Point

Cartesian Plane: Let XOX' and YOY' be two perpendicular lines. 'O' be their intersecting point called origin. XOX' is horizontal line called X-axis and YOY' is vertical line called Y-axis. The plane made by these axes is called Cartesian plane or coordinate plane.

The axes divide the plane into four parts called quadrant:

1st quadrant, 2nd quadrant, 3rd quadrant and 4th quadrant

As shown in the figure. OX is known as positive direction

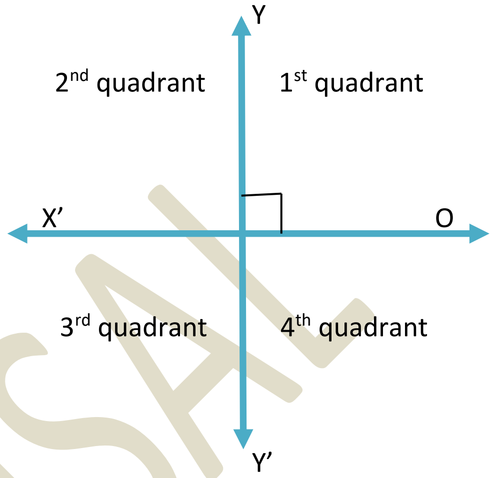
of X-axis and OX' is known as negative direction of X-axis. X

Similarly, OY is known as positive direction of Y-axis and

OY' is known as negative direction of Y-axis.

The axes XOX' and YOY' are together known as rectangular

axes or coordinate axes.



Point: A point is a mark of location on a plane. Is has no dimension i.e. no length, no breadth and no height. For example, tip of pencil, toothpick etc. A point in a plane is represented as an ordered pair of real numbers called coordinates of point.

The perpendicular distance of a point from the Y-axis is $P(x, y)$

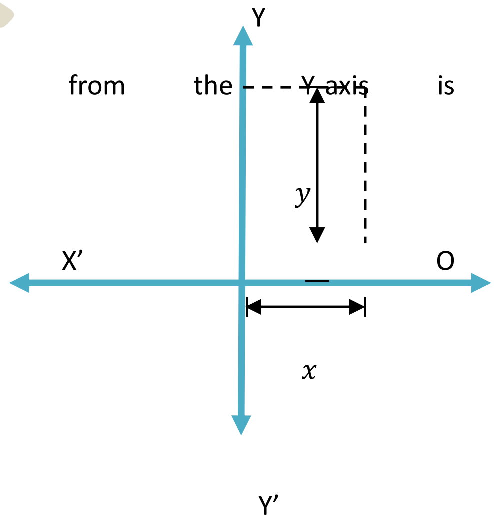
Called abscissa or x-coordinate and the perpendicular

distance of a point from the X-axis is Called ordinate or X

y-coordinate. If $P(x, y)$ be any point in the plane then x

is the abscissa of the point P and y is the ordinate of the

point P.



Note: **(i)** If distance along X-axis is measured to the right of Y-axis then it is positive and if it is measured to the left of Y-axis then it is negative.

(ii) If distance along Y-axis is measured to the above of X-axis then it is positive and if it is measured to the below of X-axis then it is negative.

(iii) The coordinates of origin 'O' are $(0,0)$.

(iv) A point on X-axis is represented as $(x, 0)$ i.e. ordinate is zero.

(v) A point on Y-axis is represented as $(0, y)$ i.e. abscissa is zero.

(vi) In 1st quadrant $x > 0$ and $y > 0$

In 2nd quadrant $x < 0$ and $y > 0$

In 3rd quadrant $x < 0$ and $y < 0$

In 4th quadrant $x > 0$ and $y < 0$.

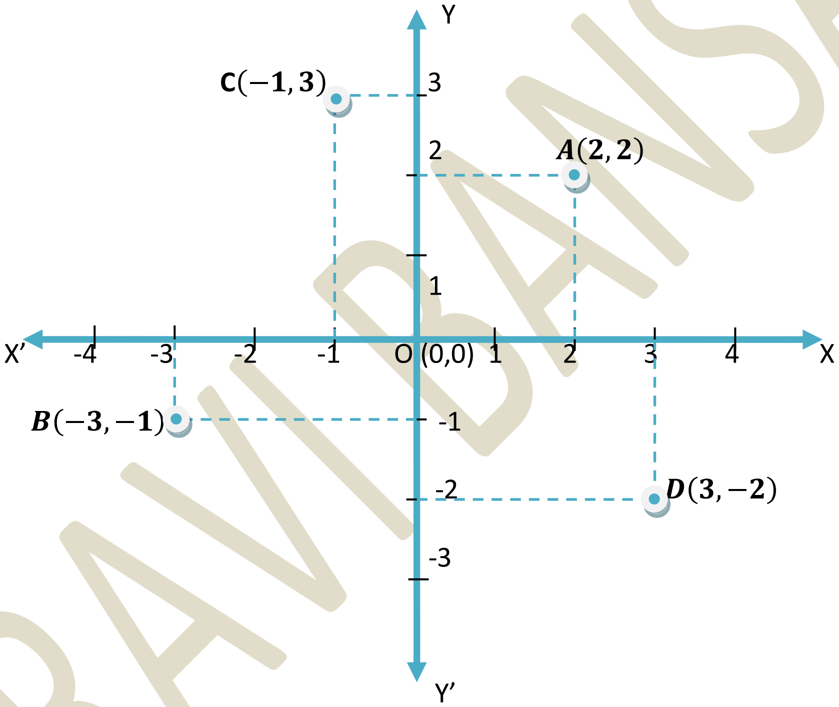
Distance between Two Points in a Plane: Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be any two points in a plane then the distance between them is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Q.1. Plot the following points and find the quadrant in which they lie:

(i) $A(2,2)$ **(ii)** $B(-3,-1)$ **(iii)** $C(-1,3)$ **(iv)** $D(3,-2)$

Sol.



By graph it is clear that

(i) Point $A(2,2)$ lies in the 1st quadrant.
(ii) Point $B(-3,-1)$ lies in the 3rd quadrant.
(iii) Point $C(-1,3)$ lies in the 2nd quadrant.
(iv) Point $D(3,-2)$ lies in the 4th quadrant.

Q.2. Without plotting, find the quadrant in which the following points lie:

(i) $A(2,-3)$ **(ii)** $B(-5,-6)$

Sol.

(i) The given point is $A(2,-3)$

Here X-coordinate = 2, which is positive and Y-coordinate = -3, which is negative.

Hence the point $A(2,-3)$ lies in 4th quadrant.

(ii) The given point is $B(-5,-6)$

Here X-coordinate = -5, which is negative and Y-coordinate = -6, which is also negative.

Hence the point $B(-5,-6)$ lies in 3rd quadrant.

Q.3. Find the distance between the following pairs of points:

(i) $(\mathbf{0},\mathbf{5})$, $(\mathbf{3},\mathbf{6})$ **(ii)** $(-\mathbf{1},\mathbf{2})$, $(\mathbf{4},\mathbf{3})$

Sol.

(i) Let **A** represents the point $(0,5)$ and **B** represents the point $(3,6)$.

So, the distance between **A** and **B** is:

$$AB=\sqrt{(3-0)^2+(6-5)^2}$$

$$=\sqrt{(3)^2+(1)^2}=\sqrt{9+1}=\sqrt{10}\ units$$

(ii) Let **A** represents the point $(-1,2)$ and **B** represents the point $(4,3)$.

So, the distance between **A** and **B** is:

$$AB=\sqrt{(4-(-1))^2+(3-2)^2}$$

$$=\sqrt{(5)^2+(1)^2}=\sqrt{25+1}=\sqrt{26}\ units$$

Q.4. Using distance formula, prove that the triangle formed by the points $\boldsymbol{A}(\mathbf{4},\mathbf{0})$, $\boldsymbol{B}(-\mathbf{1},-\mathbf{1})$ and $\boldsymbol{C}(\mathbf{3},\mathbf{5})$ is an isosceles triangle.

Sol. Given that vertices of the triangle are $A(4,0)$, $B(-1,-1)$ and $C(3,5)$.

To find the length of edges of the triangle, we will use the distance formula:

Distance between **A** and **B** is

$$AB=\sqrt{(4-(-1))^2+(0-(-1))^2}$$

$$=\sqrt{(5)^2+(1)^2}=\sqrt{25+1}=\sqrt{26}\ units$$

Distance between **B** and **C** is

$$BC = \sqrt{(-1-3)^2 + (-1-5)^2}$$

$$= \sqrt{(-4)^2 + (-6)^2} = \sqrt{16+36} = \sqrt{52}\ units$$

Distance between **A** and **C** is

$$AC = \sqrt{(4-3)^2 + (0-5)^2}$$

$$= \sqrt{(1)^2 + (-5)^2} = \sqrt{1+25} = \sqrt{26}\ units$$

We can see that $AB = AC \neq BC$

Hence the triangle formed by the points $A(4,0)$, $B(-1,-1)$ and $C(3,5)$ is an isosceles triangle.

Q.5. Using distance formula, prove that the triangle formed by the points $\boldsymbol{A(0,0)}$**,** $\boldsymbol{B(0,2)}$ and $\boldsymbol{C(\sqrt{3},1)}$ is an equilateral triangle.

Sol. Given that vertices of the triangle are $A(0,0)$, $B(0,2)$ and $C(\sqrt{3},1)$.

To find the length of edges of the triangle, we will use the distance formula:

Distance between **A** and **B** is

$$AB = \sqrt{(0-0)^2 + (0-2)^2}$$

$$= \sqrt{(0)^2 + (-2)^2} = \sqrt{0+4} = \sqrt{4} = 2\,units$$

Distance between **B** and **C** is

$$BC = \sqrt{\left(0-\sqrt{3}\right)^2 + (2-1)^2}$$

$$= \sqrt{\left(-\sqrt{3}\right)^2 + (1)^2} = \sqrt{3+1} = \sqrt{4} = 2\,units$$

Distance between **A** and **C** is

$$AC = \sqrt{\left(0-\sqrt{3}\right)^2 + (0-1)^2}$$

$$= \sqrt{\left(-\sqrt{3}\right)^2 + (-1)^2} = \sqrt{3+1} = \sqrt{4} = 2\,units$$

We can see that $AB = BC = AC$

Hence the triangle formed by the points $A(0,0)$, $B(0,2)$ and $C(\sqrt{3},1)$ is an equilateral triangle.

Midpoint between two points: If (x_1, y_1) and (x_2, y_2) be any two points then midpoint between these points is given by $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Q.6. Find the mid points between the following pairs of points:

(i) $(\mathbf{2}, \mathbf{3}), (\mathbf{8}, \mathbf{5})$ **(ii)** $(\mathbf{6}, \mathbf{3}), (\mathbf{6}, -\mathbf{9})$

Sol.

(i) The given points are $(2,3)$ and $(8,5)$.

So, the mid-point between these points is given by:

$$\left(\frac{2+8}{2}, \frac{3+5}{2}\right) = \left(\frac{10}{2}, \frac{8}{2}\right) = (5, 4)$$

(ii) The given points are $(6,3)$ and $(6,-9)$.

So, the mid-point between these points is given by:

$$\left(\frac{6+6}{2}, \frac{3+(-9)}{2}\right) = \left(\frac{12}{2}, \frac{3-9}{2}\right) = \left(\frac{12}{2}, \frac{-6}{2}\right) = (6, -3)$$

Q.7. If the mid-point between two points is $(3,5)$ and one point between them is $(-1,2)$, find the other point.

Sol. Let the required point is (a, b).

So, according to given statement $(3,5)$ is the mid-point of $(-1,2)$ and (a, b).

$$\Rightarrow \quad (3,5) = \left(\frac{-1+a}{2}, \frac{2+b}{2}\right)$$

$$\Rightarrow \quad \frac{-1+a}{2} = 3 \quad \& \quad \frac{2+b}{2} = 5$$

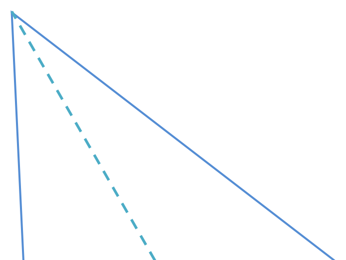
$$\Rightarrow \quad -1+a = 6 \quad \& \quad 2+b = 10$$

$$\Rightarrow \quad a = 7 \quad \& \quad b = 8$$

Hence the required point is $(7,8)$.

Centroid of a Triangle: The centroid of a triangle is the intersection point of the three medians of the triangle. In other words, the **average** of the three vertices of the triangle is called the centroid of the triangle.

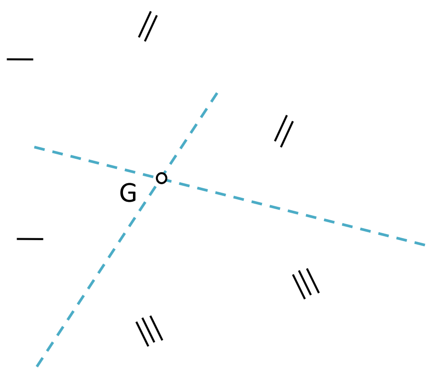
i.e. If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are three



vertices of a triangle then the centroid of the triangle is given by:

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

In this figure the point G is the centroid of the Triangle.



Q.8. Vertices of the triangles are given below, find the centroid of the triangles:

(i) $(5,2)$, $(5,4)$, $(8,6)$ **(ii)** $(4,-3)$, $(-4,8)$, $(5,7)$

Sol.

(i) The given vertices of the triangle are $(5,2)$, $(5,4)$ and $(8,6)$.

So, the centroid of the triangle is

$$\left(\frac{5+5+8}{3}, \frac{2+4+6}{3}\right)=\left(\frac{18}{3}, \frac{12}{3}\right)=(6,4)$$

(ii) The given vertices of the triangle are $(4,-3)$, $(-4,8)$ and $(5,7)$.

So, the centroid of the triangle is

$$\left(\frac{4-4+5}{3}, \frac{-3+8+7}{3}\right)=\left(\frac{5}{3}, \frac{12}{3}\right)=\left(\frac{5}{3}, 4\right)$$

Q.9. If centroid of the triangle is $(10,18)$ and two vertices of the triangle are $(1,-5)$ and $(3,7)$, find the third vertex of the triangle.

Sol. Let the required vertex of the triangle is (a,b).

So, according to given statement and definition of centroid, we get

$$\Rightarrow \quad (10,18)=\left(\frac{1+3+a}{3}, \frac{-5+7+b}{3}\right)$$

$$\Rightarrow \quad \frac{1+3+a}{3}=10 \quad \& \quad \frac{-5+7+b}{3}=18$$

$$\Rightarrow \quad 1+3+a=30 \quad \& \quad -5+7+b=54$$

$$\Rightarrow \quad a=26 \quad \& \quad b=52$$

Hence the required vertex of triangle is $(26,52)$.

<u>Area of a Triangle with given vertices</u>: If (x_1, y_1), (x_2, y_2) and (x_3, y_3) are vertices of a triangle then area of triangle is given by

$$\Delta = \pm\frac{1}{2}\left[(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_1 - x_1y_3)\right]$$

To remember this we can take help of figure given below:

$$\Delta = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix}$$

<u>Alternate Method</u>: We can find the area of triangle by determinant if all the three vertices are given. If (x_1, y_1), (x_2, y_2) and (x_3, y_3) are vertices of a triangle then area of triangle is given by

$$\Delta = \pm\frac{1}{2}\begin{vmatrix} x_1 - x_2 & x_1 - x_3 \\ y_1 - y_2 & y_1 - y_3 \end{vmatrix}$$

Note: **(i)** Area is always non-negative. So take the suitable sign that gives the non-negative value.

(ii) If $\Delta = 0$ then the three points don't form triangle and these are **collinear** points.

Q.10. Vertices of the triangles are given below, find the area of the triangles:

(i) $(\mathbf{3}, \mathbf{2})$, $(\mathbf{5}, \mathbf{4})$, $(\mathbf{7}, \mathbf{2})$ **(ii)** $(\mathbf{1}, \mathbf{3})$, $(\mathbf{-4}, \mathbf{5})$, $(\mathbf{3}, \mathbf{-4})$

Sol.

(i) The given vertices of the triangle are (3,2) , (5,4) and (7, 2).

Comparing these points with (x_1, y_1), (x_2, y_2) and (x_3, y_3) respectively, we get

$x_1 = 3, y_1 = 2, x_2 = 5, y_2 = 4, x_3 = 7, y_3 = 2$

So, area of the triangle is given by

$$\Delta = \pm\frac{1}{2}\left[(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_1 - x_1y_3)\right]$$

i.e. $$\Delta = \pm\frac{1}{2}\left[(3\times 4 - 5\times 2) + (5\times 2 - 7\times 4) + (7\times 2 - 3\times 2)\right]$$

$\Rightarrow$ $$\Delta = \pm\frac{1}{2}\left[(12 - 10) + (10 - 28) + (14 - 6)\right]$$

$\Rightarrow$ $$\Delta = \pm\frac{1}{2}\left[2 - 18 + 8\right] = \pm\frac{1}{2}\left[-8\right] = 4\, sq.units$$

Alternate Method

The given vertices of the triangle are $(3,2)$, $(5,4)$ and $(7,2)$.

Comparing these points with (x_1, y_1), (x_2, y_2) and (x_3, y_3) respectively, we get

$x_1 = 3, y_1 = 2, x_2 = 5, y_2 = 4, x_3 = 7, y_3 = 2$

So, area of the triangle is given by

$$\Delta = \pm\frac{1}{2}\begin{vmatrix} x_1 - x_2 & x_1 - x_3 \\ y_1 - y_2 & y_1 - y_3 \end{vmatrix}$$

i.e. $$\Delta = \pm\frac{1}{2}\begin{vmatrix} 3-5 & 3-7 \\ 2-4 & 2-2 \end{vmatrix}$$

$\Rightarrow$ $$\Delta = \pm\frac{1}{2}\begin{vmatrix} -2 & -4 \\ -2 & 0 \end{vmatrix}$$

$\Rightarrow$ $$\Delta = \pm\frac{1}{2}[(-2)(0)-(-2)(-4)]$$

$\Rightarrow$ $$\Delta = \pm\frac{1}{2}[0-8] = 4\,sq.units$$

(ii) The given vertices of the triangle are $(1,3)$, $(-4,5)$ and $(3,-4)$.

Comparing these points with (x_1, y_1), (x_2, y_2) and (x_3, y_3) respectively, we get

$x_1 = 1, y_1 = 3, x_2 = -4, y_2 = 5, x_3 = 3, y_3 = -4$

So, area of the triangle is given by

$$\Delta = \pm\frac{1}{2}[(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_1 - x_1y_3)]$$

i.e. $$\Delta = \pm\frac{1}{2}[\{1\times 5 - (-4)\times 3\} + \{-4\times(-4) - 3\times 5\} + \{3\times 3 - 1\times(-4)\}]$$

$\Rightarrow$ $$\Delta = \pm\frac{1}{2}[(5+12) + (16-15) + (9+4)]$$

$\Rightarrow$ $$\Delta = \pm\frac{1}{2}[17+1+13] = \pm\frac{1}{2}[31] = 15.5\,sq.units$$

Q.11. Prove that the triplet of points $(\mathbf{4}, \mathbf{7})$, $(\mathbf{0}, \mathbf{1})$ **,** $(\mathbf{2}, \mathbf{4})$ is collinear.

Sol. The points are $(4,7)$, $(0,1)$ and $(2,4)$.

Comparing these points with (x_1, y_1), (x_2, y_2) and (x_3, y_3) respectively, we get

$x_1 = 4, y_1 = 7, x_2 = 0, y_2 = 1, x_3 = 2, y_3 = 4$

So by formula of area of the triangle, we get

$$\Delta = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix}$$

i.e.

$$\Delta = \pm\frac{1}{2}\begin{vmatrix} 4 & 7 \\ 0 & 1 \\ 2 & 4 \\ 4 & 7 \end{vmatrix}$$

$\Rightarrow$ $$\Delta = \pm\frac{1}{2}[(4-0)+(0-2)+(14-16)]$$

$\Rightarrow$ $$\Delta = \pm\frac{1}{2}[4-2-2]=0$$

which shows that the given points are collinear.

Straight Line

Definition of Straight Line: A path traced by a point travelling in a constant direction is called a straight line.

OR

The shortest distance between two points is called a straight line.

General Equation of Straight Line: A straight line in XY plane has general form

$$ax + by + c = 0$$

where a is the coefficient of x, b is the coefficient of y and c is the constant term.

Note: **(i)** Any point (x_1, y_1) lies on the line $ax + by + c = 0$ if it satisfies the equations of the line **i.e.** if we substitute the values x_1 at the place of x and y_1 at the place of y in the equation of line, the result $ax_1 + by_1 + c$ becomes zero.

(ii) X-axis is usually represented horizontally and its equation is $y = 0$.

(iii) Y-axis is usually represented vertically and its equation is $x = 0$.

(iv) $x = k$ represents the line parallel to Y-axis, where k is some constant .

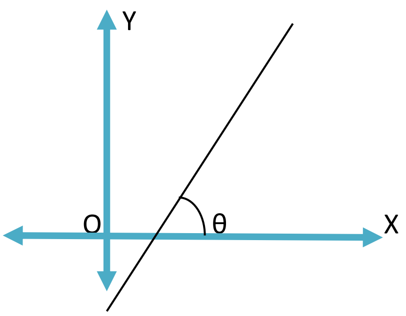
(v) $y = k$ represents the line parallel to X-axis, where k is some constant .

Slope of a Straight Line: Slope of straight line

Measures how slanted the line is relative to

the horizontal.

It is usually represented by $\boldsymbol{m}$.



To find Slope of a Straight Line:

(i) If a non-vertical line making an angle θ with positive X-axis then the slope m of the line is given by $m = tan\theta$.

(ii) If a non-vertical line passes through two points (x_1, y_1) and (x_2, y_2) then the slope m of the line is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$.

(iii) If equation of a straight line is $ax + by + c = 0$, then its slope m is given by $m = -\frac{a}{b}$.

Note: **(i)** Slop of a horizontal line is always zero **i.e.** slope of a line parallel to X-axis is zero as $m = tan0° = 0$.

(ii) Slop of a vertical line is always infinity **i.e.** slope of a line perpendicular to X-axis is infinity as $= tan90° = \infty$.

(iii) Let L_1 and L_2 represents two straight lines. Let m_1 and m_2 be slopes of L_1 and L_2 respectively. We say that L_1 and L_2 are parallel lines iff $m_1 = m_2$ **i.e.** slopes are equal. We say that L_1 and L_2 are perpendicular iff $m_1 . m_2 = -1$ **i.e.** product of slopes is equal to -1.

Q.1. Find the slope of the straight lines which make following angles:

(i) 45° **(ii) 120°**

with the positive direction of X-axis.

Sol.

(i) Let m be the slope of the straight line and θ be the angle which the straight line makes with the positive direction of X-axis.

Therefore $\theta = 45°$ and $m = tan\theta$

$\Rightarrow \quad m = \tan 45^\circ$

$\Rightarrow \quad m = 1$

which is the required slope.

(ii) Let m be the slope of the straight line and θ be the angle which the straight line makes with the positive direction of X-axis.

Therefore $\theta = 120°$ and $m = tan\theta$

$\Rightarrow \quad m = \tan 120^\circ$

$\Rightarrow \quad m = \tan\left(180^\circ - 60^\circ\right)$

$\Rightarrow \quad m = -\tan\left(60^\circ\right)$

$\Rightarrow \quad m = -\sqrt{3}$

which is the required slope.

Q.2. Find the slope of the straight lines which pass through the following pairs of points:

(i) $(\mathbf{2}, \mathbf{5})$ **,** $(\mathbf{6}, \mathbf{17})$ **(ii)** $(-\mathbf{8}, \mathbf{7})$ **,** $(\mathbf{3}, -\mathbf{5})$

Sol.

(i) Given that the straight line passes through the points (2,5) and (6,17).

Comparing these points with respectively (x_1, y_1) and (x_2, y_2) respectively, we get

$x_1 = 2$, $y_1 = 5$, $x_2 = 6$ and $y_2 = 17$

Let m be the slope of the straight line.

Therefore $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$\Rightarrow \quad m = \dfrac{17-5}{5-2} = \dfrac{12}{3}$

$\Rightarrow \quad m = 4$

which is the required slope.

(ii) Given that the straight line passes through the points $(-8,7)$ and $(3,-5)$.

Comparing these points with respectively (x_1, y_1) and (x_2, y_2) respectively, we get

$x_1 = -8$, $y_1 = 7$, $x_2 = 3$ and $y_2 = -5$
Let m be the slope of the straight line.

Therefore $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$\Rightarrow \quad m = \dfrac{-5-7}{3-(-8)} = \dfrac{-12}{3+8}$

$\Rightarrow \quad m = -\dfrac{12}{11}$

which is the required slope.

Q.3. Find the slopes of the following straight lines:

(i) $2x + 4y + 5 = 0$ **(ii)** $x - 3y + 9 = 0$

Sol.

(i) Given that equation of the straight line is $2x + 4y + 5 = 0$.
Comparing this equation with $ax + by + c = 0$, we get
$a = 2$, $b = 4$ and $c = 5$
Let m be the slope of given straight line.

Therefore, $m = -\dfrac{a}{b}$

$\Rightarrow \quad m = -\dfrac{2}{4}$

$\Rightarrow \quad m = -\dfrac{1}{2}$

which is the required slope.

(ii) Given that equation of the straight line is $-3y + 9 = 0$.
Comparing this equation with $ax + by + c = 0$, we get
$a = 1$, $b = -3$ and $c = 9$
Let m be the slope of given straight line.

Therefore, $m = -\left(\dfrac{1}{-3}\right)$

$\Rightarrow \quad m = \dfrac{1}{3}$

which is the required slope.

Q.4. Find the equation of straight line which is parallel to X-axis passes through $(1,5)$.

Sol. Equation of straight line parallel to X-axis is given by

$$y = k \tag{4.1}$$

Given that the straight line passes through the point $(1,5)$.

Put $x = 1$ and $y = 5$ in (4.1), we get

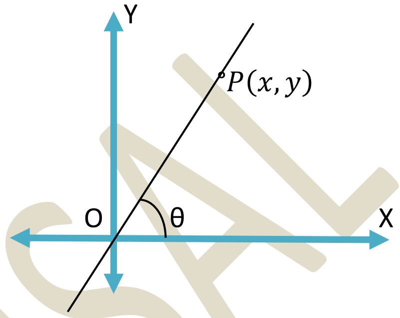
$$5 = k$$

So, $y = 5$ be the required equation of straight line.

<u>**Equation of Straight Line Passing Through Origin**</u>:

If a non-vertical line passes through origin and m be its slope. $P(x, y)$ be any point on the line, then equation of straight line is

$$\boldsymbol{y = mx}$$



Q.5. Find the equation of straight line having slope equal to 5 and passes through origin.

Sol. Let m be the slope of required line. Therefore $m = 5$.

Also it is given that the required line passes through the origin.

We know that equation of straight line passes through origin is $y = mx$, where m be the slope of the line.

So, $y = 5x$ be the required equation of straight line.

Q.6. Find the equation of straight line which passes through origin and makes an angle $60°$ with the positive direction of X-axis.

Sol. Let m be the slope of required line.

Therefore $m = \tan 60^\circ$

$$\Rightarrow \quad m = \sqrt{3}$$

Also it is given that the required line passes through the origin.

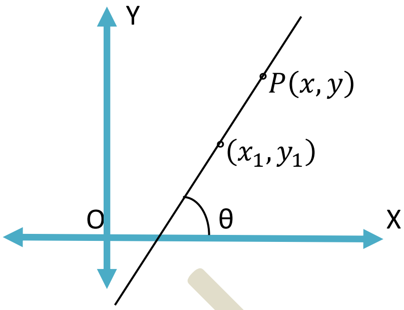
We know that equation of straight line passes through origin is $y = mx$, where m be the slope of the line.

So, $y = \sqrt{3}x$ be the required equation of straight line.

Equation of Straight Line in Point-Slope form:

Let a non-vertical line passes through a point (x_1, y_1) and m be its slope. $P(x, y)$ be any point on the line, then equation of straight line is

$$\boldsymbol{y - y_1 = m(x - x_1)}$$



Q.7. Find the equation of straight line having slope equal to 9 and passes through the point $(1,5)$.

Sol. Let m be the slope of required line. Therefore $m = 9$.

Also it is given that the required line passes through the point $(1,5)$.

We know that equation of straight line in point slope form is $y - y_1 = m(x - x_1)$.

$\Rightarrow \quad y - 5 = 9(x - 1)$

$\Rightarrow \quad y - 5 = 9x - 9$

$\Rightarrow \quad 9x - y - 9 + 5 = 0$

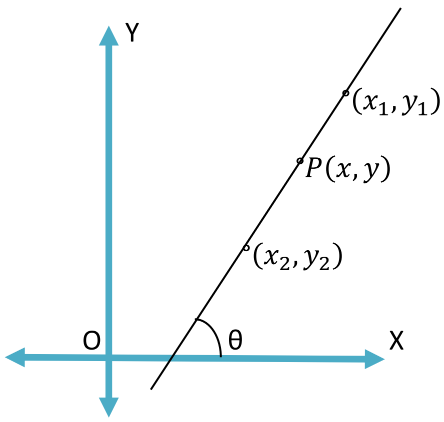
$\Rightarrow \quad 9x - y - 4 = 0$

which is the required equation of straight line.

Equation of Straight Line in Two Points form:

Let a non-vertical line passes through two points (x_1, y_1) and (x_2, y_2). $P(x, y)$ be any point on the line, then equation of straight line is

$$\boldsymbol{y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)}$$



Q.8. Find the equation of straight line passes through the points $(2, -2)$ and $(0,6)$.

Sol. Given that the straight line passes through the points $(2, -2)$ and $(0,6)$.

Comparing these points with (x_1, y_1) and (x_2, y_2) respectively, we get

$x_1 = 2$, $y_1 = -2$, $x_2 = 0$ and $y_2 = 6$.

We know that equation of straight line in two points slope form is

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1).$$

$$\Rightarrow \quad y - (-2) = \left(\frac{6 - (-2)}{0 - 2}\right)(x - 2)$$

$$\Rightarrow \quad y + 2 = \left(\frac{6 + 2}{-2}\right)(x - 2)$$

$$\Rightarrow \quad y + 2 = -4(x - 2)$$

$$\Rightarrow \quad y + 2 = -4x + 8$$

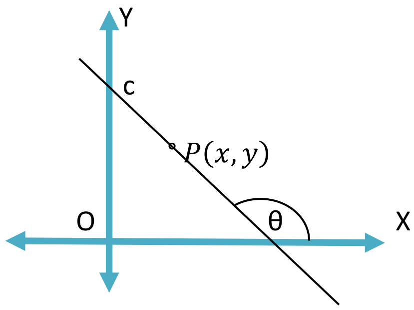
$$\Rightarrow \quad 4x + y - 6 = 0$$

which is the required equation of straight line.

Equation of Straight Line in Slope-Intercept form:

Let a non-vertical line having slope m and its y-intercept is equal to c. $P(x, y)$ be any point on the line, then equation of straight line is

$$\boldsymbol{y = mx + c}$$



Note: **(i)** If intercept c is given above the X-axis or above the origin then it is positive.

(ii) If intercept c is given below the X-axis or below the origin then it is negative.

Q.9. Find the equation of straight line having slope 3 and cuts of an intercept -2 on Y-axis.

Sol. Given that the slope m of straight line is 3 and Y-intercept is -2 i.e. $c = -2$.

We know that equation of straight line in slope-intercept form is

$$y = mx + c$$

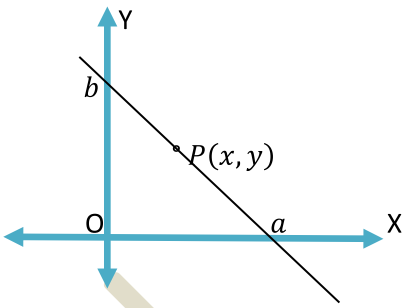
$$\Rightarrow \quad y = 3x - 2$$

$$\Rightarrow \quad 3x - y - 2 = 0$$

which is the required equation of straight line.

Equation of Straight Line in Intercept form:

Let a non-vertical line having intercepts a and b on X-axis and Y-axis respectively. $P(x, y)$ be any point on the line, then equation of straight line is



$$\frac{x}{a} + \frac{y}{b} = 1$$

Q.10. Find the equation of straight line which makes intercepts 2 and 5 on X-axis and Y-axis respectively.

Sol. Given that X-intercept is 2 and Y-intercept is 5

i.e. $a = 2$ and $b = 5$

We know that equation of straight line in Intercept form is

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\Rightarrow \quad \frac{x}{2} + \frac{y}{5} = 1$$

$$\Rightarrow \quad \frac{5x + 2y}{10} = 1$$

$$\Rightarrow \quad 5x + 2y = 10$$

$$\Rightarrow \quad 5x + 2y - 10 = 0$$

which is the required equation of straight line.

Q.11. Find the equation of straight line which passes through $(1, -4)$ and makes intercepts on axes which are equal in magnitude and opposite in sign.

Sol. Let the intercepts on the axes are p and $-p$

i.e. $a = p$ and $b = -p$

We know that equation of straight line in Intercept form is

$$\frac{x}{a} + \frac{y}{b} = 1$$

$\Rightarrow \qquad \frac{x}{p}+\frac{y}{-p}=1$

$\Rightarrow \qquad \frac{x}{p}-\frac{y}{p}=1$

$\Rightarrow \qquad \frac{x-y}{p}=1$

$\Rightarrow \qquad x-y=p$ (11.1)

Given that this line passes through $(1,-4)$.

Therefore put $x=1$ and $y=-4$ in (11.1), we get

$1-(-4)=p$

$\Rightarrow \qquad p=5$

Using this value in (11.1), we get

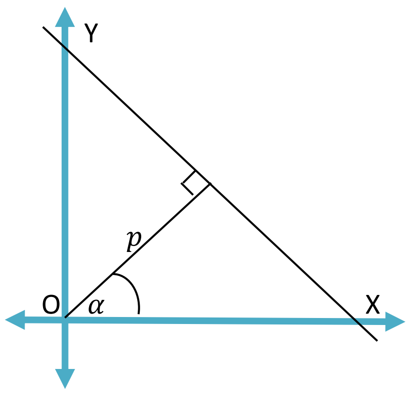
$x-y=5$

$\Rightarrow \qquad x-y=5$

which is the required equation of straight line.

Equation of Straight Line in Normal form:

Let p be the length of perpendicular from the origin to the straight line and α be the angle which this perpendicular makes with the positive direction of X-axis. (x, y) be any point on the line, then equation of straight line is



$$p = x\,cos\alpha + y\,sin\alpha$$

Q.12. Find the equation of straight line such that the length of perpendicular from the origin to the straight line is 2 and the inclination of this perpendicular to the X-axis is 120°.

Sol. We know that equation of straight line in Normal form is

$$x\cos\alpha + y\sin\alpha = p \qquad (12.1)$$

where p be the length of perpendicular from the origin to the straight line and α be the angle which this perpendicular makes with the positive direction of X-axis.

Here $p=2$ and $\alpha=120^\circ$.

Put these values in (12.1), we get

$$x\cos 120^\circ + y\sin 120^\circ = 2$$

$$\Rightarrow \quad x\cos\left(180^\circ - 60^\circ\right) + y\sin\left(180^\circ - 60^\circ\right) = 2$$

$$\Rightarrow \quad -x\cos\left(60^\circ\right) + y\sin\left(60^\circ\right) = 2$$

$$\Rightarrow \quad -x\left(\frac{1}{2}\right) + y\left(\frac{\sqrt{3}}{2}\right) = 2$$

$$\Rightarrow \quad \frac{-x+\sqrt{3}\,y}{2} = 2$$

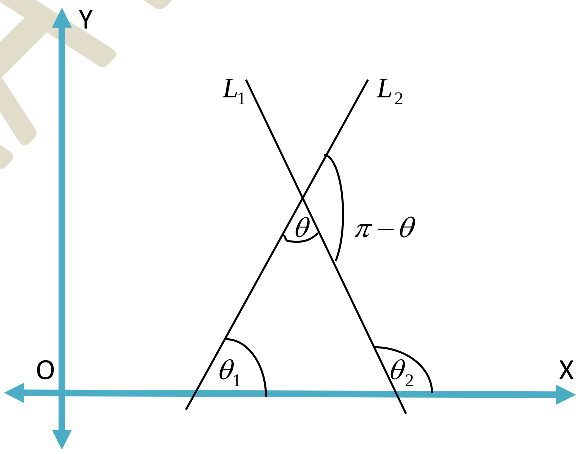
$$\Rightarrow \quad -x+\sqrt{3}\,y = 4$$

$$\Rightarrow \quad -x+\sqrt{3}\,y - 4 = 0$$

which is the required equation of straight line.

<u>Angle Between Two Straight Lines</u>: Two intersecting lines always intersects at two angles in which one angle is acute angle and other angle is obtuse angle. The sum of both the angles is 180° i.e. they are supplementary to each other. For example, if one angle between intersecting lines is 60° then other angle is $180^\circ - 60^\circ = 120^\circ$.

Generally, we take acute angle as the angle between the lines.



Let L_1 & L_2 be straight lines and m_1 & m_2 be their slopes respectively.

Let θ_1 & θ_2 be the angles which L_1 & L_2 make with positive X-axis respectively.

Therefore $m_1 = \tan(\theta_1)$ & $m_2 = \tan(\theta_2)$.

Let θ be the acute angle between lines, then

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \qquad or \qquad \tan\theta = \left|\frac{m_2 - m_1}{1 + m_2 m_1}\right|$$

Q.13. Find the acute angle between the lines whose slopes are 1 and 0.

Sol. Given that slopes of lines are 1 and 0.

Let $m_1 = 1$ and $m_2 = 0$.

Also let θ be the acute angle between lines.

Therefore, $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\Rightarrow \quad \tan\theta = \left|\frac{1-0}{1+(1)(0)}\right|$

$\Rightarrow \quad \tan\theta = \left|\frac{1}{1+0}\right|$

$\Rightarrow \quad \tan\theta = 1$

$\Rightarrow \quad \tan\theta = \tan\left(\frac{\pi}{4}\right)$

$\Rightarrow \quad \theta = \frac{\pi}{4}$

which is the required acute angle.

Q.14. Find the acute angle between the lines whose slopes are $2+\sqrt{3}$ and $2-\sqrt{3}$.

Sol. Given that slopes of lines are $2+\sqrt{3}$ and $2-\sqrt{3}$.

Let $m_1 = 2+\sqrt{3}$ and $m_2 = 2-\sqrt{3}$.

Also let θ be the acute angle between lines.

Therefore, $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\Rightarrow \qquad \tan\theta = \left| \dfrac{(2+\sqrt{3})-(2\ \sqrt{3})}{1+(2+\sqrt{3})(2-\sqrt{3})} \right|$

$\Rightarrow \qquad \tan\theta = \left| \dfrac{2+\sqrt{3}-2+\sqrt{3}}{1+(4-3)} \right|$

$\Rightarrow \qquad \tan\theta = \left| \dfrac{2\sqrt{3}}{2} \right|$

$\Rightarrow \qquad \tan\theta = \sqrt{3}$

$\Rightarrow \qquad \tan\theta = \tan\left(\dfrac{\pi}{3}\right)$

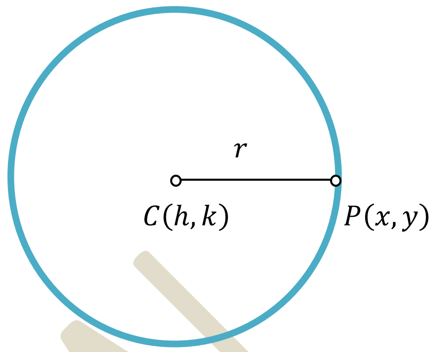
$\Rightarrow \qquad \theta = \dfrac{\pi}{3}$

which is the required acute angle.

Circle

Circle: Circle is the locus of a point which moves in a plane such that its distance from a fixed point always remains constant. The fixed point is called the centre of the circle and the constant distance is called the radius of the circle.

In figure, $C(h,k)$ be the centre of the circle, r be the radius of the circle and $P(x,y)$ be the moving point on the circumference of the circle.



Standard Equation of the Circle: Let $C(h,k)$ be the centre of the circle, r be the radius of the circle and $P(x,y)$ be any point on the circle, then equation of circle is

$$(x-h)^2+(y-k)^2=r^2 \qquad (1)$$

which is known as standard equation of circle. This is also known as central form of equation of circle.

General Equation of Circle: An equation of the form $x^2+y^2+2gx+2fy+c=0$ is known as general equation of circle, where g , f and c are arbitrary constants.

To Convert General Equation into Standard Equation: Let the general equation of circle is

$$x^2+y^2+2gx+2fy+c=0$$

(2)

$$\Rightarrow \quad (x^2+2gx)+(y^2+2fy)+c=0$$

$$\Rightarrow \quad (x^2+2gx+g^2-g^2)+(y^2+2fy+f^2-f^2)+c=0$$

$$\Rightarrow \quad (x+g)^2-g^2+(y+f)^2-f^2+c=0$$

$$\Rightarrow \quad (x+g)^2+(y+f)^2=f^2+g^2-c$$

$$\Rightarrow \quad (x+g)^2+(y+f)^2=\left(\sqrt{g^2+f^2-c}\right)^2$$

which is the required standard form.

Comparing it with $(x-h)^2+(y-k)^2=r^2$, we get

$h=-g$, $k=-f$ and $r=\sqrt{g^2+f^2-c}$.

Hence, centre of given circle (2) is $(-g,-f)$ and radius is $\sqrt{g^2+f^2-c}$.

We observe that the centre of circle (2) is $\left(-\frac{1}{2}\times Coefficient\ of\ x, -\frac{1}{2}\times Coefficient\ of\ y\right)$.

Equation of Circle in Diametric Form: Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be the end points of diameter of a circle then equation of circle is given by

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

Q.1. Find the centre and radius of the following circles:

(i) $\boldsymbol{x^2+y^2+2x+4y-4=0}$ **(ii)** $\boldsymbol{2x^2+2y^2+5x-6y+2=0}$

Sol.

(i) Given that equation of circle is

$x^2+y^2+2x+4y-4=0$ (1.1.1)

Compare (1.1.1) with $x^2+y^2+2gx+2fy+c=0$, we get

$2g=2$, $2f=4$ and $c=-4$

i.e. $g=1$, $f=2$ and $c=-4$.

We know centre of circle is given by $(-g,-f)$

and radius r is given by $\sqrt{g^2+f^2-c}$.

Therefore, centre of circle (1.1.1) is $(-1,-2)$

and radius r of circle (1.1.1) is

$$r=\sqrt{1^2+2^2-(-4)}$$

$$\Rightarrow \quad r=\sqrt{1+4+4}$$

$$\Rightarrow \quad r=\sqrt{9}=3$$

(ii) Given that equation of circle is

$2x^2+2y^2+5x-6y+2=0$

Diving this equation by 2, we get

$x^2+y^2+\frac{5}{2}x-3y+1=0$ (1.2.1)

Compare (1.2.1) with $x^2+y^2+2gx+2fy+c=0$, we get

$2g=\frac{5}{2}$, $2f=-3$ and $c=1$

i.e. $g=\frac{5}{4}$, $f=-\frac{3}{2}$ and $c=1$.

We know centre of circle is given by $(-g,-f)$

and radius r is given by $\sqrt{g^2+f^2-c}$.

Therefore, centre of circle (1.2.1) is $\left(-\frac{5}{4},\frac{3}{2}\right)$

and radius r of circle (1.2.1) is

$$r=\sqrt{\left(\frac{5}{4}\right)^2+\left(-\frac{3}{2}\right)^2-1}$$

$$\Rightarrow \quad r=\sqrt{\frac{25}{16}+\frac{9}{4}-1}$$

$$\Rightarrow \quad r=\sqrt{\frac{25+36-16}{16}}=\sqrt{\frac{45}{16}}$$

$$\Rightarrow \quad r=\frac{3\sqrt{5}}{4}$$

Q.2. Find the equations of circles if their centres and radii are as follow:

(i) $(\mathbf{0},\mathbf{0})$ **, 2** **(ii)** $(\mathbf{2},\mathbf{0})$ **, 5**

Sol.

(i) Given that centre of circle is $(0,0)$ and radius is 2 i.e. $h=0$, $k=0$ and $r=2$.
We know that the equation of circle, when centre and radius is given, is

$$(x-h)^2+(y-k)^2=r^2$$

$$\Rightarrow \quad (x-0)^2+(y-0)^2=2^2$$

$$\Rightarrow \quad x^2+y^2=4$$

$$\Rightarrow \quad x^2+y^2-4=0$$

which is the required equation of circle.

(ii) Given that centre of circle is $(2,0)$ and radius is 5 i.e. $h=2$, $k=0$ and $r=5$.
We know that the equation of circle, when centre and radius is given, is

$$(x-h)^2+(y-k)^2=r^2$$

$\Rightarrow$ $$(x-2)^2+(y-0)^2=5^2$$

$\Rightarrow$ $$x^2+4-4x+y^2=25$$

$\Rightarrow$ $$x^2+y^2-4x-21=0$$

which is the required equation of circle.

Q.3. Find the equations of circles if end points of their diameters are as follow:

(i) $(\mathbf{1},\mathbf{5})$ **and** $(\mathbf{3},\mathbf{6})$ **(ii)** $(\mathbf{1},\mathbf{0})$ **and** $(-\mathbf{2},-\mathbf{5})$

Sol.

(i) Given that end points of diameter of circle are $(1,5)$ and $(3,6)$.
Comparing these points with (x_1, y_1) and (x_2, y_2) respectively, we get
$x_1 = 1$, $y_1 = 5$, $x_2 = 3$ and $y_2 = 6$.
We know that the equation of circle in diametric form is

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

$\Rightarrow$ $$(x-1)(x-3)+(y-5)(y-6)=0$$

$\Rightarrow$ $$x^2-3x-x+3+y^2-6y-5y+30=0$$

$\Rightarrow$ $$x^2+y^2-4x-11y+33=0$$

which is the required equation of circle.

(ii) Given that end points of diameter of circle are $(1,0)$ and $(-2,-5)$.
Comparing these points with (x_1, y_1) and (x_2, y_2) respectively, we get
$x_1 = 1$, $y_1 = 0$, $x_2 = -2$ and $y_2 = -5$.
We know that the equation of circle in diametric form is

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

$\Rightarrow$ $$(x-1)(x-(-2))+(y-0)(y-(-5))=0$$

$\Rightarrow$ $$(x-1)(x+2)+y(y+5)=0$$

$\Rightarrow$ $$x^2+2x-x-2+y^2+5y=0$$

$\Rightarrow$ $$x^2+y^2+x+5y-2=0$$

which is the required equation of circle.