**Differentiation**

**Increment**: Increment is the quantity by which the value of variable changes. It may be positive or negative. e.g. suppose the value of a variable  changes from 5 to 5.3 then 0.3 is the increment in . Similarly, if the value of variable  changes from 5 to 4.5 then -0.5 is the increment in .

Usually represents the increment in , represents the increment in , represents the increment in  etc.

**Derivative or Differential Co-efficient**: If  is a function of . Let  be the increment in  and  be the corresponding increment in , then  is called the derivative or differential co-efficient of  with respect to  and is dented by 

i.e. 

**Differentiation**:

Let  (1)

Let  be the increment in  and  be the corresponding increment in , then

 (2)

Subtracting equation (1) from equation (2), we get





Dividing both sides by , we get



Taking limit  on both sides, we get



If this limit exists, we write it as



where .

This is called the differentiation or derivative of the function  with respect to .

**Notations**: The first order derivative of the function  with respect to  can be represented in the following ways:



Similarly, the first order derivative of  with respect to  can be represented as:



**Some Properties of Differentiation**:

If  are  differentiable functions, then

1.  where is some constant.
2.  where is some constant.
3. 
4. 
5. 

This property is known as Product Rule of differentiation.

1. provided that

This property is known as Quotient Rule of differentiation.

**Some Basic Formulas of Differentiation**:

1.  this is known as power formula, here  is any real number.
2. 
3. 
4. 
5. 
6. 
7. 
8. 
9. 
10. 
11. 

**Q.1.** Differentiate with respect to .

**Sol.** Given that 

Differentiating it with respect to , we get



**Q.2.** Differentiate with respect to .

**Sol.** Given that 

Differentiating it with respect to , we get





**Q.3.** Differentiate with respect to .

**Sol.** Given that 

Differentiating it with respect to , we get





**Q.4.** Differentiate with respect to .

**Sol.** Given that 

Differentiating it with respect to , we get





**Q.5.** Differentiate with respect to .

**Sol.** Given that 

Differentiating it with respect to , we get





**Q.6.** Differentiate with respect to .

**Sol.** Given that 

Differentiating it with respect to , we get





**Chain Rule**: If  and  are two differentiable functions then



So, we may generalize our basic formulas as:

1.  here  is any real number.
2.  etc.

**Questions based on Chain Rule:**

**Q.7.** Differentiate with respect to .

**Sol.** Given that 

Differentiating it with respect to , we get





**Q.8.** Differentiate with respect to .

**Sol.** Given that 

Differentiating it with respect to , we get



**Q.9.** Differentiate with respect to .

**Sol.** Given that 

Differentiating it with respect to , we get



**Questions based on Product Rule:**

**Q.11.** Differentiate with respect to .

**Sol.** Given that 

Differentiating it with respect to , we get





**Q.12.** Differentiate with respect to .

**Sol.** Given that 

Differentiating it with respect to , we get









**Questions based on Quotient Rule:**

**Q.13.** Differentiate with respect to .

**Sol.** Given that 

Differentiating it with respect to , we get





**Q.14.** Differentiate with respect to .

**Sol.** Given that 

Differentiating it with respect to , we get











**Logarithmic Differentiation :**

Let  and  are two differentiable function and 

To differentiate , first we take logarithm of :





Differentiating it with respect to , we get











**Questions based on Derivative of or Logarithmic Differentiation :**

**Q.15.** Differentiate with respect to .

**Sol.** Given that 

Taking logarithm on both sides, we get





Differentiating it with respect to , we get













**Q.16.** Differentiate with respect to .

**Sol.** Given that 

Taking logarithm on both sides, we get





Differentiating it with respect to , we get











**Questions based on Derivative of Infinite Series form :**

**Q.17.** Differentiate with respect to .

**Sol.** Let 





Differentiating it with respect to , we get











**Q.18.** Differentiate with respect to .

**Sol.** Let



Taking logarithm on both sides, we get





Differentiating it with respect to , we get













**Successive Differentiation or Higher Order Derivative:**

Let  be a differentiable function, then  represents the first order derivative of  with respect to .If we may further differentiate it i.e.  ,then it is called second order derivative of  with respect to . Some other way to represent second order derivative of  with respect to :  .

So, successive derivatives of  with respect to can be represented as



**Q.19.** If , find .

**Ans.** Given that 

Differentiating with respect to  , we get





Again differentiating with respect to  , we get





**Q.20.** If , find .

**Ans.** Given that 

Differentiating with respect to  , we get









Again differentiating with respect to  , we get









