin\theta$	$\cos(90^\circ + \theta) = -\sin\theta$
$\tan(90^\circ - \theta) = \cot\theta$	$\tan(90^\circ + \theta) = -\cot\theta$
$\text{Cosec}(90^\circ - \theta) = \sec\theta$	$\text{Cosec}(90^\circ + \theta) = \sec\theta$
$\sec(90^\circ - \theta) = \text{Cosec}\,\theta$	$\sec(90^\circ + \theta) = -\text{Cosec}\,\theta$
$\cot(90^\circ - \theta) = \tan\theta$	$\cot(90^\circ + \theta) = -\tan\theta$

3rd

$\sin(270^\circ - \theta) = -\cos\theta$	$\sin(270^\circ + \theta) = -\cos\theta$
$\cos(270^\circ - \theta) = -\sin\theta$	$\cos(270^\circ + \theta) = \sin\theta$
$\tan(270^\circ - \theta) = \cot\theta$	$\tan(270^\circ + \theta) = -\cot\theta$
$\text{Cosec}(270^\circ - \theta) = -\sec\theta$	$\text{Cosec}(270^\circ + \theta) = -\sec\theta$
$\sec(270^\circ - \theta) = -\text{Cosec}\,\theta$	$\sec(270^\circ + \theta) = \text{Cosec}\,\theta$
$\cot(270^\circ - \theta) = \tan\theta$	$\cot(270^\circ + \theta) = -\tan\theta$

$\sin\theta$	$\cos\theta$	$\tan\theta$
$\frac{P}{H}$	$\frac{B}{H}$	$\frac{P}{B}$
$\text{Cosec}\,\theta$	$\sec\theta$	$\cot\theta$

$\sin = \frac{P}{H}$ $\quad \cos\theta = \frac{B}{H}$ $\quad \tan\theta = \frac{P}{B}$

$\text{Cosec} = \frac{H}{P}$ $\quad \sec\theta = \frac{H}{B}$ $\quad \cot\theta = \frac{B}{P}$

Scanned with CamScanner

Formula :-

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

$180^\circ = \pi$

$1^\circ = \frac{\pi}{180}$ radians

π radian $= 180^\circ$

1 radia $= \left(\frac{180}{\pi}\right)^\circ$

$\frac{2\pi}{6} = \frac{2 \times 180}{6_3}\ 60 \quad = 60^\circ$

$45^\circ = 180 = \pi$ radian

$1^\circ = \frac{\pi}{180^\circ}$

$\frac{45 \times \pi}{180\ 4} = \frac{\pi}{4}\ 4$

$90^\circ = 100$ grades $\quad \boxed{90^\circ = 100^g}$

$1^\circ = \frac{100}{90}$ grades

$\boxed{100^g = 90^\circ}$

$1 = \left(\frac{90}{100}\right)^\circ$

★ Allied angles :- To angle are said to be allied angle when there sum or difference is either zero or a multiple of 90°.

Scanned with CamScanner

* Complimentary angles :- To angle whose sum is 90° are called compliment of each other.
The angle θ and $90^\circ - \theta$ are complimentary angle.

* Supplimentary angle :- To angle whose sum is 180° are called supplementary of each other.
The angle θ and $180^\circ - \theta$ are supplementary angle.

1. $\sin(A+B) = \sin A \cos B + \cos A \sin B$
$\sin(A-B) = \sin A \cos B - \cos A \sin B$

2. $\cos(A+B) = \cos A \cos B - \sin A \sin B$
$\cos(A-B) = \cos A \cos B + \sin A \sin B$

3. $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

$\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

4. $\cot(A+B) = \dfrac{\cot B \cdot \cot A - 1}{\cot B + \cot A}$

$\cot(A-B) = \dfrac{\cot B \cdot \cot A + 1}{\cot B - \cot A}$

5. $\sin 2A = 2 \sin A \cos A$
$\cos 2A = \cos^2 A - \sin^2 A$

Scanned with CamScanner

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$
$$2\cos A\sin B = \sin(A+B) - \sin(A-B)$$
$$2\cos A\cos B = \cos(A-B) + \cos(A+B)$$
$$2\sin A\sin B = \cos(A-B) - \cos(A+B)$$

$$\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{D-C}{2}\right)$$

$$\cos C - \cos D = 2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{D-C}{2}\right)$$

	0	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

$\pi^c = 180° = 200^g$

* English System / Sexagesimal system / Degree measure (ESD)

French system / centesimal system / Grade measure (FCG)

Scanned with CamScanner

Circular system / Radian measure (CR)

English system
1 rt angle = 90°
1° = 60' (60 minutes)
1' 60" (60 seconds)

French system
1 rt angle = 100g
1g = 100' (100 mint)
1' = 100" (100 second)

Ques 1. Write the following angles in circular measure.

i) 75°

D → R $\quad \boxed{\pi^c = 180° = 200g}$

$180° = \pi^c$

$1° = \frac{\pi^c}{180}$

$\Rightarrow 75° = \frac{75^5 \times \pi^c}{180_{12}} = \frac{5\pi^c}{12}$

(ii) 140°

D ⇒ R

$180° = \pi^c$

$1° = \frac{\pi^c}{180}$

$\Rightarrow 140° \Rightarrow \frac{140 \times \pi^c}{180} = \frac{7\pi^c}{9}$

Ques 2. Find the centesimal means of the angle whose radian measure

i) $\frac{4\pi^c}{5}$

R ⇒ G

$\pi^c = 200°$

$1^c = \frac{200g}{\pi}$

$\Rightarrow \frac{4\pi^c}{5} = \frac{4\pi}{5} \times \frac{200g}{\pi} = 160g$

(ii) $\frac{3\pi^c}{10}$

R = G

$\pi^c = 200°$

$\Rightarrow 1^c = \frac{200g}{\pi}$

$= \frac{3\pi^c}{10} \Rightarrow \frac{3\pi}{10} \times \frac{200}{\pi}$

$= 60g$

Scanned with CamScanner

Page No.
Date

Ans 3. Write the following angle in sexagesimal measure whose radian measure are:

i) $\frac{\pi^c}{5}$

$R = G$

$\pi^c = 180^\circ$

$\Rightarrow 1^c = \frac{180}{\pi}$

$\Rightarrow \frac{\pi^c}{5} = \frac{\pi}{5} \times \frac{180^{36}}{\pi} = 36^\circ$

(ii) $\frac{\pi^c}{6}$

$R = G$

$\Rightarrow \pi^c = 180^\circ$

$\Rightarrow 1^c = \frac{180}{\pi}$

$\Rightarrow \frac{\pi^c}{6} = \frac{\pi}{6} \times \frac{180^{30}}{\pi} = 30^\circ$

Ans 4. Convert the following degree measure into circular measure.

i) 60°

$D \Rightarrow R$

$180^\circ = \pi^c$

$1 = \frac{\pi^c}{180^\circ}$

$\Rightarrow 60^\circ = \frac{60 \times \pi^c}{180_3} = \frac{\pi^c}{3}$

(ii) 120°

$D \Rightarrow R$

$180^\circ = \pi^c$

$1 = \frac{\pi^c}{180^\circ}$

$\Rightarrow 120^\circ = \frac{120^2 \times \pi^c}{180_3} = \frac{2\pi^c}{3}$

iii) 150°

$D \Rightarrow R$

$180^\circ = \pi^c$

$1 = \frac{\pi^c}{180^\circ}$

$150^\circ \Rightarrow \frac{150^5 \times \pi^c}{180_6} = \frac{5\pi^c}{6}$

(iv) 210°

$D \Rightarrow R$

$180^\circ = \pi^c$

$1 = \frac{\pi^c}{180^\circ}$

$210^\circ = \frac{210^7 \times \pi^c}{180_6} = \frac{7\pi^c}{6}$

Scanned with CamScanner

(v) 270°

D ⇒ R

$180^\circ = \pi^c$

$1 = \frac{\pi^c}{180^\circ}$

$270^\circ \Rightarrow \frac{270 \times \pi^c}{180} = \frac{9\pi^c}{6}$

(vi) 330°

D ⇒ R

$180^\circ = \pi^c$

$1 = \frac{\pi^c}{180^\circ}$

$330^\circ \Rightarrow \frac{330 \times \pi^c}{180} = \frac{11\pi^c}{6}$

Ques 5. Write the following angle into sexagesimal measure whose radian measure.

i) $\frac{\pi^c}{6}$

R ⇒ D

$\pi^c = 180^\circ$

$1^c = \frac{180^\circ}{\pi}$

$\frac{\pi^c}{6} \Rightarrow \frac{\pi}{6} \times \frac{180^\circ}{\pi} = 30^\circ$

ii) $\frac{4\pi^c}{5}$

R = D

$\pi^c = 180^\circ$

$1^c = \frac{180}{\pi}$

$\frac{4\pi^c}{5} \Rightarrow \frac{4\pi}{5} \times \frac{180^\circ}{\pi} = 144^\circ$

iii) $\frac{\pi^c}{9}$

R ⇒ D

$\pi^c = 180^\circ$

$1^c = \frac{180^\circ}{\pi}$

$\frac{\pi^c}{9} = \frac{\pi}{9} \times \frac{180^\circ}{\pi} = 20^\circ$

(iv) $\frac{3\pi^c}{2}$

R = D

$\pi^c = 180^\circ$

$1^c = \frac{180^\circ}{\pi}$

$\frac{3\pi^c}{2} \Rightarrow \frac{3\pi}{2} \times \frac{180}{\pi} = 270^\circ$

(v) $\frac{\pi^c}{2}$

R ⇒ D

$\pi^c = 180^\circ$

$1^c = \frac{180^\circ}{\pi}$

$\frac{\pi^c}{2} \Rightarrow \frac{\pi}{2} \times \frac{180^\circ}{\pi} = 90^\circ$

(vi) $\frac{2\pi^c}{3}$

R ⇒ D

$\pi^c = 180^\circ$

$\therefore 1^c = \frac{180^\circ}{\pi}$

$\frac{2\pi^c}{3} = \frac{2\pi}{3} \times \frac{180}{\pi} = 120^\circ$

Scanned with CamScanner

Page No.
Date

Ans 6. Convert the following radian measure into centesimal measure.

i) $\frac{\pi^c}{5}$

$R \Rightarrow G$

$\pi^c = 200$

$1^c = \frac{200}{\pi}$

$\frac{\pi^c}{5} \Rightarrow \frac{\pi}{5} \times \frac{40\ 200}{\pi} = 40g$

(ii) $\frac{3\pi^c}{5}$

$R \Rightarrow G$

$\pi^c = 200^g$

$1^c = \frac{200^g}{\pi}$

$\frac{3\pi^c}{5} = \frac{3\pi}{5} \times \frac{40\ 200^g}{\pi}$

$= 120g$

iii) $\frac{7\pi^c}{10}$

$R \Rightarrow G$

$\pi^c = 200^g$

$1^c = \frac{200^g}{\pi}$

$\frac{7\pi^c}{10} = \frac{7\pi}{10} \times \frac{200^g}{\pi} = 140g$

(iv) $\frac{\pi^c}{50}$

$R = G$

$\pi^c = 200^g$

$1^c = \frac{200^g}{\pi}$

$= \frac{\pi^c}{50} \Rightarrow \frac{\pi}{50} \times \frac{4\ 200^g}{\pi}$

$= 4g$

(v) $\frac{7\pi^c}{5}$

$R \Rightarrow G$

$\pi^c = 200^g$

$1^c = \frac{200^g}{\pi}$

$\frac{7\pi^c}{5} \Rightarrow \frac{7\pi}{5} \times \frac{40\ 200^g}{\pi} = 280g$

(vi) $\frac{5\pi^c}{2}$

$R = G$

$\pi^c = 200^g$

$1^c = \frac{200^g}{\pi}$

$\frac{5\pi^c}{2} = \frac{5\pi}{2} \times \frac{100\ 200^g}{\pi}$

$= 500g$

Scanned with CamScanner

Page No.
Date

Ques 7. Write the following angle in circular measure whose degree measure is

(i) 100g

$G \Rightarrow R$

$200 = \pi^c$

$1^g = \frac{\pi}{200}$

$100^g = 100 \times \frac{\pi}{200}$

$= \frac{\pi}{2}$

(ii) 300g

$G \Rightarrow R$

$200 = \pi^c$

$1^g = \frac{\pi}{200}$

$300^g = 300 \times \frac{\pi}{200} = \frac{3\pi}{2}$

(iii) 400g

$G \Rightarrow R$

$200 = \pi^c$

$1^g = \frac{\pi}{200}$

$400g = 400 \times \frac{\pi}{200}$

$= 2\pi$

(iv) 25g

$G = R$

$200 = \pi^c$

$1^g = \frac{\pi}{200}$

$25g = 25 \times \frac{\pi}{200} = \frac{\pi}{8}$

(v) 800g

$G \Rightarrow R$

$200 = \pi^c$

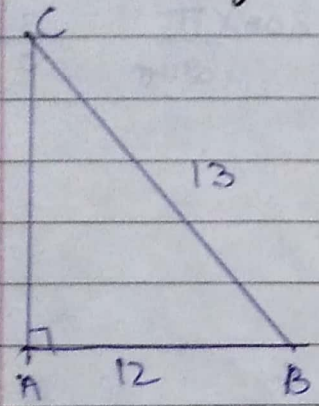
$1^g = \frac{\pi}{200}$

$800g = 800 \times \frac{\pi}{200} = 4\pi$

Scanned with CamScanner

$\sin\theta$	$\cos\theta$	$\tan\theta$
P	b	P
h	h	b
$\operatorname{cosec}\theta$	$\sec\theta$	$\cot\theta$

Qus 1. In a $\triangle ABC$, at angle at A = if AB = 12, AC = 5 and BC = 13 find the value of $\sin B$, $\cos B$, $\tan B$

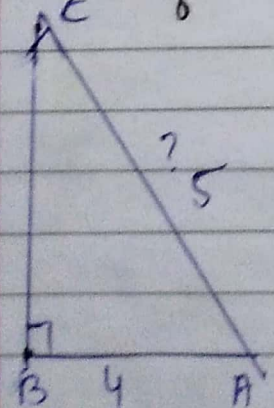


Base = 12
Perpendicular (P) = 5
hypotenus (H) = 13

$\sin B = \frac{P}{H} = \frac{5}{13}$ $\cos B = \frac{B}{H} = \frac{12}{13}$

$\tan\theta = \frac{P}{B} = \frac{5}{12}$

Qus 2. In $\triangle ABC$ at angle at B if AB = 4, BC = 3 find the value of $\sin A$ $\cos A$ and $\cot A$

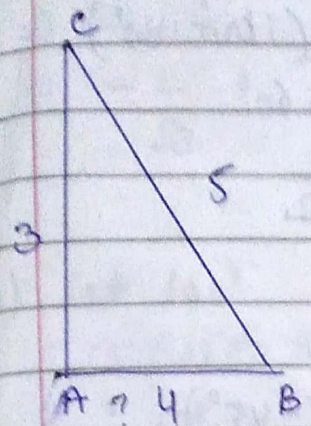


Base (B) = 4
Perpendicular (P) = 3
hyp (h) = ?
we know that
$AC^2 = AB^2 + BC^2$
$AC^2 = 4^2 + 3^2$
$AC^2 = 16 + 9$
$AC^2 = 25$
$AC = \sqrt{25}$
$AC = 5$

$\sin A = \frac{P}{H} = \frac{3}{5}$, $\cos A = \frac{B}{H} = \frac{4}{5}$, $\cot A = \frac{B}{P} = \frac{4}{3}$

Scanned with CamScanner

Ans 3. If $\sin A = \frac{3}{5}$, find the value of $\cos A$, $\tan A$, $\operatorname{cosec} A$ and $\sec A$



$P = 3$

$h = 5$

$B = 4$

We know that

$BC^2 = BA^2 + AC^2$

$BC^2 = 4^2 + 3^2$ $\quad 5^2 = BA^2 + 3^2$

$BC^2 = 16 + 9$ $\quad 25 = BA^2 + 9$

$BC^2 = 25$ $\quad BA^2 = 25 - 9$

$BA^2 = 16$

$BA = \sqrt{16}$

$BA = 4$

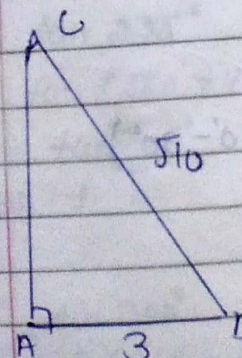
$\cos A = \frac{b}{H} = \frac{4}{5}$

$\tan A = \frac{P}{B} = \frac{3}{4}$

$\operatorname{cosec} A = \frac{H}{P} = \frac{5}{3}$

$\sec A = \frac{H}{B} = \frac{5}{4}$

Ans 4. If $\operatorname{cosec} A = \sqrt{10}$ find the value of $\sin A$, $\cos A$, $\cot A$, cosec.



$P = 1$

$h = \sqrt{10}$

$B = ?$

We know that

$BC^2 = AB^2 + AC^2$

$(\sqrt{10})^2 = AB^2 + 1^2$

$10 = AB^2 + 1$

$AB^2 = 10 - 1$

$AB^2 = \sqrt{9}$

$AB = 3$

$\sin A = \frac{P}{H} = \frac{1}{\sqrt{10}}$

$\cos A = \frac{b}{H} = \frac{3}{\sqrt{10}}$

$\cot A = \frac{b}{P} = \frac{3}{1}$

$\operatorname{cosec} A = \frac{h}{b} = \frac{\sqrt{10}}{1}$

Scanned with CamScanner

1 Evaluate the following

(i) $\sin 120$
$= \sin(180^\circ - 60^\circ)$
$\Rightarrow \sin 60^\circ$
$= \frac{\sqrt{3}}{2}$

(ii) $\cos 1200^\circ$
$= \cos(180^\circ - 60^\circ)$
$= -\cos 60^\circ$
$= -\frac{1}{2}$

(iii) $\tan 120^\circ$
$= \tan(180^\circ - 60^\circ)$
$= -\tan 60^\circ$
$= -\sqrt{3}$

(iv) $\sin 135^\circ$
$= \sin(180^\circ - 45^\circ)$
$= \sin 45^\circ$
$= \frac{1}{\sqrt{2}}$

(v) $\cos 135^\circ$
$= \cos(180^\circ - 45^\circ)$
$= -\cos 45^\circ$
$= -\frac{1}{\sqrt{2}}$

(vi) $\tan 135^\circ$
$= \tan(180^\circ - 45^\circ)$
$= -\tan 45^\circ$
$= -1$

(vii) $\sin 150^\circ$
$= \sin(180^\circ - 30^\circ)$
$\Rightarrow \sin 30^\circ$
$\Rightarrow \frac{1}{2}$

(viii) $\cos 150^\circ$
$= \cos(180^\circ - 30^\circ)$
$= -\cos 30^\circ$
$\Rightarrow -\frac{\sqrt{3}}{2}$

(ix) $\tan 150^\circ$
$= \tan(180^\circ - 30^\circ)$
$= -\tan 30^\circ$
$\Rightarrow -\frac{1}{\sqrt{3}}$

(x) $\operatorname{cosec} 150^\circ$
$= \operatorname{cosec}(180^\circ - 30^\circ)$
$= \operatorname{cosec} 30^\circ$
$\Rightarrow 2$

Scanned with CamScanner

xi) $\sec 150^\circ$
$= \sec(180^\circ - 30^\circ)$
$\Rightarrow -\sec 30^\circ$
$= -\frac{2}{\sqrt{3}}$

(xii) $\cot 150^\circ$
$= \cot(180^\circ - 30^\circ)$
$= -\cot 30^\circ$
$\Rightarrow -\sqrt{3}$

(xiii) $\sin 225^\circ$
$= \sin(180^\circ + 45^\circ)$
$= -\sin 45^\circ$
$= -\frac{1}{\sqrt{2}}$

(xiv) $\cos 225^\circ$
$= \cos(180^\circ + 45^\circ)$
$= -\cos 45^\circ$
$= -\frac{1}{\sqrt{2}}$

xv) $\tan 225^\circ$
$= \tan(180^\circ + 45^\circ)$
$\Rightarrow \tan 45^\circ$
$\Rightarrow 1$

xvi) $\sin 300^\circ$
$= \sin(360^\circ - 60^\circ)$
$= -\sin 60^\circ$
$= -\frac{\sqrt{3}}{2}$

xvii) $\cos 300^\circ$
$= \cos(360^\circ - 60^\circ)$
$= \cos 60^\circ$
$= \frac{1}{2}$

(xviii) $\tan 300^\circ$
$= \tan(360^\circ - 60^\circ)$
$= -\tan 60^\circ$
$= -\sqrt{3}$

(xix) $\sin 15^\circ$
$= \sin(45^\circ - 30^\circ)$
$= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$
$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$
$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$
$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

(xx) $\cos 15^\circ$
$= \cos(60^\circ - 45^\circ)$
$\Rightarrow \cos 60^\circ \cos 45^\circ + \sin 60^\circ \sin 45^\circ$
$= \frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}$
$= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{3}}$
$= \frac{1+\sqrt{3}}{2\sqrt{2}}$

Scanned with CamScanner

Page No.
Date

xxi) $\tan 15^\circ$

$= \tan(60^\circ - 45^\circ)$

$= \dfrac{\tan 60^\circ - \tan 45^\circ}{1 + \tan 60^\circ \cdot \tan 45^\circ}$

$\Rightarrow \dfrac{\sqrt{3} - 1}{1 + \sqrt{3} \times 1} = \dfrac{\sqrt{3} - 1}{1 + \sqrt{3}}$

xxii) $\sin 75^\circ$

$= \sin(45^\circ + 30^\circ)$

$\Rightarrow \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$

$\dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{3}}{2} + \dfrac{1}{\sqrt{2}} \times \dfrac{1}{2}$

$= \dfrac{\sqrt{3}}{2\sqrt{2}} + \dfrac{1}{2\sqrt{2}}$

$= \dfrac{\sqrt{3}+1}{2\sqrt{2}}$

xxiii) $\tan 75^\circ$

$= \tan(45^\circ + 30^\circ)$

$= \dfrac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \cdot \tan 30^\circ} = \dfrac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \times \frac{1}{\sqrt{3}}}$

$= \dfrac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}}$ Ans

Scanned with CamScanner

xxiv) $\cos 75^\circ$

$= \cos(45^\circ + 30^\circ)$

$= \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} \Rightarrow \frac{\sqrt{3}-1}{2\sqrt{2}}$ Ans

xxv) $\sin 105^\circ$

$\sin(60^\circ + 45^\circ)$

$\Rightarrow \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ$

$\Rightarrow \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} \Rightarrow \frac{\sqrt{3}+1}{2\sqrt{2}}$ Ans

xxvi) $\cos 105^\circ$

$\Rightarrow \cos(60^\circ + 45^\circ)$

$\Rightarrow \cos 60^\circ \cos 45^\circ - \sin 60^\circ \cdot \sin 45^\circ$

$\Rightarrow \frac{1}{2} \times \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}} \Rightarrow \frac{1-\sqrt{3}}{2\sqrt{2}}$ Ans

xxvii) $\tan 105^\circ$

$\Rightarrow \tan(60^\circ + 45^\circ)$

$\Rightarrow \frac{\tan 60^\circ + \tan 45^\circ}{1 - \tan 60^\circ \cdot \tan 45^\circ}$

$\Rightarrow \frac{\sqrt{3}+1}{1-\sqrt{3}\times 1} \Rightarrow \frac{\sqrt{3}+1}{1-\sqrt{3}}$ Ans

Scanned with CamScanner

xxviii) $\sin(-690^\circ)$

$= \sin(360^\circ + 360^\circ - 30^\circ)$

$= -\sin(-30^\circ)$

$= \sin 30^\circ$

$= \frac{1}{2}$

xxix) $\cos(-690^\circ)$

$= \cos(360^\circ + 360^\circ - 30^\circ)$

$= \cos(-30^\circ)$

$= \cos 30^\circ$

$= \frac{\sqrt{3}}{2}$

xxx) $\tan(-690^\circ)$

$= \tan(360^\circ + 360^\circ - 30^\circ)$

$= -\tan(-30^\circ)$

$= \tan 30^\circ$

$\Rightarrow \frac{1}{\sqrt{3}}$

xxxi) $\sin(-750^\circ)$

$= \sin(360^\circ + 360^\circ + 30^\circ)$

$= \sin 30^\circ$

$\Rightarrow \frac{1}{2}$

xxxii) $\cos(-270^\circ)$

$= \cos\theta$

$= \cos(360^\circ - 90^\circ)$

$= \cos(-90^\circ)$

$= 0$

xxxiii) $\tan(765^\circ)$

Scanned with CamScanner

Ans 1 Evaluate the following

(i) $\frac{\cos 37^\circ}{\sin 53^\circ}$

$= \frac{\cos(90^\circ - 53^\circ)}{\sin 53^\circ}$

$= \frac{\sin 53^\circ}{\sin 53^\circ} = 1$

(ii) $\sin 39^\circ - \cos 51^\circ$

$= \sin(90^\circ - 51^\circ) - \cos 51^\circ$

$= +\cos 51^\circ - \cos 51^\circ$

$= 0$

(iii) $\frac{\sin 41^\circ}{\cos 49^\circ}$

$= \frac{\sin 41^\circ}{\cos(90^\circ - 41^\circ)}$

$= \frac{\sin 41^\circ}{\sin 41^\circ} = 1$

(iv) $\frac{\tan 54^\circ}{\cot 36^\circ}$

$= \frac{\tan(90^\circ - 36^\circ)}{\cot 36^\circ}$

$\Rightarrow \frac{\cot 36^\circ}{\cot 36^\circ} = 1$

(v) $\frac{\operatorname{cosec} 30^\circ}{\sec 58^\circ}$

$= \frac{\operatorname{cosec}(90^\circ - 58^\circ)}{\sec 58^\circ}$

$= \frac{\sec 58^\circ}{\sec 58^\circ}$

(vi) $\operatorname{cosec} 25^\circ - \sec 65^\circ$

$= \operatorname{cosec}(90^\circ - 65^\circ) - \sec 65^\circ$

$= +\sec 65^\circ - \sec 65^\circ$

$= 0$

(vii) $\cot 34^\circ - \tan 56^\circ$

$= \cot(90^\circ - 56^\circ) - \tan 56^\circ$

$= +\tan - \tan 56^\circ$

$= 0$

(viii) $\frac{\sin 36^\circ}{\cos 54^\circ} - \frac{\sin 54^\circ}{\cos 36^\circ}$

$= \frac{\sin(90^\circ - 54^\circ)}{\cos 54^\circ} - \frac{\sin(90^\circ - 36^\circ)}{\cos 36^\circ}$

$= \frac{\cos 54^\circ}{\cos 54^\circ} - \frac{\cos 36^\circ}{\cos 36^\circ} \Rightarrow 1 - 1 = 0$

Scanned with CamScanner

Ques 8 Evaluate the following

(i) $\cos 68^\circ \cos 8^\circ + \sin 68^\circ \sin 8^\circ$

$\cos(68^\circ - 8^\circ)$ $\because \cos(A-B) = \cos A \cos B + \sin A \sin B$

$= \cos 60^\circ$

$= \frac{1}{2}$

(ii) $\cos 50^\circ \cdot \cos 10^\circ - \sin 50^\circ \cdot \sin 10^\circ$

$= \cos(50^\circ + 10^\circ)$ $(\because) \cos(A+B) = \cos A \cos B - \sin A \sin B$

$= \cos 60^\circ$

$= \frac{1}{2}$

(iii) $\sin 22^\circ \cdot \cos 38^\circ + \cos 22^\circ \cdot \sin 38^\circ$

$= \sin(22^\circ + 38^\circ)$ $\because \sin(A+B) = \sin A \cos B + \cos A \sin B$

$\Rightarrow \sin 60^\circ$

$= \frac{\sqrt{3}}{2}$

(iv) $\sin 80^\circ \cdot \cos 20^\circ - \cos 80^\circ \cdot \sin 20$

$= \sin(80^\circ - 20^\circ)$ $\because \sin(A-B) = \sin A \cos B - \cos A \sin B$

$= \sin 60^\circ$

$= \frac{\sqrt{3}}{2}$

(v) $\frac{\tan 66^\circ + \tan 69^\circ}{1 - \tan 66^\circ \cdot \tan 69^\circ}$

$= \tan(66^\circ + 69^\circ)$ $\because \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

$= \tan(135^\circ)$

$= \tan(180^\circ - 45^\circ)$

$= -\tan 45^\circ$

$= -1$

Scanned with CamScanner

vii) $\frac{\tan 70^\circ - \tan 25^\circ}{1 + \tan 70^\circ \cdot \tan 25^\circ}$

$= \tan(70^\circ - 25^\circ)$ $\because \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

$= \tan 45^\circ$

$= 1$

Ans 3 Prove that

i) $\frac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ} = \tan 56^\circ$

$\frac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ} \Rightarrow \frac{\frac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ}}{\frac{\cos 11^\circ - \sin 11^\circ}{\cos 11^\circ}}$

$\Rightarrow \frac{\frac{\cos 11^\circ}{\cos 11^\circ} + \frac{\sin 11^\circ}{\cos 11^\circ}}{\frac{\cos 11^\circ}{\cos 11^\circ} - \frac{\sin 11^\circ}{\cos 11^\circ}}$

$\Rightarrow \frac{1 + \tan 11^\circ}{1 - \tan 11^\circ}$

$= \tan(45^\circ + 11^\circ)$ $\because \tan(45^\circ + \theta) = \frac{1 + \tan\theta}{1 - \tan\theta}$

$= \tan(56^\circ)$

ii) $\tan 88^\circ = \frac{\cos 17^\circ - \sin 17^\circ}{\cos 17^\circ + \sin 17^\circ}$

$= \frac{\cos 17^\circ - \sin 17^\circ}{\cos 17^\circ + \sin 17^\circ} \Rightarrow \frac{\frac{\cos 17^\circ - \sin 17^\circ}{\cos 17^\circ}}{\frac{\cos 17^\circ + \sin 17^\circ}{\cos 17^\circ}}$

$$= \frac{\cos 17^\circ}{\cos 17^\circ} - \frac{\sin 17^\circ}{\cos 17^\circ}$$

$$\Rightarrow \frac{\frac{\cos 17^\circ}{\cos 17^\circ} - \frac{\sin 17^\circ}{\cos 17^\circ}}{\frac{\cos 17^\circ}{\cos 17^\circ} + \frac{\sin 17^\circ}{\cos 17^\circ}}$$

$$\Rightarrow \frac{1 - \tan 17^\circ}{1 + \tan 17^\circ}$$

$$\Rightarrow \tan(45^\circ - 17^\circ)$$

$$\Rightarrow \tan 28^\circ \quad \because \tan(45^\circ - \theta) = \frac{1 - \tan\theta}{1 + \tan\theta}$$

(iii) $\tan 58^\circ = \dfrac{\cos 13^\circ + \sin 13^\circ}{\cos 13^\circ - \sin 13^\circ}$

$$\frac{\cos 13^\circ + \sin 13^\circ}{\cos 13^\circ - \sin 13^\circ} \Rightarrow \frac{\frac{\cos 13^\circ + \sin 13^\circ}{\cos 13^\circ}}{\frac{\cos 13^\circ - \sin 13^\circ}{\cos 13^\circ}}$$

$$\Rightarrow \frac{\frac{\cos 13^\circ}{\cos 13^\circ} + \frac{\sin 13^\circ}{\cos 13^\circ}}{\frac{\cos 13^\circ}{\cos 13^\circ} - \frac{\sin 13^\circ}{\cos 13^\circ}}$$

$$\Rightarrow \frac{1 + \tan 13^\circ}{1 - \tan 13^\circ}$$

$\Rightarrow \tan(45^\circ + 13)$ $(\because) \tan(45^\circ + \theta) = \dfrac{1 + \tan\theta}{1 - \tan\theta}$

$= \tan 58^\circ$

Scanned with CamScanner

iv) $\frac{\cos 14^\circ - \sin 14^\circ}{\cos 14^\circ + \sin 14^\circ} = \tan 31^\circ$

$\frac{\cos 14^\circ - \sin 14^\circ}{\cos 14^\circ + \sin 14^\circ} \Rightarrow \frac{\frac{\cos 14^\circ - \sin 14^\circ}{\cos 14^\circ}}{\frac{\cos 14^\circ + \sin 14^\circ}{\cos 14^\circ}}$

$\Rightarrow \frac{\frac{\cos 14^\circ}{\cos 14^\circ} - \frac{\sin 14^\circ}{\cos 14^\circ}}{\frac{\cos 14^\circ}{\cos 14^\circ} + \frac{\sin 14^\circ}{\cos 14^\circ}}$

$\Rightarrow \frac{1 - \tan 14^\circ}{1 + \tan 14^\circ}$

$= \tan(45^\circ - 14^\circ)$ $\left[\because \tan(45^\circ - \theta) = \frac{1 - \tan\theta}{1 + \tan\theta}\right]$

$= \tan 31^\circ$

Ques 4. Prove that

(i) $\tan 3A \cdot \tan 2A \cdot \tan A = \tan 3A - \tan 2A - \tan A$

$\tan(3A) = \tan(2A + A)$

$\frac{\tan 3A}{1} = \frac{\tan 2A + \tan A}{1 - \tan 2A \cdot \tan A}$

$= \tan 3A - \tan 3A \cdot \tan 2A \cdot \tan A = \tan 2A + \tan A$

$= \tan 3A - \tan 2A - \tan A = \tan 3A \cdot \tan 2A \cdot \tan A$

Hence proved

ii) $\tan 13A \cdot \tan 9A \cdot \tan 4A = \tan 13A - \tan 9A - \tan 4A$

$\tan(13A) = \tan(9A + 4A)$

$= \frac{\tan 13A}{1} = \frac{\tan 9A + \tan 4A}{1 - \tan 9A \cdot \tan 4A}$

$= \tan 13A - \tan 13A \cdot \tan 9A \cdot \tan 4A = \tan 9A + \tan 4A$

$= \tan 13A - \tan 9A - \tan 4A = \tan 13A \cdot \tan 9A \cdot \tan 4A$

Hence proved

iii) $\tan 22A \cdot \tan 12A \cdot \tan 10A = \tan 22A - \tan 12A - \tan 10A$

$\tan 22A = \tan(12A + 10A)$

$= \frac{\tan 22A}{1} = \frac{\tan 12A + \tan 10A}{1 - \tan 12A \cdot \tan 10A}$

$\Rightarrow \tan 22A - \tan 22A \cdot \tan 12A \cdot \tan 10A = \tan 12A + \tan 10A$

$\Rightarrow \tan 22A - \tan 12A - \tan 10A = \tan 22A \cdot \tan 12A \cdot \tan 10A$

Hence proved

(iv) $\tan 15\theta - \tan 9\theta - \tan 6\theta = \tan 15\theta \cdot \tan 9\theta \cdot \tan 6\theta$

$\tan 15\theta = \tan(9\theta + 6\theta)$

$= \frac{\tan 15\theta}{1} = \frac{\tan 9\theta + \tan 6\theta}{1 - \tan 9\theta \cdot \tan 6\theta}$

$= \tan 15\theta - \tan 15\theta \cdot \tan 9\theta \cdot \tan 6\theta = \tan 9\theta + \tan 6\theta$

$= \tan 15\theta - \tan 9\theta - \tan 6\theta = \tan 15\theta \cdot \tan 9\theta \cdot \tan 6\theta$

Hence proved

Ques 5. If $\tan A = \sqrt{3}$ and $\tan B = 2 - \sqrt{3}$ form them $(A - B$

Ques 5. If A and B are acutes angles and $\sin A = \frac{1}{\sqrt{10}}$, $\sin B = \frac{1}{\sqrt{5}}$ proved that $A + B = \frac{\pi}{4}$

Scanned with CamScanner

$\cos^2\theta = 1 - \sin^2\theta$

$\cos\theta = \pm\sqrt{1 - \sin^2\theta}$

$\cos A = \sqrt{1 - \sin^2 A}$

$\cos B = \sqrt{1 - \sin^2 B}$

Now

$\cos A = \sqrt{1 - \sin^2 A}$

$= \sqrt{1 - \left(\frac{1}{\sqrt{10}}\right)^2}$

$= \sqrt{\frac{1}{1} - \frac{1}{10}}$

$= \sqrt{\frac{10-1}{10}} = \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}}$

$\cos A = \frac{3}{\sqrt{10}}$

$\cos B = \sqrt{1 - \left(\frac{1}{5}\right)^2}$

$= \sqrt{1 - \frac{1}{5}}$

$= \sqrt{\frac{5-1}{5}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$

$\cos B = \frac{2}{\sqrt{5}}$

$A + B = \frac{\pi}{4}$

Now $\sin(A+B)$

$= \sin A \cos B + \cos A \sin B$

$= \frac{1}{\sqrt{10}} \times \frac{2}{\sqrt{5}} + \frac{3}{\sqrt{10}} \times \frac{1}{\sqrt{5}}$

Scanned with CamScanner

$$= \frac{2}{\sqrt{50}} + \frac{2}{\sqrt{50}}$$

$$= \frac{2+3}{\sqrt{50}} = \frac{5}{\sqrt{50}}$$

$$\Rightarrow \sin(A+B) = \frac{5}{\sqrt{25}\times 2} = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\sin(A+B) = \frac{1}{\sqrt{2}}$$

$$\sin(A+B) = \sin 45^\circ$$

$$A+B = 45^\circ = \frac{\pi}{4}$$

Hence proved

Q.no 6. Proved that $\tan 2\theta - \tan\theta = \tan\theta \sec 2\theta$

L.H.S $\tan 2\theta - \tan\theta$

$$= \frac{\sin 2\theta}{\cos 2\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\sin 2\theta \cdot \cos\theta - \sin\theta \cdot \cos 2\theta}{\cos 2\theta \cdot \cos\theta}$$

$$= \frac{\sin(2\theta - \theta)}{\cos 2\theta - \cos\theta}$$

$$= \frac{\sin\theta}{\cos 2\theta \cdot \cos\theta}$$

$$= \tan\theta \times \sec 2\theta$$

$$= R.H.S$$

Scanned with CamScanner

Ques 7. Proved that $\sin 105^\circ + \cos 105^\circ = \cos 45^\circ$

$\sin 105^\circ = \sin(45^\circ + 60^\circ)$

$= \sin 45^\circ \cos 60^\circ + \cos 45^\circ \cdot \sin 60^\circ$

$= \frac{1}{\sqrt{2}} \times \frac{1}{2} + \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}$

$= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$

$= \frac{1+\sqrt{3}}{2\sqrt{2}}$

$\cos 105^\circ = \cos(45^\circ + 60^\circ)$

$= \cos 45^\circ \cdot \cos 60^\circ - \sin 45^\circ \cdot \sin 60^\circ$

$= \frac{1}{\sqrt{2}} \times \frac{1}{2} - \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}$

$= \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}$

$= \frac{1-\sqrt{3}}{2\sqrt{2}}$

Now $= \sin 105^\circ + \cos 105^\circ$

$= \frac{1+\sqrt{3}}{2\sqrt{2}} + \frac{1-\sqrt{3}}{2\sqrt{2}}$

$= \frac{1+\sqrt{3}+1-\sqrt{3}}{2\sqrt{2}}$

$= \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} = \cos 45^\circ$

Scanned with CamScanner

Other process:-

$\sin 105^\circ + \cos 105^\circ$

$\sin 105^\circ + \cos(90^\circ + 15^\circ)$

$\sin 105^\circ - \sin 15^\circ$

$2\cos\left(\frac{105+15}{2}\right)\sin\left(\frac{105-15}{2}\right)$

$= 2\cos\left(\frac{120}{2}\ 60\right)\sin\left(\frac{90}{2}\ 45\right)$

$= 2\cos 60 \sin 45^\circ$

$= 2 \times \frac{1}{2} \times \frac{1}{\sqrt{2}}$

$= \frac{1}{\sqrt{2}} \cos 45^\circ$

Ques 8. Prove that the following

(i) $\tan 70^\circ = \tan 20^\circ + 2\tan 50^\circ$

$\tan 70^\circ = \tan(20^\circ + 50^\circ)$

$\frac{\tan 70^\circ}{1} = \frac{\tan 20^\circ + \tan 50^\circ}{1 - \tan 20^\circ \cdot \tan 50^\circ}$

$= \tan 70^\circ - \tan 70^\circ \tan 20^\circ \tan 50^\circ = \tan 20^\circ + \tan 50^\circ$

$= \tan 70^\circ = \tan 20^\circ + \tan 50^\circ + \tan 70^\circ \tan 20^\circ + \tan 50^\circ$

$= \tan 70^\circ = \tan 20^\circ + \tan 50^\circ + \tan(90^\circ - 20^\circ) \cdot \tan 20^\circ + \tan 50^\circ$

$= \tan 70^\circ = \tan 20^\circ + \tan 50^\circ + \cot 20^\circ \tan 20^\circ \tan 50^\circ$

$= \tan 70^\circ = \tan 20^\circ + \tan 50^\circ + \frac{1}{\tan 20^\circ} \times \tan 20^\circ \tan 50^\circ$

$= \tan 70^\circ = \tan 20^\circ + \tan 50^\circ + \tan 50^\circ$

$= \tan 70^\circ = \tan 20^\circ + 2\tan 50^\circ$

Scanned with CamScanner

ii) $\tan 65^\circ = \tan 25^\circ + 2\tan 40^\circ$

$\tan 65^\circ = \tan(25^\circ + 40^\circ)$

$\frac{\tan 65^\circ}{1} = \frac{\tan 25^\circ + \tan 40^\circ}{1 - \tan 25^\circ \cdot \tan 40^\circ}$

$= \tan 65^\circ - \tan 65^\circ \tan 25^\circ \cdot \tan 40^\circ = \tan 25^\circ + \tan 40^\circ$

$\Rightarrow \tan 65^\circ = \tan 25^\circ + \tan 40^\circ \tan 65^\circ + \tan 25^\circ + \tan 40^\circ \cdot \tan 25^\circ \tan 40^\circ$

$\Rightarrow \tan 65^\circ = \tan 25^\circ + \tan 40^\circ + \tan(90^\circ - 25^\circ) \tan 25^\circ \tan 40^\circ$

$\Rightarrow \tan 65^\circ = \tan 25^\circ + \tan 40^\circ + \cot 25^\circ \cdot \tan 25^\circ \tan 40^\circ$

$= \tan 65^\circ = \tan 25^\circ + \tan 40^\circ + \frac{1}{\tan 25^\circ} \times \tan 25^\circ \tan 40^\circ$

$\tan 65^\circ = \tan 25^\circ + \tan 40^\circ + \tan 40^\circ$

$\tan 65^\circ = \tan 25^\circ + 2\tan 40^\circ$

iii) $\tan 55^\circ = \tan 35^\circ + 2\tan 20^\circ$

$\tan 55^\circ = \tan(35^\circ + 20^\circ)$

$\frac{\tan 55^\circ}{1} = \frac{\tan 35^\circ + \tan 20^\circ}{1 - \tan 35^\circ \cdot \tan 20^\circ}$

$\Rightarrow \tan 55^\circ - \tan 55^\circ \tan 35^\circ \cdot \tan 20^\circ = \tan 35^\circ + \tan 20^\circ$

$\Rightarrow \tan 55^\circ = \tan 35^\circ + \tan 20^\circ \cdot \tan 55^\circ \tan 35^\circ \cdot \tan 20^\circ$

$= \tan 55^\circ = \tan 35^\circ + \tan 20^\circ + \tan(90^\circ - 35^\circ) \tan 35^\circ \tan 20^\circ$

$= \tan 55^\circ = \tan 35^\circ + \tan 20^\circ + \cot 35^\circ \cdot \tan 35^\circ \cdot \tan 20^\circ$

$= \tan 55^\circ = \tan 35^\circ + \tan 20^\circ + \frac{1}{\tan 35^\circ} \times \tan 35^\circ \cdot \tan 20^\circ$

$\tan 55^\circ = \tan 35^\circ + \tan 20^\circ + \tan 20^\circ$

$\tan 55^\circ = \tan 35^\circ + 2\tan 20^\circ$

Ques 9. Express the following as a sum

i) $2 \sin 5x \cos 3x$

$\sin(5x+3x) + \sin(5x-3x)$

$= \sin 8x + \sin 2x$

ii) $2 \sin 4x \sin 3x$

~~$\sin(5x+3x) + \sin(5x$~~

$\cos(4x-3x) - \cos(4x+3x)$

$\cos x - \cos 7x$

iii) $8 \cos 8x \cos 4x$

$4 \times 2 \cos(8x-4x) + \cos(8x+4x)$

$4[\cos 4x + \cos 12x]$

$4\cos 4x + 4\cos 12x$

iv) $2 \sin 4\theta \cos 2\theta$

$\sin(4\theta+2\theta) + \sin(4\theta-2\theta)$

$= \sin 6\theta + \sin 2\theta$

(v) $2 \sin\theta \cos 3\theta$

$\sin(\theta+3\theta) + \sin(\theta-3\theta)$

$\sin 4\theta + \sin 2\theta$

(vi) $\cos 6\theta \cos 4\theta$

$\frac{1}{2} \times 2(\cos(6\theta-4\theta) + \cos(6\theta+4\theta))$

$= \frac{1}{2}\cos 2\theta + \cos 10$

Ans 10 Express the following as the product of T-ratios :-

i) $\sin 14x + \sin 2x$
$\quad C \quad D$
$= 2\sin\left(\frac{14x+2x}{2}\right)\cos\left(\frac{14x-2x}{2}\right)$

$\left(\because\right) \sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

$= 2\sin 8x \cos 6x$

ii) $\cos 10^\circ - \cos 50^\circ$
$\quad C \quad D$
$= 2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{D-C}{2}\right)$

$= 2\sin\left(\frac{10+50}{2}\right)\sin\left(\frac{50-10}{2}\right)$

$= 2\sin 30 \sin 20$

iii) $\sin 80^\circ - \sin 20^\circ$
$= 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$2\cos\left(\frac{50+20}{2}\right)\sin\left(\frac{20-50}{2}\right)$

$2\cos 35^\circ \sin(-15)$
$2\cos 35^\circ \sin 15^\circ$

iv) $\sin 7\theta + \sin 3\theta$
$= 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

$2\sin\left(\frac{7\theta+3\theta}{2}\right)\cos\left(\frac{7\theta-3\theta}{2}\right)$

$2\sin\left(\frac{10\theta}{2}\right)\cos\left(\frac{4\theta}{2}\right)$

$2\sin 5\theta \cos 2\theta$

Scanned with CamScanner

(v) $\cos 5\theta + \cos 3\theta$

$2\cos\left(\frac{5\theta+3\theta}{2}\right)\cos\left(\frac{3\theta-5\theta}{2}\right)$

$2\cos\left(\frac{8\theta}{2}\right)\cos\left(\frac{-2\theta}{2}\right)$

$2\cos 4\theta \cos(-\theta)$

$2\cos 4\theta \cos\theta$

(vi) $\sin 5\theta - \sin\theta$

$2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$2\cos\left(\frac{5\theta+\theta}{2}\right)\sin\left(\frac{5\theta-\theta}{2}\right)$

$2\cos\left(\frac{6\theta}{2}\right)\sin\left(\frac{4\theta}{2}\right)$

$2\cos 3\theta \sin 2\theta$

(vii) $\cos 2\theta - \cos 4\theta$

$-2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{D-C}{2}\right)$

$2\sin\left(\frac{2\theta+4\theta}{2}\right)\sin\left(\frac{4\theta-2\theta}{2}\right)$

$2\sin\left(\frac{6\theta}{2}\right)\sin\left(\frac{2\theta}{2}\right)$

$2\sin 3\theta \sin\theta$

Ques 11. Prove that the following

i) $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{1}{16}$

L.H.S

$\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ$

$\cos 80^\circ \cos 60^\circ \cos 40^\circ \cos 20^\circ$

$= \cos 80^\circ \times \frac{1}{2} \times \cos 40^\circ \cos 20^\circ$

Scanned with CamScanner

$= \frac{1}{2} \cos 80^\circ \cos 40^\circ \cos 20^\circ$

$= \frac{1}{2} \cos 80^\circ \times \frac{1}{2} \times (2 \cos 40^\circ \cdot \cos 20^\circ$

$\frac{1}{2} \times \frac{1}{2} \times \cos 80^\circ \times [\cos(40^\circ - 20^\circ) + \cos(40^\circ + 20^\circ)]$

$\frac{1}{4} \times \cos 80^\circ \times [\cos 20^\circ + \cos 60^\circ]$

$= \frac{1}{4} \cos 80^\circ \times [\cos 20^\circ + \frac{1}{2}]$

$= \frac{1}{4} [\cos 80^\circ \cos 20^\circ + \frac{1}{2} \cos 80^\circ]$

$\frac{1}{4} [\frac{1}{2} \times (2 \cos 80^\circ \cos 20^\circ) + \frac{1}{2} \cos 80^\circ]$

$= \frac{1}{4} \times \frac{1}{2} (2 \cos 80^\circ \cos 20^\circ + \cos 80^\circ]$

$= \frac{1}{8} (\cos 60^\circ (80^\circ - 20^\circ) + \cos(80^\circ + 20^\circ) + \cos 80^\circ]$

$= \frac{1}{8} (\cos 60^\circ + \cos 100^\circ + \cos 80^\circ]$

$= \frac{1}{8} [\frac{1}{2} + \cos(180^\circ - 80^\circ) + \cos 80^\circ)$

$\frac{1}{8} [\frac{1}{2} + -\cos 80^\circ + \cos 80^\circ]$

$= \frac{1}{8} \times \frac{1}{2} = \frac{1}{16}$ R.H.S

(iii) $\sin 10^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{8}$

$\sin 10^\circ \sin 50^\circ \sin 70^\circ$

$\sin 70^\circ \sin 50^\circ \sin 10^\circ$

$\sin 70^\circ \times \frac{1}{2} \times (2 \sin 50^\circ \sin 10^\circ)$

$\because 2 \sin A \sin B = \cos(A-B) - \cos(A+B)$

Scanned with CamScanner

$= \frac{1}{2} \sin 70^\circ \times [\cos(50^\circ - 10^\circ) - \cos(50^\circ + 10^\circ)]$

$= \frac{1}{2} \sin 70^\circ \times [\cos 40^\circ - \cos 60^\circ]$

$= \frac{1}{2} \sin 70^\circ \times [\cos 40^\circ - \frac{1}{2}]$

$= \frac{1}{2} [\sin 70^\circ \cdot \cos 40^\circ - \frac{1}{2} \sin 70^\circ]$

$\frac{1}{2} [\frac{1}{2} \times 2 \sin 70^\circ \cos 40^\circ - \frac{1}{2} \sin 70^\circ]$

$= \frac{1}{2} \times \frac{1}{2} [2 \sin 70^\circ \cos 40^\circ - \sin 70^\circ]$

$\because 2 \sin A \cos B = \sin(A+B) + \sin(A-B)$

$\frac{1}{4} [\sin(70^\circ + 40^\circ) + \sin(70^\circ - 40^\circ) - \sin 70^\circ]$

$\frac{1}{4} [\sin 110^\circ + \sin 30^\circ - \sin 70^\circ]$

$\frac{1}{4} [\sin(180^\circ - 70^\circ) + \frac{1}{2} - \sin 70^\circ]$

$= \frac{1}{4} [\cancel{\sin 70^\circ} + \frac{1}{2} - \cancel{\sin 70^\circ}]$

$= \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$ R.H.S

(iii) $\cos 10^\circ \cos 50^\circ \cos 70^\circ = \frac{\sqrt{3}}{8}$

$\cos 10^\circ \cos 50^\circ \cos 70^\circ$

$\cos 70^\circ \cos 50^\circ \cos 10^\circ$

$= \cos 70^\circ \times \frac{1}{2} (2 \cos 50^\circ \cos 10^\circ)$ $\because 2 \cos A \cos B = \cos(A-B) + \cos(A+B)$

$= \frac{1}{2} \cos 70^\circ \times [\cos(50^\circ - 10^\circ) + \cos(50^\circ + 10^\circ)]$

$= \frac{1}{2} \cos 70^\circ \times [\cos 40^\circ + \cos 60^\circ]$

Scanned with CamScanner

$\frac{1}{2}\cos 70^\circ \times \left[\cos 40^\circ + \frac{1}{2}\right]$

$= \frac{1}{2}\left(\cos 70^\circ \cdot \cos 40^\circ + \frac{1}{2}\cos 70^\circ\right]$

$= \frac{1}{2}\left[\frac{1}{2} \times 2\cos 70^\circ \cos 40^\circ - \frac{1}{2}\cos 70^\circ\right]$

$= \frac{1}{2} \times \frac{1}{2}\left[2\cos 70^\circ \cos 40^\circ - \cos 70^\circ\right]$

$= \frac{1}{4}\left[\cos(70^\circ - 40^\circ) + \cos(70^\circ + 40^\circ) - \cos 70^\circ\right]$ $\because 2\cos A \cos B = \cos(A-B) + \cos(A+B)$

$= \frac{1}{4}\left[\cos 30^\circ + \cos 110^\circ - \cos 70^\circ\right]$

$= \frac{1}{4}\left[\frac{\sqrt{3}}{2} + \cos(180^\circ - 110^\circ) - \cos 70^\circ\right]$

$= \frac{1}{4}\left[\frac{\sqrt{3}}{2} + \cancel{\cos 70^\circ} - \cancel{\cos 70^\circ}\right]$

$= \frac{1}{4} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{8}$ R.H.S

iv) $\sin 10^\circ \sin 50^\circ \sin 60^\circ \sin 70^\circ = \frac{\sqrt{3}}{16}$

L.H.S

$\sin 10^\circ \sin 50^\circ \sin 60^\circ \sin 70^\circ$

$\sin 70^\circ \sin 60^\circ \sin 50^\circ \sin 10^\circ$

$\sin 70^\circ \times \frac{\sqrt{3}}{2} \times \sin 50^\circ \sin 10^\circ$

$= \frac{\sqrt{3}}{2}\sin 70^\circ \times \frac{1}{2} \times (2\sin 50^\circ \sin 10^\circ)$ $\because 2\sin A \sin B = \cos(A-B) - \cos(A+B)$

$= \frac{\sqrt{3}}{2} \times \frac{1}{2} \times \sin 70^\circ \times \left[\cos(50^\circ - 10^\circ) - \cos(50^\circ + 10^\circ)\right]$

$= \frac{\sqrt{3}}{4} \times \sin 70^\circ \times \left[\cos 40^\circ - \cos 60^\circ\right]$

$= \frac{\sqrt{3}}{4} \times \sin 70^\circ \left[\cos 40^\circ - \frac{1}{2}\right]$

Scanned with CamScanner

$\frac{\sqrt{3}}{4}\left[\sin 70^\circ \cos 40^\circ - \frac{1}{2}\sin 70^\circ\right]$

$\frac{\sqrt{3}}{4}\left[\frac{1}{2}\times 2\sin 70^\circ \cos 40^\circ - \frac{1}{2}\sin 70^\circ\right]$

$\frac{\sqrt{3}}{4}\times\frac{1}{2}\left[2\sin 70^\circ \cos 40^\circ - \sin 70^\circ\right]$ $\because 2\sin A\cos B = \sin(A+B) + \sin(A-B)$

$= \frac{\sqrt{3}}{8}\left[\sin(70^\circ+40^\circ) + \sin(70^\circ-40^\circ) - \sin 70^\circ\right]$

$\frac{\sqrt{3}}{8}\left[\sin 110^\circ + \sin 30^\circ - \sin 70^\circ\right]$

$\frac{\sqrt{3}}{8}\left[\sin 110^\circ + \frac{1}{2} - \sin 70^\circ\right]$

$\frac{\sqrt{3}}{8}\left[\sin(180^\circ - 70^\circ) + \frac{1}{2} - \sin 70^\circ\right]$

$= \frac{\sqrt{3}}{8}\left[\sin 70^\circ + \frac{1}{2} - \sin 70^\circ\right]$

$= \frac{\sqrt{3}}{8}\times\frac{1}{2} = \frac{\sqrt{3}}{16}$ R.H.S

Qus 12. Prove that the following

(i) $\frac{\sin A + \sin B}{\cos A + \cos B} = \tan\left(\frac{A+B}{2}\right)$

L.H.S

$\frac{\sin A + \sin B}{\cos A + \cos B} = \frac{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{B-A}{2}\right)}$

$= \frac{\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}$

$= \tan\left(\frac{A+B}{2}\right)$ R.H.S

Scanned with CamScanner

ii) $\frac{\cos 8x - \cos 6x}{\sin 17x - \sin 3x} = \frac{-\sin x}{\cos 10x}$

L.H.S

$$\frac{\cos 8x - \cos 6x}{\sin 17x - \sin 3x} = \frac{\sin\left(\frac{8x+6x}{2}\right)\sin\left(\frac{6x-8x}{2}\right)}{\cos\left(\frac{17x+3x}{2}\right)\sin\left(\frac{17x-3x}{2}\right)}$$

$$= \frac{\sin\left(\frac{14x}{2}\right)\sin\left(\frac{-2x}{2}\right)}{\cos\left(\frac{20x}{2}\right)\sin\left(\frac{14x}{2}\right)}$$

$$= \frac{\sin 7x \sin(-x)}{\cos 10x \sin 7x}$$

$$= \frac{-\sin x}{\cos 10x} \quad R.H.S$$

iii) $\frac{\sin 7A + \sin 3A}{\sin 7A + \cos 3A} = \tan 5A$

L.H.S

$$\frac{\sin 7A + \sin 3A}{\sin 7A + \cos 3A} = \frac{\sin\left(\frac{7A+3A}{2}\right)\cos\left(\frac{7A-3A}{2}\right)}{\cos\left(\frac{7A+3A}{2}\right)\cos\left(\frac{3A-7A}{2}\right)}$$

$$= \frac{\sin\left(\frac{10A}{2}\right)\cos\left(\frac{4A}{2}\right)}{\cos\left(\frac{10A}{2}\right)\cos\left(\frac{-4A}{2}\right)}$$

$$= \frac{\sin 5A \cos 2A}{\cos 5A \cos 2A} = \tan 5A \quad R.H.S$$

Scanned with CamScanner

(iv) $\frac{\cos A - \cos 3A}{\sin 3A - \sin A} = \tan 2A$

L.H.S

$$\frac{\cos A - \cos 3A}{\sin 3A - \sin A} = \frac{2\sin\left(\frac{A+3A}{2}\right)\sin\left(\frac{3A-A}{2}\right)}{2\cos\left(\frac{A+3A}{2}\right)\sin\left(\frac{A-3A}{2}\right)}$$

$$= \frac{\sin\left(\frac{A+3A}{2}\right)\sin\left(\frac{3A-A}{2}\right)}{\cos\left(\frac{A+3A}{2}\right)\sin\left(\frac{3A-A}{2}\right)}$$

$$= \frac{\sin\left(\frac{4A}{2}\right)}{\cos\left(\frac{4A}{2}\right)} = \frac{\sin 2A}{\cos 2A} = \tan 2A$$

(v) $\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$

L.H.S

$$\frac{(\cos 4x + \cos 2x) + \cos 3x}{(\sin 4x + \sin 2x) + \sin 3x}$$

$$= \frac{2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{2x-4x}{2}\right) + \cos 3x}{2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \sin 3x}$$

$$= \frac{2\cos\left(\frac{6x}{2}\right)\cos\left(\frac{-2x}{2}\right) + \cos 3x}{2\sin\left(\frac{6x}{2}\right)\cos\left(\frac{2x}{2}\right) + \sin 3x}$$

$$= \frac{2\cos 3x\cos(-x) + \cos 3x}{2\sin 3x\cos x + \sin 3x}$$

Scanned with CamScanner

$\cos 3x(2\cos x + 1)$
$\sin 3x(2\cos x + 1)$

$\frac{\cos 3x}{\sin 3x} = \cot 3x$ R.H.S

Q13. Prove that the following:

(i) $\sin 47^\circ + \cos 77^\circ = \cos 17^\circ$

L.H.S

$\sin 47^\circ + \cos 77^\circ$

$\sin 47^\circ + \cos(90^\circ - 13^\circ)$

$\sin 47^\circ + \sin 13^\circ$

$2\sin\left(\frac{47^\circ + 13^\circ}{2}\right)\cos\left(\frac{47^\circ - 13^\circ}{2}\right)$

$2\sin\left(\frac{60^{30}}{2}\right)\cos\left(\frac{34^{17}}{2}\right)$

$2\sin 30^\circ \cos 17^\circ$

$2 \times \frac{1}{2}\cos 17^\circ$

$= \cos 17^\circ$ R.H.S

(ii) $\sin 51^\circ + \cos 81^\circ = \cos 21^\circ$

$\sin 51^\circ + \cos(90^\circ - 9^\circ)$

$\sin 51^\circ + \cos 9^\circ$

$2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

$2\sin\left(\frac{51^\circ + 9^\circ}{2}\right)\cos\left(\frac{51^\circ - 9^\circ}{2}\right)$

$2\sin\left(\frac{60^{30}}{2}\right)\cos\left(\frac{42^{21}}{2}\right)$

$2\sin 30^\circ \cos 21^\circ$

$2 \times \frac{1}{2}\cos 21^\circ$

$= \cos 21^\circ$ R.H.S

Scanned with CamScanner

iii) $\cos A + \cos(120^\circ - A) + \cos(120^\circ + A) = 0$

$2\cos\left(\frac{A + 120^\circ - A}{2}\right)\cos\left(\frac{120^\circ - A - A}{2}\right) + \cos(120^\circ + A)$

$2\cos 60^\circ \cdot \cos\left(\frac{120^\circ - 2A}{2}\right) + \cos(120^\circ + A)$

$2 \times \frac{1}{2}\cos(60 - A) + \cos(120^\circ + A)$

$= \cos(60^\circ - A) + \cos(120^\circ + A)$

$= 2\cos\left(\frac{60^\circ - A + 120^\circ + A}{2}\right)\cos\left(\frac{120^\circ + A - 60^\circ + A}{2}\right)$

$= 2\cos\left(\frac{180}{2}\right) \times \cos\left(\frac{60^\circ + 2A}{2}\right)$

$= 2\cos 90^\circ \times \cos\left(\frac{60^\circ + 2A}{2}\right)$

$= 2 \times 0 \times \cos(30 + A)$

$= 0$ R.H.S

Application

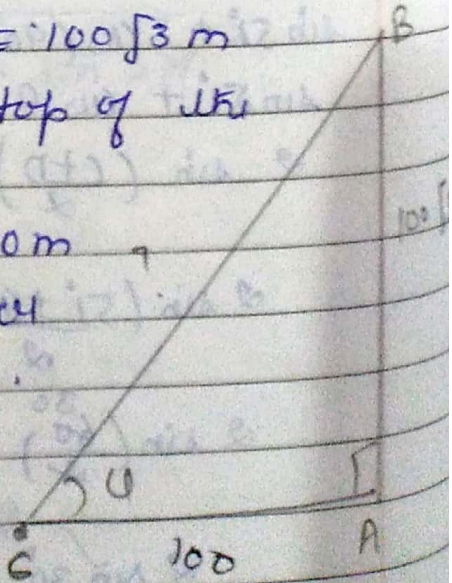
Q. A tower is $100\sqrt{3}$ m high find the angle of elevation of its top from a point 100 m away from its foot.

Ans Let AB = height of the tower $= 100\sqrt{3}$ m where A is foot and B is top of the tower.

Let C is the point which is 100 m away from the foot of tower and let θ is required angle.

By given statement AC = 100 m

Now $\tan\theta = \frac{P}{B} = \frac{AB}{AC}$



Scanned with CamScanner

$= \tan\theta = \frac{160\sqrt{3}\,m}{100\,m}$

$= \tan\theta = \sqrt{3}$

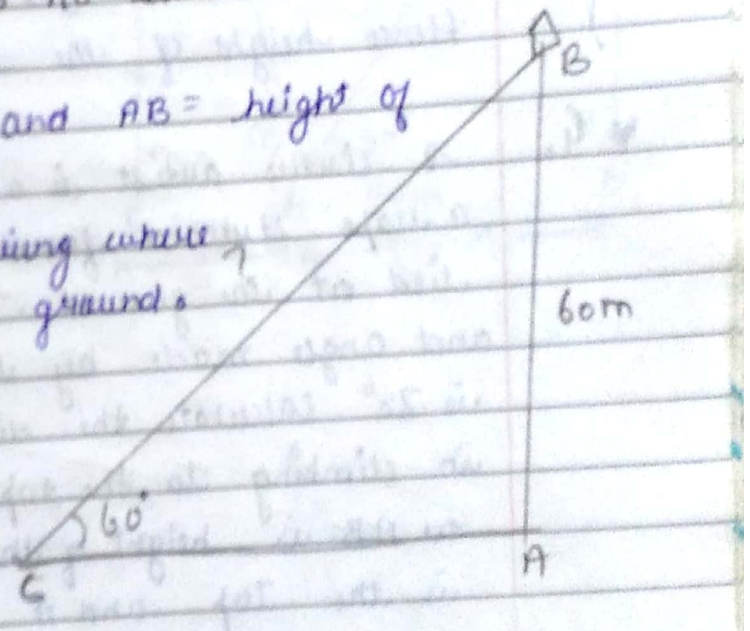
$= \tan\theta = \tan 60^\circ$

$= \theta = 60^\circ$

Ques. A kite is flying at a height of 60m above the ground the string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. find the length of the string assuming that there is no slack in the string.

Ans. Let B represent the kite and AB = height of the kite = 60m

Let BC is length of the string where C is the point on the ground.



$\sin 60^\circ = \frac{P}{H} = \frac{AB}{BC}$

$\frac{\sqrt{3}}{2} = \frac{60m}{BC}$

$= \sqrt{3}\,BC = 120m$

$BC = \frac{120}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{120\sqrt{3}}{3} = 40\sqrt{3}$

Ques 3. The string of a kite is 100m long and it makes angles of 60° with the horizontal find the height of kite assuming the there is no slak in the string.

Scanned with CamScanner

Let B represent the kite and AB = height of the pole = ?

Let BC = the length of the string = 100 m

$$\sin 60^\circ = \frac{P}{H} = \frac{AB}{BC}$$

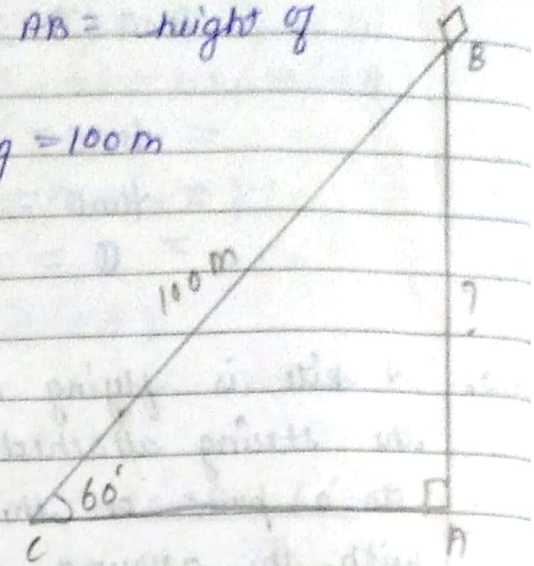
$$\frac{\sqrt{3}}{2} = \frac{AB}{100\,m}$$

$$= \frac{50}{100\sqrt{3}\,m}{2}$$

$$= 50 = AB$$

$$= AB = 50\sqrt{3}$$

Hence height of the kite is $50\sqrt{3}$ m.



Q. A circus artist is a climbing from the ground along a rope stretched from the top of vertical pole and tied at the ground. The height of the pole is 12 m and angle made by the rope with the ground level is 30° calculate the distance covered by the artist in climbing to the top of the pole.

∴ AB is height of the pole = 12 m where B is the top and A is the foot of the pole and BC is the length of the rope.

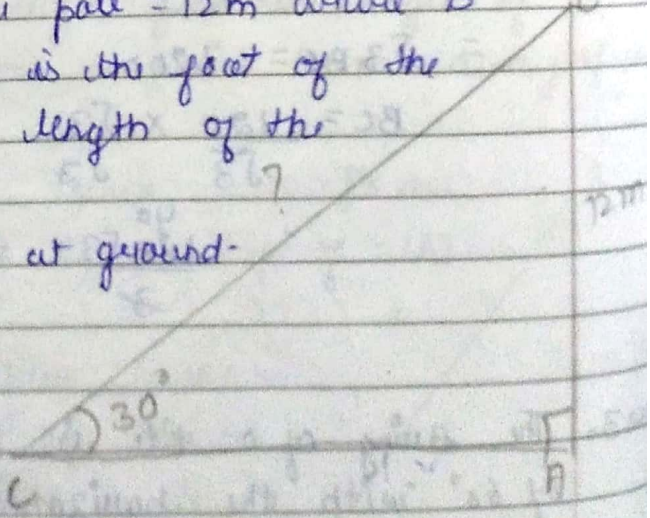
where C is the point at ground.

$$\sin 30^\circ = \frac{P}{H} = \frac{AB}{BC}$$

$$\frac{1}{2} = \frac{12\,m}{BC}$$

$$BC = 24\,m$$

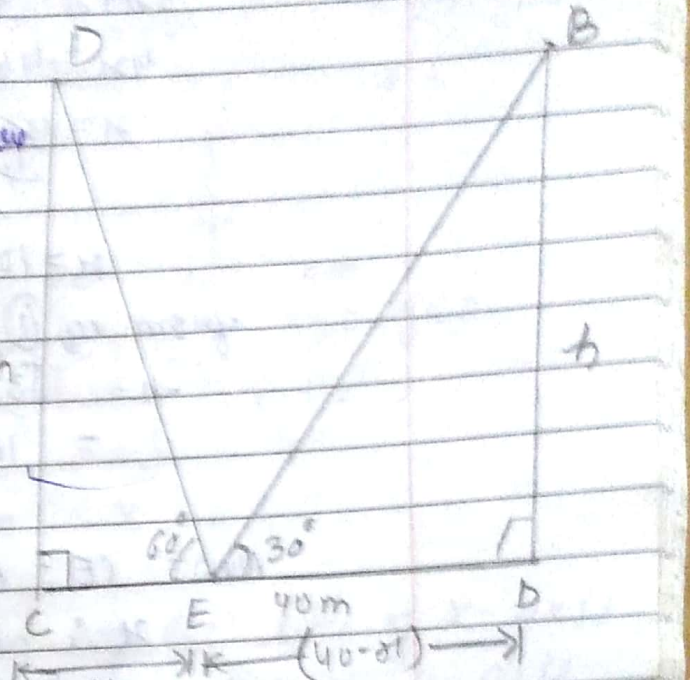
Hence the distance covered by the artist ing climbing the top is 24 m



Scanned with CamScanner

150

Two poles of equal heights stands on either side of a road which is 40m wide. At a point on the road between the poles the angle of elevation of the top of the poles are 60° and 30° find their hight and the position of the point.



AB = CD = h = hight of the pole whose B and D are tops of the poles

Now AC = 40m

Let E be the point on the road between the top poles such that CE = x m

EA = (40-x) m

and ∠CED = 60°

∠AEB = 30°

In ΔDCE

$$\tan 60^\circ = \frac{h}{x\,m}$$

$$\sqrt{3} = \frac{h}{x\,m}$$

$$\sqrt{3}x\,m = h \quad -(i)$$

In ΔBAE

$$\tan 30^\circ = \frac{h}{(40-x)\,m}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{(40-x)\,m}$$

$$\frac{(40-x)m}{\sqrt{3}} = h \quad -(2)$$

From eq (1) and (2) we have

Scanned with CamScanner

$\sqrt{3}x = \frac{40-x}{\sqrt{3}}$

$= (\sqrt{3})^2 x = 40 - x$

$= 3x = 40 - x$

$= 3x + x = 40$

$4x = 46$

$x = \frac{40}{4}\ 10$

$x = 10$

from eq (1)

$h = \sqrt{3} \times 10m$

$= 10\sqrt{3}\ m$

Now = CE : EA

$x : 40 - x$

$10 : 40 - 10$

$10 : 30$

$1 : 3$

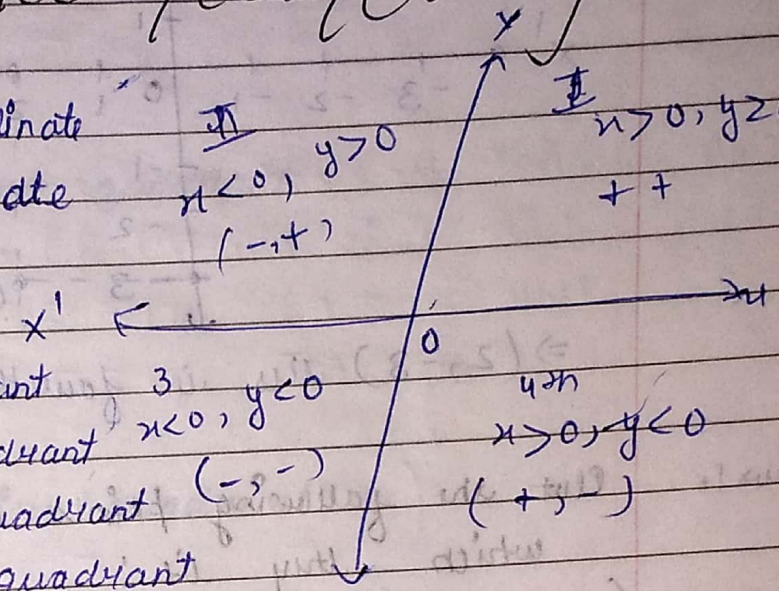
Hence the height of the pole is $10\sqrt{3}$ m and at point E divides CA into ratio 1 : 3.

Scanned with CamScanner

Unit = 3

Co-ordinate Geometry

Point (x, y) — y-coordinate or ordinate; x-coordinate or abscissa



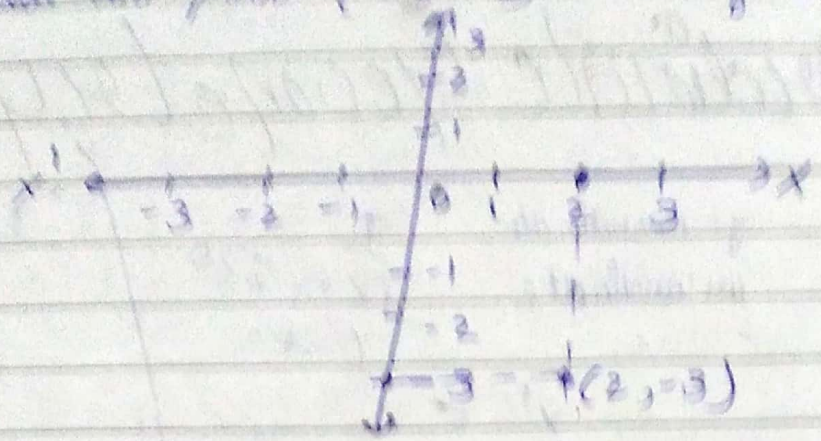
$(2, 3)$ lies in 1st quadrant
$(-2, 3)$ lies in 2nd quadrant
$(-2, -3)$ lies in 3rd quadrant
$(2, -3)$ lies in 4th quadrant
$(5, 0)$ lies on x-axis / lies on positive side of x-axis
$(-5, 0)$ lies on x-axis / lies on negative side of x-axis
$(0, 5)$ lies on Y-axis / lies on positive side of y-axis
$(0, -5)$ lies on Y axis / lies on negative side of y-axis
$(0, 0)$ lies is origin

(i) What is the location $(2, 3)$?
Here x coordinate $= 2 > 0$
and y coordinate $= 3 > 0$
$\Rightarrow (2, 3)$ lies in first quadrant

(ii) What is the location $(-5, 0)$?
Here x coordinate $= -5 < 0$
and y coordinate $= 0 > 0$
$\Rightarrow (-5, 0)$ lies in Second quadrant

Scanned with CamScanner

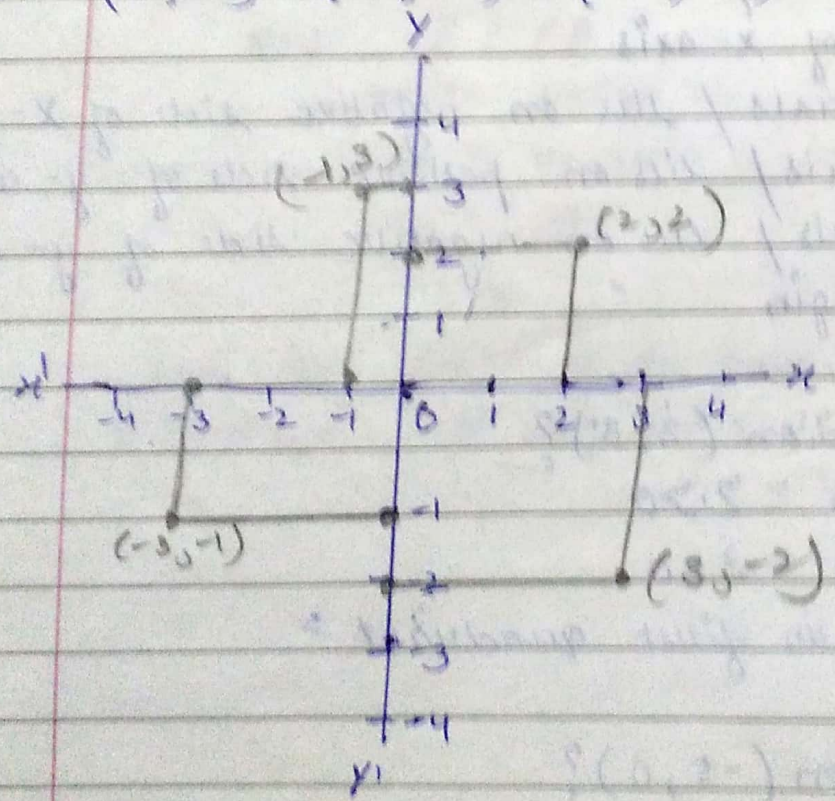
Plot the point (2, -3) and find its quadrant.



⇒ (2, -3) lies in fourth quadrant

Ques 1. Plot the following points and find the quadrant in which they lie:

i. (2, 2), (-3, -1), (-1, 3) and (3, -2)



i) (2, 2) lies in first quadrant
(-3, -1) lies in third quadrant
(-1, 3) lies in second quadrant
(3, -2) lies in fourth quadrant

Scanned with CamScanner

Page No.
Date

Ans: Without pointing find the location / quadrant in which the following points lie:

i) $(2, -4)$
x-coordinate $= 2 > 0$
y-coordinate $= -4 < 0$
$\Rightarrow (2, -4)$ lies in fourth quadrant

ii) $(-5, -6)$
x-coordinate $= -5 < 0$
y-coordinate $= -6 < 0$
$\Rightarrow (-5, -6)$ lies in third quadrant

iii) $(4, 3)$
x-coordinate $= 4 > 0$
y-coordinate $= 3 > 0$
$\Rightarrow (4, 3)$ lies in 1st quadrant

iv) $(-1, 5)$
x-coordinate $= -1 < 0$
y-coordinate $= 5 > 0$
$\Rightarrow (-1, 5)$ lies in second quadrant

v) $(0, 9)$
x-coordinate $= 0 > 0$
y-coordinate $= 9 > 0$
$\Rightarrow (0, 9)$ lies in 1st quadrant

vi) $(-3, 0)$
x-coordinate $= -3 < 0$
y-coordinate $= 0 > 0$
$\Rightarrow (-3, 0)$ lies in second quadrant

Scanned with CamScanner

Mid point :- If (x, y) and (x_2, y_2) be any two points, then mid point between these point is given by $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Qus 1 Find the mid point between $(1, 3)$ and $(-5, 7)$

Ans Given point are $(1, 3)$ and $(-5, 7)$

Here $x_1 = 1 \quad y_1 = 3$

$x_2 = -5 \quad y_2 = 7$

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{1+(-5)}{2}, \frac{3+7}{2}\right)$$

$$= \left(\frac{1-5}{2}, \frac{3+7}{2}\right)$$

$$= \left(\frac{-4}{2}, \frac{10}{2}\right)$$

$$= (-2, 5)$$

(ii) $(2, 3), (8, 5)$

Given point are $(2, 3)$ and $(8, 5)$

Here $x_1 = 2 \quad y_1 = 3$

$x_2 = 8 \quad y_2 = 5$

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$\left(\frac{2+8}{2}, \frac{3+5}{2}\right)$$

$$\left(\frac{10}{2}, \frac{8}{2}\right)$$

$$= (5, 4)$$

Scanned with CamScanner

(iii) (6,3) (6,-9)

Given points are (6,3) and (6,-9)

Here $x_1 = 6$ $y_1 = 3$

$x_2 = 6$ $y_2 = -9$

$$\left(\frac{6+6}{2}, \frac{3-9}{2}\right)$$

$$\left(\frac{12}{2}, \frac{-6}{2}\right)$$

$$(6, -3)$$

(iv) (-2,-4) (3,-6)

Given points are (-2,-4) and (3,-6)

Here $x_1 = -2$ $y_1 = -4$

$x_2 = 3$ $y_2 = -6$

$$\left(\frac{-2+3}{2}, \frac{-4-6}{2}\right)$$

$$\left(\frac{1}{2}, \frac{-10}{2}\right)$$

$$\left(\frac{1}{2}, -5\right)$$

(v) (0,8) (6,0)

Given points are (0,8) and (6,0)

Here $x_1 = 0$ $y_1 = 8$

$x_2 = 6$ $y_2 = 0$

$$\left(\frac{0+6}{2}, \frac{8+0}{2}\right)$$

$$\left(\frac{6}{2}, \frac{8}{2}\right)$$

$$(3, 4)$$

Scanned with CamScanner

Ques 2. If mid-point between two points is $(3,-6)$ and one point between them is $(5,2)$ find the other point.

Ans Let the other point is (a,b)

one point is $(5,2)$

and mid point between these point is $(3,-6)$

$\therefore \left(\frac{a+5}{2}, \frac{b+2}{2}\right) = (3,-6)$

$= \frac{a+5}{2} = 3$ and $\frac{b+2}{2} = -6$

$= a+5 = 6$ and $b+2 = -12$

$= a = 6-5$ and $b = -12-2$

$a = 1$ and $b = -14$

$\therefore$ required point is $(1,-14)$

Ques 3. If the mid point between two points is $(3,5)$ and one point between them is $(-1,2)$ find the other point.

Ans Let the other point is (a,b)

one point is $(-1,2)$

and mid point between these point is $(3,5)$

$\therefore \left(\frac{a-1}{2}, \frac{b+2}{2}\right) = (3,5)$

$\frac{a-1}{2} = 3$ and $\frac{b+2}{2} = 5$

$\Rightarrow a-1 = 6$ and $b+2 = 10$

$a = 6+1$ and $b = 10-2$

$a = 7$ and $b = 8$

$\therefore$ required point is $(7,8)$

Scanned with CamScanner

Distance formula :- If (x_1, y_1) and (x_2, y_2) be any two points then distance between these points is given by

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Q.1. Find the distance between the points $(2, -3)$ and $(4, -5)$

Sol. Given points are $(2, -3)$ and $(4, -5)$

$x_1 = 2 \quad y_1 = -3$

$x_2 = 4 \quad y_2 = -5$

Distance between these point =

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(4-2)^2+(-5-3)^2}$$

$$= \sqrt{(2)^2+(-2)^2}$$

$$= \sqrt{4+4} = \sqrt{8} \cdot \sqrt{4\times 2}$$

ii) $(0, 5)$ $(3, 6)$

Given point are $(0, 5)$ $(3, 6)$

$x_1 = 0 \quad y_1 = 5$

$x_2 = 3 \quad y_2 = 6$

$$\sqrt{(3-0)^2+(6-5)^2}$$

$$= \sqrt{(3)^2+(1)^2}$$

$$= \sqrt{9+1}$$

$$= \sqrt{10} = \sqrt{5\times 2}$$

Scanned with CamScanner

Ques 2. Using distance formula, prove that the triangle formed by the points A (4, 0), B (−1, −1) and C (3, 5) is an isosceles triangle.

Ans $AB = \sqrt{(-1-4)^2 + (-1-0)^2}$

$= \sqrt{(-5)^2 + (-1)^2}$

$= \sqrt{25+1} = \sqrt{26}$

$BC = \sqrt{(3+1)^2 + (5+1)^2}$

$= \sqrt{(4)^2 + (6)^2}$

$= \sqrt{16+36} = \sqrt{52}$

$CA = \sqrt{(4-3)^2 + (0-5)^2}$

$= \sqrt{(1)^2 + (-5)^2}$

$= \sqrt{1+25} = \sqrt{26}$

Ques 3. Using distance formula, prove that the triangle formed by the points. A (0, 0), B (0, 2) and C ($\sqrt{3}$, 1) is an equilateral triangle

$AB = \sqrt{(0-0)^2 + (2-0)^2}$

$= \sqrt{(0)^2 + (2)^2}$

Scanned with CamScanner

$\sqrt{0+4} = \sqrt{4} = 2$

$BC = \sqrt{(\sqrt{3}-0)^2 + (1-2)^2}$

$= \sqrt{(\sqrt{3})^2 + (-1)^2}$

$= \sqrt{3+1} = \sqrt{4} = 2$

$CA = \sqrt{(0-\sqrt{3})^2 + (0-1)^2}$

$= \sqrt{(\sqrt{3})^2 + (-1)^2}$

$= \sqrt{3+1} = \sqrt{4} = 2$

Centroid of a triangle :- Intersecting point of all the three medians of the triangle is called centroid of triangle.

or If (x_1, y_1), (x_2, y_2) and (x_3, y_3) be the vertices of the triangle, then centroid of the triangle is given by $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

Ans 1. Vertices of the triangle are given below, find the centroid of the triangle.

(i) (5, 2), (5, 4) and (8, 6)

given that vertices of the triangle are (5, 2), (5, 4) and (8, 6)

Here $x_1 = 5$ $y_1 = 2$

$x_2 = 5$ $y_2 = 4$

$x_3 = 8$ $y_3 = 6$

Scanned with CamScanner

$\therefore$ Centroid $= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

$= \left(\frac{5+5+8}{3}, \frac{2+4+6}{3}\right)$

$= \left(\frac{18}{3}, \frac{12}{3}\right)$

$= (6, 4)$

(ii) $(4,-3), (-4,8)$ and $(5,7)$

given that vertices of the triangle are $(4,-3), (-4,8)$ and $(5,7)$

$\therefore$ Centroid $= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

$= \left(\frac{4+(-4)+5}{3}, \frac{-3+8+7}{3}\right)$

$= \left(\frac{4-4+5}{3}, \frac{12}{3}\right)$

$= \left(\frac{5}{3}, 4\right)$

(iii) $(2,-4), (0,-10)$ and $(4,5)$

given that vertices of the triangle are $(2,-4), (0,-10)$ and $(4,5)$.

Here $x_1 = 2 \quad y_1 = -4$

$x_2 = 0 \quad y_2 = -10$

$x_3 = 4 \quad y_3 = 5$

Centroid $= \left(\frac{2+0+4}{3}, \frac{-4+(-10)+5}{3}\right)$

$= \left(\frac{6}{3}, \frac{-4-10+5}{3}\right)$

$= \left(\frac{6}{3}, \frac{-9}{3}\right) = (2, -3)$

Scanned with CamScanner

If centroid of a triangle is (10, 18) and two vertices of the triangle are (1, -5) and (3, 7) find the third vertex of the triangle.

given that centroid = (10, 18)

and two vertices of the triangle are (1, -5) and (3, 7)

let third vertex is (a, b)

∴ by defination of centroid

$$\left(\frac{1+3+a}{3}, \frac{-5+7+b}{3}\right) = (10, 18)$$

$$= \left(\frac{4+a}{3}, \frac{2+b}{3}\right) = (10, 18)$$

$$\Rightarrow \frac{4+a}{3} = 10 \text{ and } \frac{2+b}{3} = 18$$

$$\Rightarrow 4+a = 30 \text{ and } 2+b = 54$$

$$= a = 30-4 \text{ and } b = 54-2$$

$$= a = 26 \text{ and } b = 52$$

∴ third vertex is (26, 52)

3. If centroid of a triangle is (-5, -7) and two vertices of the triangle are (0, 6) and (-3, 2) find the third vertex of the triangle.

Given that centroid = (-5, -7)

and two vertices of the triangle are (0, 6) and (-3, 2)

let the third vertex is (a, b)

∴ by defination of centroid

$$= \left(\frac{0+(-3)+a}{3}, \frac{6+2+b}{3}\right) = (-5, -7)$$

$$= \left(\frac{0-3+a}{3}, \frac{8+b}{3}\right) = (-5, -7)$$

$$= \left(\frac{-3+a}{3}, \frac{8+b}{3}\right) = (-5, -7)$$

$$= \frac{-3+a}{3} = -5 \text{ and } \frac{8+b}{3} = -7$$

Scanned with CamScanner

$\Rightarrow -3+a=-15$ and $8+b=-21$

$\Rightarrow a=-15+3$ and $b=-21-8$

$\therefore a=-12$ and $b=-29$

$\therefore$ third vertex is $(-12, -29)$

Ques 4. If centroid of a triangle formed by the points $(1, a)$, $(4, b)$ and $(c^2, -5)$ lies on x-axis, prove that $a+b=5$

Ans Vertices of the triangle are $(1, a)$, $(4, b)$ and $(c^2, -5)$

$\therefore$ centroid $= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

$\therefore = \left(\frac{1+4+c^2}{3}, \frac{a+b-5}{3}\right)$

This lies on x-axis

$\Rightarrow \frac{a+b-5}{3}=0$

$\Rightarrow a+b-5=0$

$\Rightarrow a+b=5$

Hence proved

Ques 5. If centroid of a triangle formed by the points $(-a, a)$, (c^2, b) and $(d, 5)$ lies y-axis, prove that $c^2=a-d$

Ans Vertices of triangle are $(-a, a)$ (c^2, b) and $(d, 5)$

$\therefore$ centroid $= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

$= \left(\frac{-a+c^2+d}{3}, \frac{a+b+5}{3}\right)$

$=$ This lies on y-axis.

Scanned with CamScanner

$\Rightarrow \frac{-a+c^2+d}{3} = 0$

$\Rightarrow -a+c^2+d = 0$

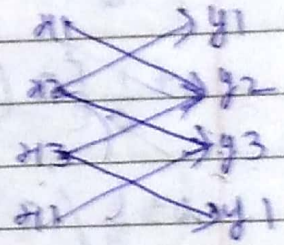
$\Rightarrow c^2 = a-d$

Hence proved

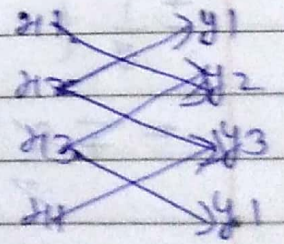
Area of Triangle :- If (x_1, y_1) (x_2, y_2) and (x_3, y_3) are the vertices of a triangle, then area of the triangle is given by

$$\Delta = \pm\frac{1}{2}\left[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)\right]$$

or $\Delta = \pm\frac{1}{2}\left[(x_1y_2 - x_2y_1) + (x_2y_3 - y_3y_2) + (x_3y_1 - x_1y_3)\right]$



or $\Delta = \pm\frac{1}{2}\left[(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)\right]$



or $\Delta = \pm\frac{1}{2}\begin{vmatrix} x_1-x_2 & x_1-x_3 \\ y_1-y_2 & y_1-y_3 \end{vmatrix}$

or $\Delta = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

Scanned with CamScanner

Note:- i) Area is always non negative so we have to mention the suitable sign.

ii) If area is zero then the zero-three point are collinear.

Ques:- Vertices of triangle are given below find the area of the triangle

i) (3, 2), (5, 4) and (7, 2)

Here $x_1 = 3$ $y_1 = 2$

$x_2 = 5$ $y_2 = 4$

$x_3 = 7$ $y_3 = 2$

$$\Delta = \frac{1}{2}\left(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right)$$

$$\Delta = \frac{1}{2}\left[3(4-2) + 5(2-2) + 7(2-4)\right.$$

$$\Delta = \frac{1}{2}\left(3(2) + 5(0) + 7(-2)\right.$$

$$\Delta = \frac{1}{2}\left(6 + 0 - 14\right)$$

$$= \pm\frac{1}{2}(-8)$$

$$= \mp\frac{8}{2}4 = 4 \text{ Sq. units.}$$

(ii) (1, 3), (-4, 5) and (3, -4)

Here $x_1 = 1$ $y_1 = 3$

$x_2 = -4$ $y_2 = 5$

$x_3 = 3$ $y_3 = -4$

$$\Delta = \frac{1}{2}\left(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right)$$

$$\Delta = \frac{1}{2}\left(1(5 - (-4) + (-4)(-4 - 3) + 3(3 - 5)\right.$$

Scanned with CamScanner

$\Delta = \frac{1}{2}[1(5+4) - 4(-7) + 2(-2)]$

$\Delta = \frac{1}{2}[9 + 28 - 6]$

$\Delta = \frac{1}{2}[9 + 22]$

$\Delta = \frac{1}{2}[31]$

$= \pm \frac{31}{2}$ sq units

(iii) (-2, 1), (2, -3) and (5, 5)

Here $x_1 = -2$ $y_1 = 1$

$x_2 = 2$ $y_2 = -3$

$x_3 = 5$ $y_3 = 5$

$\Delta = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$\Delta = \frac{1}{2}[-2(-3-5) + 2(5-1) + 5(1+3)]$

$\Delta = \frac{1}{2}[-2(-8) + 2(4) + 5(4)]$

$\Delta = \frac{1}{2}[16 + 8 + 20]$

$\Delta = \frac{1}{2}[44]$

$\pm \frac{44}{2}$ 22 = 22 sq units

Scanned with CamScanner

(ii) $(0,2)$, $(3,6)$ and $(7,-5)$

Here $x_1 = 0$ $y_1 = 2$

$x_2 = 3$ $y_2 = 6$

$x_3 = 7$ $y_3 = -5$

$$\Delta = \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y - y_2)\right]$$

$$\Delta = \frac{1}{2}\left[0(6-(-5)) + 3(-5-2) + 7(2-6)\right]$$

$$\Delta = \frac{1}{2}\left[0(6+5) + 3(-7) + 7(-4)\right]$$

$$\Delta = \frac{1}{2}\left[0 - 21 - 28\right]$$

$$\Delta = \frac{1}{2}\left[-49\right]$$

$$= \mp\frac{49}{2} \text{ sq units}$$

Straight line

* Straight line :- Shortest distance between two points in a plane is called straight line.

* General equation of straight line :- General equation of straight line in xy-plane is $ax + by + c = 0$, where a, b and c are arbitrary constants.

* Slop of straight line :- Slop of a straight line measures with tangent of the angle of straight line to the horizontal.
It is usually represented by m.

Scanned with CamScanner

To find slop of a straight line :

(i) If a non-vertical line makes an angle θ with the horizontal then slop $m = \tan\theta$.

(ii) If a non-vertical straight line passes through two points (x_1, y_1) and (x_2, y_2) than slop $m = \frac{y_2 - y_1}{x_2 - x_1}$

(iii) If equation of straight line is $ax + by + c = 0$, then slop of the line is $m = -\frac{a}{b}$ or $m = -\frac{\text{coeffi of } x}{\text{coefficient of } y}$

Note : (i) A point (x_1, y_1) lies on the straight line $ax + by + c = 0$ if and only if it satisfied the equation of straight line that is $ax_1 + by_1 + c = 0$

(ii) equation of x-axis is $y = 0$

(iii) equation of y-axis is $x = 0$

(iv) $x = k$, where k is constant, is a straight line parallel to y-axis.

(v) $y = k$, where k is some constant, is a straight line parallel to x-axis.

Note related to slop :- (i) slop of horizontal line is always zero, because $m = \tan 0^\circ = 0$

(ii) slop of vertical line is always infinity because $m = \tan 90^\circ = \infty$

(iii) If L_1 and L_2 be any two straight line and L_1 and L_2 and their slop respectively, then $L_1 \parallel L_2$ if and always if $m_1 = m_2$ and $L_1 \perp L_2$ if and only if $m_1 m_2 = -1$

Ques. Find the slopes of the straight lines which make the following with the horizontal (+ve x-axis)

Scanned with CamScanner

(i) 45°

Ans $\theta = 45°$

slope $= m = \tan\theta$

$m = \tan\theta$

$= 1$

(ii) 120°

$\theta = 120°$

slope $= m = \tan\theta$

$m = \tan 120°$

$= \tan(180° - 60°)$

$= -\tan 60°$

$m = -\sqrt{3}$

(iii) 30°

Ans $\theta = 30°$

slope $= m = \tan\theta$

$m = \tan 30°$

$m = \frac{1}{\sqrt{3}}$

(iv) 150°

Ans $\theta = 150°$

slope $= m = \tan\theta$

$m = \tan 150°$

$m = \tan(180° - 30°)$

$= -\tan 30°$

$= -\frac{1}{\sqrt{3}}$

(v) 210°

$\theta = 210°$

slope $= m = \tan\theta$

$m = \tan 210°$

$m = \tan(270° - 60°)$

$m = \tan 60°$

$m = \sqrt{3}$

(vi) 60°

$\theta = 60°$

slope $= m = \tan\theta$

$m = \tan 60°$

$m = \sqrt{3}$

Ques 2. Find the slopes of straight line which pass through the following pairs of points.

(i) (2, 5) and (6, 17)

Ans Given that straight line passes through the point (2, 5) and (6, 17)

Now $x_1 = 2 \quad y_1 = 5$

$x_2 = 6 \quad y_2 = 17$

$\therefore$ slop $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{17 - 5}{6 - 2} = \frac{12}{4} = 3$

Scanned with CamScanner

(ii) $(-8,7)$ and $(3,-5)$

Sol Given that straight line passes through the point $(-8,7)$ and $(3,-5)$

Here $x_1 = -8$ $y_1 = 7$

$x_2 = 3$ $y_2 = -5$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-7}{3+8} = \frac{-12}{11}$$

(iii) $(0,-6)$ and $(7,9)$

Sol Given that straight line passes through the point $(0,-6)$ and $(7,9)$

Here $x_1 = 0$ $y_1 = -6$

$x_2 = 7$ $y_2 = 9$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9+6}{7-0} = \frac{15}{7}$$

(iv) $(-11,-5)$ and $(-3,-10)$

Sol Given that straight line passes through the point $(-11,-5)$ and $(-3,-10)$

Here $x_1 = -11$ $y_1 = -5$

$x_2 = -3$ $y_2 = -10$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-10+5}{-3+11} = \frac{-5}{8}$$

(v) $(0,0)$ and $(10,-12)$

Sol Given that straight line passes through the point $(0,0)$ and $(10,-12)$

Here $x_1 = 0$ $y_1 = 0$

$x_2 = 10$ $y_2 = -12$

Scanned with CamScanner

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-12 - 0}{10 - 0} = \frac{-12}{10} = -\frac{6}{5}$$

Q3. Find the slope of straight line

(i) $2x + 4y + 5 = 0$

Ans Given equation of straight line is:

$2x + 4y + 5 = 0$

a = coefficient $x = 2$

b = coefficient $y = 4$

$$\text{slop } m = \frac{-a}{b} = \frac{-2}{4} = \frac{-1}{2}$$

(ii) $x - 3y + 9 = 0$

Ans given equation of straight line is:

$x - 3y + 9 = 0$

a = coeff of $x = 1$

b = coeff of $y = -3$

$$m = \frac{-a}{b} = \frac{-1}{-3} = \frac{1}{3}$$

(iii) $5y - 10x + 1 = 0$

Ans given equation of straight line is:

$5y - 10x + 1 = 0$

a = coeff of $x = -10$

b = coeff of $y = 5$

$$m = \frac{-a}{b} = \frac{-(-10)}{5} = \frac{10}{5} = 2$$

(iv) $-2x - 6y = 0$

Ans Given equation of straight line is:

$-2x - 6y = 0$

a = coeff of $x = -2$

b = coeff of $y = -6$

$$m = \frac{-a}{b} = \frac{-(-2)}{-6} = \frac{2}{-6} = \frac{-1}{3}$$

Scanned with CamScanner

$x = 5$

(iii) given equation of straight line is:

$x - 5 = 0$

a = coeff of $x = 1$

b = coeff of $y = 0$

$$m = \frac{-a}{b} = \frac{-1}{0} = \infty$$

(iv) $y = -6$

Ans given equation of straight line is:

$y = -6$

a = coeff of $x = 0$

b = coeff of $y = 1$

$$m = \frac{-a}{b} = \frac{-0}{1} \Rightarrow 0$$

Ques 4. Find the equation of straight line which is parallel its x-axis and passes through $(1, 5)$.

Ans Equation of line $\parallel$ to x-axis is

$y = k$ — ①

This line passes through $(1, 5)$

∴ Put $x = 1$ and $y = 5$ in eq ① we get

$5 = k$

Put $k = 5$ in equations ① we get

$y = 5$

Ques 5. Find the equation of straight line which is parallel to Y-axis and pas through $(-3, -7)$.

Ans Equation of line $\parallel$ to Y-axis is

$x = k$ — ①

This line passes through $(-3, -7)$

∴ Put $x = -3$ and $y = -7$ in eq ① we get

$-3 = k$

Put $k = -3$ in eq ① we get

$x = -3$

Scanned with CamScanner

Page No.
Date

Formula Equation of straight line passing through origin

$$y = mx$$

where m is the slop.

Ques 1. Find the equation of straight line having slop equal to 5 and passes through origin.

Ans Equation of line passing through a origin is

$$y = mx$$

where m is the slop = 5 (given)

$\therefore y = 5x$

$\Rightarrow -5x + y = 0$

which is required a $y = 5x$

Ques 2. Find the equation of straight line ~~passing~~ having slop equal to -10 and passes through origin.

Ans Equation of line passing ~~through~~ a origin is

$$y = mx$$

where m is the slop $= -10$ (given)

$\therefore y = -10x$

$\Rightarrow -10x - y = 0$

Ques 3. Find the equation of straight line ~~having slop equal to through~~ passes origin makes an angle 60° with the positive direction X- axis.

Ans Equation of the line passing through origin is

$$y = mx \quad \text{—①}$$

where m is slope

Here $\theta = 60^\circ$

$\therefore m = \tan\theta$

$= \tan 60^\circ$

$m = \sqrt{3}$

Put the $m = \sqrt{3}$ in eq ① we get

Scanned with CamScanner

$y = \sqrt{3}x$

Ques. Find the equation of straight line passes through a origin makes an angle 135° with the positive direction of x-axis

Sol. Equation of the passing through origin is

$$y = mx \quad \text{--- (1)}$$

where m is slop

Here $\theta = 135°$

$m = \tan\theta$

$m = \tan 135°$

$m = \tan(180° - 45°)$

$m = -\tan 45°$

$m = -1$

Put $m = -1$ in eq (1) we get

$$y = -x$$

★ Equation of straight line in point slope form :- If a straight line passes through (x_1, y_1) and having slope equal to m, then equation of straight line is :

$$y - y_1 = m(x - x_1)$$

Ques 1. Find equation of straight line having slop equal to 9 and passes through the point $(1, 5)$

Sol. Given that line passes through $(1, 5)$ and having slop $= 9$.

Here $x_1 = 1$, $y_1 = 5$, $m = 9$

∴ By point slop form

$$y - y_1 = m(x - x_1)$$

$$y - 5 = 9(x - 1)$$

$$y - 5 = 9x - 9$$

$$y - 5 - 9x + 9 = 0$$

Scanned with CamScanner

Page No.
Date

Qus 2. Find the equation straight line passing through $(-4,-2)$ and having slop (-8)

Ans Given that line passes through $(-4,-2)$ and having slo
-8

Here $x_1 = -4$, $y_1 = -2$ and $m = -8$

∴ by point slope form

$$y - y_1 = m(x - x_1)$$
$$y + 2 = -8(x + 4)$$
$$y + 2 = -8x - 32$$
$$y + 2 + 8x + 32 = 0$$
$$8x + y + 34 = 0$$

Qus 3. Find the equation of straight line passing through $(0,-8)$ and makes an angle 30° with the positive direction of X-axis.

Ans Given line passes through $(0,-8)$

$x_1 = 0$, $y_1 = -8$

Also $\theta = 30^\circ$

∴ slope $= m = \tan\theta$

$$m = \tan 30^\circ$$
$$m = \frac{1}{\sqrt{3}}$$

by point slope form

$$y - y_1 = m(x - x_1)$$
$$y + 8 = \frac{1}{\sqrt{3}}(x - 0)$$
$$y + 8 = \frac{1}{\sqrt{3}}(x)$$
$$\sqrt{3}y + 8\sqrt{3} = x$$
$$-x + \sqrt{3}y + 8\sqrt{3} = 0$$

Scanned with CamScanner

Page No.
Date

Ques 4. Find the equation of straight line passing through $(-9, 0)$ and make an angle $150°$

Ans Given line passes through $(-9, 0)$

$x_1 = -9 \quad y_1 = 0$

Also $\theta = 150°$

Slope $= m = \tan\theta$

$m = \tan 150°$

$m = \tan(180° - 30°)$

$m = -\tan 30$

$m = -\frac{1}{\sqrt{3}}$

by point slope form

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{-1}{\sqrt{3}}(x + 9)$

$y = \frac{-1}{\sqrt{3}}(x + 9)$

$\sqrt{3}y = -x - 9$

$x + \sqrt{3}y + 9 = 0$

★ Two point form:- If straight line passes through the points (x_1, y_1) and (x_2, y_2) then equation of line is

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

Ques 1. Find the equation of straight line passes through the points $(2, -2)$ and $(0, 6)$.

Ans Give line passes through $(2, -2)$ and $(0, 6)$

Now $x_1 = 2 \quad y_1 = -2$

$x_2 = 0 \quad y_2 = 6$

$\therefore$ equation of line is

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

Scanned with CamScanner

$\Rightarrow y-(-2)=\left(\frac{6-(-2)}{0-2}\right)(x-2)$

$\Rightarrow y+2=\left(\frac{8}{-2}\right)(x-2)$

$\Rightarrow y+2=-4(x-2)$

$\Rightarrow y+2=-4x+8$

$\Rightarrow 4x+y+2-8=0$

$\Rightarrow 4x+y-6=0$

ans 2. Find the equation of straight line passes through the points $(0,8)$ and $(5,0)$

Ans Given line passes through $(0,8)$ and $(5,0)$

Now $x_1=0 \quad y_1=8$

$x_2=5 \quad y_2=0$

∴ equation of line is

$y-y_1=\left(\frac{y_2-y_1}{x_2-x_1}\right)(x-x_1)$

$\Rightarrow y-8=\left(\frac{0-8}{5-0}\right)(x-0)$

$\Rightarrow y-8=\left(\frac{-8}{5}\right)(x-0)$

$\Rightarrow 5y-40=-8x$

$\Rightarrow 8x+5y-40=0$

ans 3. Find the equation of straight line passes through the points $(7,-4)$ and $(-1,5)$

Ans Given line passes through $(7,-4)$ and $(-1,5)$

Now $x_1=7 \quad y_1=-4$

$x_2=-1 \quad y_2=5$

∴ equation of line is

$y-y_1=\left(\frac{y_2-y_1}{x_2-x_1}\right)(x-7)$

Scanned with CamScanner

$$y-(-4)=\left(\frac{5-(-4)}{-1-7}\right)(x-7)$$

$$\Rightarrow y+4=\left(\frac{9}{-8}\right)(x-7)$$

$$\Rightarrow -8y-32=9(x-7)$$

$$-8y-32=9x-63$$

$$\Rightarrow 9x+8y+32-63$$

$$\Rightarrow 9x+8y-31=0$$

* equation of straight line in slop- intercept form

$$y=mx+c$$

where m is slope and c is y-intercept of the line.

Note i) If intercept c is given above the the x-axis or above the origin, then it is positive.

ii) If intercept c is given below the x-axis or below the origin, then it is negative

Qus 1 Find the equation of straight line passes through the points (2, -2) and (0, 6)

Sol Given line passes through (2, -2) and (0, 6)

here $x_1=2$ $y_1=-2$

$x_2=0$ $y_2=6$

∴ equation of line is

$$y-y_1=\left(\frac{y_2-y_1}{x_2-x_1}\right)(x-x_1)$$

$$\Rightarrow y-(-2)=(-$$

Qus 1. Find the equation of straight line having slop 3 and cuts of an intercept -2 on y-axis

Sol here m slope = 3

and c = y-intercept = -2

Scanned with CamScanner

∴ by slope point intercept form

$y = mx + c$

$y = 3x + 6 - 2$

$\Rightarrow -3x + y + 2 = 0$

Ques 2. Find the equation of straight line having slope -6 cuts of an intercept 5 on y-axis above the origin.

Ans $m = -6$

$c = 5$

∴ by slop-intercept form

$\Rightarrow y = mx + c$

$y = -6x + 5$

$6x + y - 5 = 0$

Ques 3. Find the equation of straight line having slope 2 cuts of an intercept 9 on y-axis below the origin.

Ans $m = 2$

$c = -9$

∴ by slop-intercept form

$\Rightarrow y = mx + c$

$y = 2x - 9$

$-2x + y + 9 = 0$

Ques 4. Find the equation of straight line which makes an angle 45° with x-axis and cuts of an intercept 8 on y-axis below the x-axis.

Ans Here $\theta = 45°$

$c = y$-intercept $= -8$

and slope $= m = \tan\theta$

$m = \tan 45°$

$m = 1$

∴ by slop-intercept form

Scanned with CamScanner

$y = mx + c$

$y = x - 8$

$-x + y + 8 = 0$

Ques 5. Find the equation of straight line which makes an angle $60°$ with x-axis and cuts of intercept 5 on y-axis above the x-axis.

Ans Here $\theta = 60°$

$c = y$-intercept $= 5$

and slope $= m = \tan\theta$

$m = \tan 60°$

$m = \sqrt{3}$

∴ by slop - intercept form

$y = mx + c$

$y = \sqrt{3}x + 5$

$-\sqrt{3}x + y - 5 = 0$

* Intercept form :- $\frac{x}{a} + \frac{y}{b} + 1 = 0$

Ques 1. Find equation of straight line with make intercept 2 and 5 on x-axis and y-axis respectively.

Ans Here $a = x$-intercept $= 2$

$b = y$-intercept $= 5$

∴ equation of line in intercept form

$\frac{x}{a} + \frac{y}{b} = 1$

$\frac{x}{2} + \frac{y}{5} = 1$

$\frac{5x + 2y}{10} = 1$

$5x + 2y = 10$

$5x + 2y - 10 = 0$

Scanned with CamScanner

Page No.
Date

Ques 2. Find equation of straight line with makes intercept 3 and -15 on x-axis and y-axis respectively.

Ans Here a = x-axis intercept = 3

b = y-intercept = -15

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{3} - \frac{y}{15} = 1$$

$$\frac{5x - y}{15} = 1$$

$$5x - y = 15$$

$$5x - y - 15 = 0$$

Ques 3. Find equation of straight line which passes through (1, -4) and makes intercept on axes which are equal in mag- and opposite in

Ans Let x-intercept = a

y-intercept = $-a$ i.e $b = -a$

equation of line in intercept form is

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{a} + \frac{y}{-a} = 1$$

$$\frac{x}{a} - \frac{y}{b} = 1$$

$$\frac{x - y}{a} = 1$$

$$x - y = a$$

This line passes through (1, -4)

put $x = 1$ and $y = -4$ in eq (1)

$$x - y = a$$

$$1 - (-4) = a$$

Scanned with CamScanner

Page No.
Date

$5 = a$

Put $a = 5$ in eq (1) we get

$x - y = 5$

$x - y - 5 = 0$

Scanned with CamScanner

* Normal form $= p = x\cos\alpha + y\sin\alpha$

Que 1: Find the equation of straight line such that the length of perpendicular from the origin the straight line is 2 and the inclination of this perpendicular to the x-axis is 120°

Sol: $p = 2$

and $\alpha = 120°$

Now equation of line in Normal form is

$p = x\cos\alpha + y\sin\alpha$

$2 = x\cos 120° + y\sin 120°$

$2 = x\cos(180°-60°) + y\sin(180°-60°)$

$2 = -x\cos 60° + y\sin 60°$

$2 = -x\times\frac{1}{2} + y\times\frac{\sqrt{3}}{2}$

$2 = \frac{-x}{2} + \frac{y\sqrt{3}}{2}$

$2 = \frac{-x + y\sqrt{3}}{2}$

$4 = -x + y\sqrt{3}$

$\Rightarrow x - y\sqrt{3} + 4 = 0$

Que 2: Find the equation of straight line such that the length of perpendicular from the origin the straight line is 7 and the inclination of this perpendicular to the X-axis is 45°.

Sol: $p = 7$

and $\alpha = 45°$

Now equation of line in normal form is

$p = x\cos\alpha + y\sin\alpha$

$7 = x\cos 45° + y\sin 45°$

$7 = x\cos(90°-45°) + y\sin(90°-45°)$

Scanned with CamScanner

$$7 = x\cos 45^\circ + y \sin 45^\circ$$

$$7 = x \times \frac{1}{\sqrt{2}} + y \times \frac{1}{\sqrt{2}}$$

$$7 = \frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}}$$

$$7 = \frac{x+y}{\sqrt{2}}$$

$$\Rightarrow 7\sqrt{2} = x + y$$

* Angle between two straight line :- If L_1 and L_2 be two straight line with slopes m_1 and m_2 respectively and θ is acute angle between these line, the

$$\tan\theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

Note :- obtus angle $= 180^\circ -$ acute angle

Ques. Find the acute angle between the straight line whose slopes are 1 and 0.

Ans. Here $m_1 = 1$

$m_2 = 0$

Let θ is acute angle between the line.

$$\tan\theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$= \left| \frac{1 - 0}{1 + (1)(0)} \right|$$

$$\Rightarrow \tan\theta = \left| \frac{1}{1} \right|$$

$$\tan\theta = 1$$

$$\tan\theta = 45^\circ$$

$$\theta = 45^\circ$$

Scanned with CamScanner

Page No.
Date

Ques 2. Find the acute angle between the straight line whose slopes are $2+\sqrt{3}$ and $2-\sqrt{3}$.

Ans Here $m_1 = 2+\sqrt{3}$

$m_2 = 2-\sqrt{3}$

Let θ is acute angle between the line

$$\tan\theta \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$= \tan\theta \left| \frac{2+\sqrt{3} - 2 - \sqrt{3}}{1 + (2+\sqrt{3})(2-\sqrt{3})} \right|$$

$$\tan\theta \left| \frac{2\sqrt{3}}{1+4-3} \right|$$

$$\tan\theta \left| \frac{2\sqrt{3}}{2} \right|$$

$$\tan\theta = \sqrt{3}$$

$$\tan\theta = 60^\circ$$

$$\theta = 60^\circ$$

Ques 3. Find the obtuse angle between the straight line slope are $\sqrt{3}$ and $\frac{1}{\sqrt{3}}$

Ans Here $m_1 = \sqrt{3}$

$m_2 = \frac{1}{\sqrt{3}}$

Let θ is obtuse angle between the line

$$\tan\theta \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$\tan\theta \left| \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + (\sqrt{3})(\frac{1}{\sqrt{3}})} \right|$$

Scanned with CamScanner

$$\tan\theta \left| \frac{\frac{3-1}{\sqrt{3}}}{2} \right|$$

$$= \left| \frac{\frac{2}{\sqrt{3}}}{2} \right|$$

$$\left| \frac{2}{\sqrt{3}} \times \frac{1}{2} \right|$$

$\tan\theta = \sqrt{3}$

$\tan\theta = 30^\circ$

$\theta = 30^\circ$

obtuse angle

$180^\circ - \theta$

$180^\circ - 30^\circ$

$= 150^\circ$

Ques 4. Find the angle between the straight ~~angle~~ line whose slope are -3 and 5

Ans Here $m_1 = -3$

$m_2 = 5$

Let θ is acute angle

$$\tan\theta \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$\left| \frac{-3-5}{1+(-3)(5)} \right|$$

$$\left| \frac{-8}{1-15} \right| = \left| \frac{-8^4}{-14_7} \right|$$

$$= \tan\theta \left| \frac{4}{7} \right| = \tan\theta = \frac{4}{7}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{4}{7}\right)$$

Scanned with CamScanner

Page No.
Date 28 | 11 | 19

Ques 1. Find the angle between the line joining the points (0,0) (2,3) and (2,-2) & (3,5)

Ans Let m_1 = slop of line joining (0,0) and (2,3)

(x_1, y_1) (x_2, y_2)

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{3-0}{2-0}$$

$$m_1 = \frac{3}{2}$$

and m_2 = slop of line joining (2,-2) and (3,5)

(x_1, y_1) (x_2, y_2)

$$m_2 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{5-(-2)}{3-2}$$

$$= \frac{7}{1}$$

$$m_2 = 7$$

Let θ is the acute angle between the lines.

$$\therefore \tan\theta \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$\left| \frac{\frac{3}{2} - \frac{7}{1}}{1 + \left(\frac{3}{2}\right)(7)} \right|$$

$$\left| \frac{\frac{3-14}{2}}{1 + \frac{21}{2}} \right|$$

$$\left| \frac{\frac{-11}{2}}{\frac{2+21}{2}} \right| \Rightarrow \left| \frac{\frac{-11}{2}}{\frac{23}{2}} \right| \Rightarrow \left| \frac{-11}{2} \times \frac{2}{23} \right|$$

Scanned with CamScanner

$$\left|\frac{-11}{23}\right|$$

$\Rightarrow \tan\theta = \frac{11}{23}$

$\theta = \tan^{-1}\left(\frac{11}{23}\right)$

Qual 2. Find the angle between the line joining the pairs $(6,-5)$, $(-2,1)$ and $(0,3)$ $(-8,6)$

Sol Let m_1 = slope of line joining $(6,-5)$ and $(-2,1)$ ($x_1\ y_1$, $x_2\ y_2$)

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \left(\frac{1+5}{-2-6}\right)$$

$$= \frac{6}{-8} \quad \frac{3}{4} \qquad m_2 = \frac{-3}{4}$$

Let m_2 = slope of line joining $(0,3)$, $(-8,6)$ ($x_1\ y_1$, $x_2\ y_2$)

$$m_2 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{6-3}{-8-0}$$

$$= \frac{3}{-8} \qquad m_2 = \frac{-3}{8}$$

Let θ is the acute angle between the line

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{\frac{-3}{4} - \left(\frac{-3}{8}\right)}{1 + \left(\frac{-3}{4}\right)\left(\frac{-3}{8}\right)}\right|$$

Scanned with CamScanner

$$= \left| \frac{-\frac{3}{4} + \frac{3}{8}}{\frac{1}{1} + \frac{9}{32}} \right|$$

$$= \left| \frac{\frac{-6+3}{8}}{\frac{32+9}{32}} \right|$$

$$\Rightarrow \left| \frac{\frac{-3}{8}}{\frac{41}{32}} \right| \Rightarrow \left| \frac{-3}{\cancel{8}} \times \frac{\cancel{32}^{4}}{41} \right|$$

$$\Rightarrow \tan\theta = \left| \frac{-12}{41} \right|$$

$$\tan\theta = \frac{12}{41}$$

$$\theta = \tan^{-1}\left(\frac{12}{41}\right)$$

Ques3. Find the angle between the line $(-2+\sqrt{3})x + y + 9 = 0$ and $(2+\sqrt{3})x - y + 20 = 0$

Ans Let m_1 = slope of line $(-2+\sqrt{3})x + y + 9 = 0$

$$= -\left(\frac{-2+\sqrt{3}}{1}\right)$$

$$m_1 = 2 - \sqrt{3}$$

m_2 = slope of line $(2+\sqrt{3})x - y + 20 = 0$

$$= -\left(\frac{2+\sqrt{3}}{-1}\right)$$

$$m_2 = 2 + \sqrt{3}$$

Let θ is the acute angle between the

Scanned with CamScanner

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{2-\sqrt{3} - 2+\sqrt{3}}{1+(2-\sqrt{3})(2+\sqrt{3})}\right|$$

$$= \left|\frac{-2\sqrt{3}}{1+4-3}\right|$$

$$= \left|\frac{-2\sqrt{3}}{2}\right|$$

$$= |-\sqrt{3}|$$

$$\tan\theta = \sqrt{3}$$

$$\theta = \sqrt{3}$$

$$\theta = 60^\circ$$

Q.4. Find the angle between the line $x + \sqrt{3}y - 8 = 0$ and $x - \sqrt{3}y + 2 = 0$

Ans Let m_1 = slope of line $x + \sqrt{3}y - 8 = 0$

$$m_1 = \frac{-1}{\sqrt{3}}$$

Let m_2 = slope of line $x - \sqrt{3}y + 2 = 0$

$$m_2 = \frac{-1}{-\sqrt{3}}$$

$$m_2 = \frac{1}{\sqrt{3}}$$

Let θ is the acute angle between line

$$\therefore \tan\theta \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{\frac{-1}{\sqrt{3}} - \frac{1}{\sqrt{3}}}{1+\left(\frac{-1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right)}\right|$$

Scanned with CamScanner

(191)

Page No.
Date

$$= \left| \frac{\frac{-1-1}{\sqrt{3}}}{1-\frac{1}{3}} \right|$$

$$= \left| \frac{\frac{-2}{\sqrt{3}}}{\frac{3-1}{3}} \right|$$

$$= \left| \frac{\frac{-2}{\sqrt{3}}}{\frac{2}{3}} \right|$$

$$= \left| \frac{-2}{\sqrt{3}} \times \frac{3}{2} \right|$$

$$= \left| \frac{-3}{\sqrt{3}} \right|$$

$$= \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$$

$$\tan\theta = \sqrt{3}$$

$$\theta = \sqrt{3}$$

$$\theta = 60^\circ$$

Scanned with CamScanner