**Differentiation**

**Increment**: Increment is the quantity by which the value of variable changes. It may be positive or negative. e.g. suppose the value of a variable  changes from 5 to 5.3 then 0.3 is the increment in . Similarly, if the value of variable  changes from 5 to 4.5 then -0.5 is the increment in .

Usually represents the increment in , represents the increment in , represents the increment in  etc.

**Derivative or Differential Co-efficient**: If  is a function of . Let  be the increment in  and  be the corresponding increment in , then  is called the derivative or differential co-efficient of  with respect to  and is dented by 

i.e. 

**Differentiation**:

Let  (1)

Let  be the increment in  and  be the corresponding increment in , then

 (2)

Subtracting equation (1) from equation (2), we get

 



Dividing both sides by , we get



Taking limit  on both sides, we get



If this limit exists, we write it as



where .

This is called the differentiation or derivative of the function  with respect to .

**Notations**: The first order derivative of the function  with respect to  can be represented in the following ways:



Similarly, the first order derivative of  with respect to  can be represented as:



**Some Properties of Differentiation**:

If  are  differentiable functions, then

1.  where $K$ is some constant.
2.  where $K$ is some constant.
3. 
4. 
5. 

This property is known as Product Rule of differentiation.

1. provided that

This property is known as Quotient Rule of differentiation.

**Some Basic Formulas of Differentiation**:

1.  this is known as power formula, here  is any real number.
2. 
3. 
4. 
5. 
6. 
7. 
8. 
9. 
10. 
11. 

**Q.1.** Differentiate with respect to .

**Sol.** Given that 

 Differentiating it with respect to , we get

 

**Q.2.** Differentiate with respect to .

**Sol.** Given that 

 Differentiating it with respect to , we get

 

 

**Q.3.** Differentiate with respect to .

**Sol.** Given that 

 Differentiating it with respect to , we get

 

 

**Q.4.** Differentiate with respect to .

**Sol.** Given that 

 Differentiating it with respect to , we get

 

 

**Q.5.** Differentiate with respect to .

**Sol.** Given that 

 Differentiating it with respect to , we get

 

 

**Q.6.** Differentiate with respect to .

**Sol.** Given that 

 Differentiating it with respect to , we get

 

 

**Chain Rule**: If  and  are two differentiable functions then

 

So, we may generalize our basic formulas as:

1.  here  is any real number.
2.  etc.

**Questions based on Chain Rule:**

**Q.7.** Differentiate with respect to .

**Sol.** Given that 

 Differentiating it with respect to , we get

 

 

**Q.8.** Differentiate with respect to .

**Sol.** Given that 

 Differentiating it with respect to , we get

 

**Q.9.** Differentiate with respect to .

**Sol.** Given that 

 Differentiating it with respect to , we get

 

**Questions based on Product Rule:**

**Q.11.** Differentiate with respect to .

**Sol.** Given that 

 Differentiating it with respect to , we get

 

 

**Q.12.** Differentiate with respect to .

**Sol.** Given that 

 Differentiating it with respect to , we get

 

 

 

 

**Questions based on Quotient Rule:**

**Q.13.** Differentiate with respect to .

**Sol.** Given that 

 Differentiating it with respect to , we get

 

 

**Q.14.** Differentiate with respect to .

**Sol.** Given that 

 Differentiating it with respect to , we get

 

 

 

 

 

**Logarithmic Differentiation :**

Let  and  are two differentiable function and 

To differentiate , first we take logarithm of :





Differentiating it with respect to , we get

 









**Questions based on Derivative of** $f(x)^{g(x)}$ **or Logarithmic Differentiation :**

**Q.15.** Differentiate with respect to .

**Sol.** Given that 

 Taking logarithm on both sides, we get





Differentiating it with respect to , we get

 











**Q.16.** Differentiate with respect to .

**Sol.** Given that 

 Taking logarithm on both sides, we get





Differentiating it with respect to , we get

 









**Questions based on Derivative of Infinite Series form :**

**Q.17.** Differentiate with respect to .

**Sol.** Let 

 



Differentiating it with respect to , we get

 









**Q.18.** Differentiate $\cos(x)^{\cos(x) ^{\cos(x) . . .}}$ with respect to .

**Sol.** Let $y=\cos(x)^{\cos(x) ^{\cos(x) . . .}}$

 

Taking logarithm on both sides, we get

 

 

Differentiating it with respect to , we get

 











**Successive Differentiation or Higher Order Derivative:**

Let  be a differentiable function, then  represents the first order derivative of  with respect to .If we may further differentiate it i.e.  ,then it is called second order derivative of  with respect to . Some other way to represent second order derivative of  with respect to :  .

So, successive derivatives of  with respect to can be represented as

 

**Q.19.** If , find .

**Ans.** Given that 

Differentiating with respect to  , we get

 



Again differentiating with respect to  , we get

 



**Q.20.** If , find .

**Ans.** Given that 

Differentiating with respect to  , we get

 







Again differentiating with respect to  , we get

 







